EL-MOASSER

By a group of supervisors

معاك

Ma3ak App

Interactive E-learning Application

THE MAIN BOOK

مجانا مع الكتاب

مسابقة المعاصر

El-Moasser Competition

1st PREP. 2023

FIRST TERM

Maths

EL-MOASSER'S GIFT

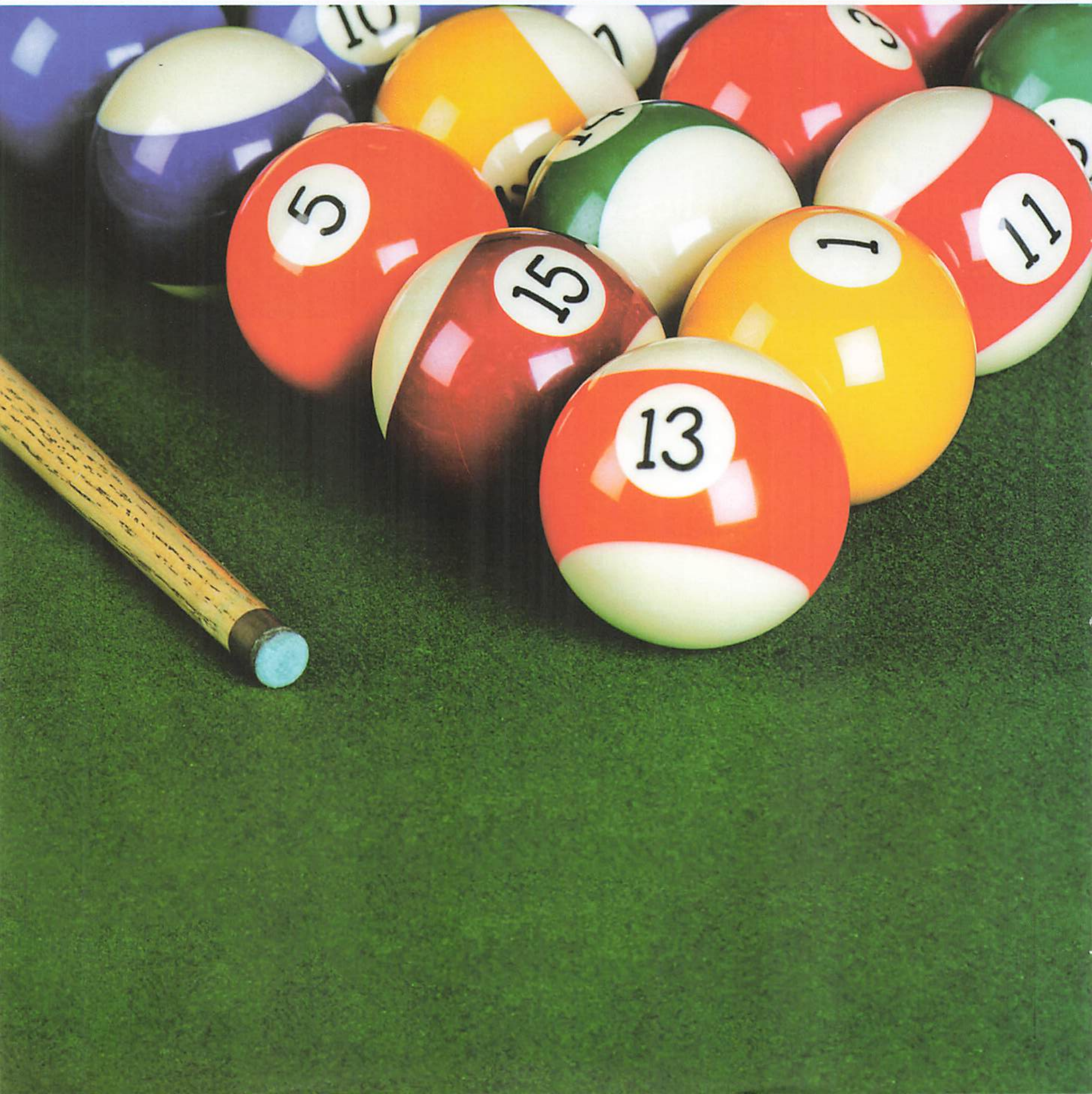
Free Ruler

Contents

First Algebra and Statistics

UNIT 1

Rational Numbers



Lessons of the unit :

1. Set of rational numbers.
2. Comparing and ordering rational numbers.
3. Adding and subtracting rational numbers.
4. Multiplying and dividing rational numbers.
5. Applications on rational numbers.

▶ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the rational number in its different forms.
- put the rational number in the simplest form.
- represent the rational numbers on the number line.
- compare two rational numbers.
- arrange a set of rational numbers.
- add rational numbers.
- recognize the properties of addition of rational numbers.
- subtract two rational numbers.
- multiply rational numbers.
- recognize the properties of multiplication of rational numbers.
- divide two rational numbers.
- solve different problems on the operations of the rational numbers.
- find a rational number lying between two rational numbers.

Al Bairony

He is one of the Arab mathematicians, he stated that :
letters and digits vary in India by local variation, he stated that the indian numbers are :
١ , ٢ , ٣ , ٤ , ٥ , ٦ , ٧ , ٨ , ٩ , ٠
and are used in Arab East and the andalusian numbers are :
0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 and are used in Al Maghreb and Andalusian.

Mohamed Ibn Ahmed Abo Al Rihany Al Bairony (Born in 363 H - 973 A.D.)

LESSON **1** | # Set of Rational Numbers

Prelude

- **You studied in the primary stage some sets of numbers as :**
  - Set of **COUNTING** numbers $= \{1 , 2 , 3 , 4 , ...\}$
  - Set of **NATURAL** numbers $\mathbb{N} = \{0 , 1 , 2 , 3 , 4 , ...\}$
  - Set of **INTEGERS** $\mathbb{Z} = \{... , -3 , -2 , -1 , 0 , 1 , 2 , 3 , ...\}$
- In this unit , you will recognize another set of numbers called

"**The set of rational numbers**" and it is denoted by the symbol " $\mathbb{Q}$ "

Rational numbers

WATCH VIDEO

The numbers : $\frac{1}{2}$, $-\frac{5}{8}$, 3 , 0 , $3\frac{1}{2}$, 0.7 , 2.5 and 15% are **rational numbers**.

Definition of the rational number

A rational number is a number that can be expressed in the form of a quotient of an integer divided by an integer other than 0

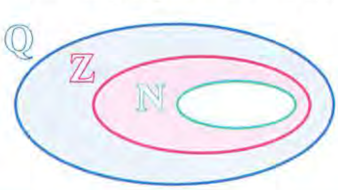
i.e. The rational numbers are all numbers can be expressed as $\frac{a}{b}$ where a and b are integers , $b \neq 0$

, where a and b are called the two terms of the rational number $\frac{a}{b}$

So **, we can express the set of rational numbers as the following :**

The set of rational numbers $\mathbb{Q} = \{x : x = \frac{a}{b} , a \in \mathbb{Z} , b \in \mathbb{Z} , b \neq 0\}$

Based on the previous definition , we can say that :

	Examples :
1 All the decimal numbers are rational numbers. because any decimal number or decimal fraction can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$	• **2.5** is a rational number can be expressed in the form $\frac{25}{10}$ or $\frac{250}{100}$ or... • **0.7** is a rational number can be expressed in the form $\frac{7}{10}$ or $\frac{70}{100}$ or...
2 All percents are rational numbers. because any percentage can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$	• **15 %** is a rational number can be expressed in the form $\frac{15}{100}$ or $\frac{150}{1000}$ or...
3 All integers are rational numbers. because any integer can be expressed in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$ **Therefore :** The set of integers is a subset of the set of rational numbers. i.e. $\mathbb{Z} \subset \mathbb{Q}$ and since $\mathbb{N} \subset \mathbb{Z}$, then $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$ and the following figure shows that.  	• **3** is a rational number can be expressed in the form $\frac{3}{1}$ or $\frac{6}{2}$ or $\frac{9}{3}$ or ... • **0** is a rational number can be expressed in the form $\frac{0}{1}$ or $\frac{0}{2}$ or $\frac{0}{3}$ or ... • **−16** is a rational number can be expressed in the form $-\frac{16}{1}$ or $-\frac{32}{2}$ or $-\frac{48}{3}$ or ...

! Remark

Each integer is a rational number , but not each rational number is an integer.

For example:

- $\frac{12}{6}$ expresses an integer because : 12 is divisible by 6 and the result is 2
- $\frac{25}{4}$ does not express an integer because : 25 is not divisible by 4

Example 1 **Show why each of the following is a rational number :**

1. $3\frac{2}{5}$
2. -0.17
3. 0.006
4. $27\ \%$

Solution Each of the previous numbers is a rational number because each of them can be expressed as $\frac{a}{b}$ where a and b are integers and $b \neq 0$ as follows :

1. $3\frac{2}{5} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5}$
2. $-0.17 = -\frac{17}{100}$
3. $0.006 = \frac{6}{1000}$
4. $27\ \% = \frac{27}{100}$

TRY by yourself 1 ***Show why each of the following is a rational number :***

$1\frac{2}{3}$, 3.07 , -51 , $30\ \%$, $102\ \%$

Final answers of try by yourself questions are at the end of each lesson to check your answer.

! Remark

If $\frac{a}{b}$ is a rational number , then $b \neq 0$

Example 2 **Choose the correct answer from the given ones :**

1. The number $\frac{5}{2X} \in \mathbb{Q}$, if $X \neq$ ....
   (a) 5 (b) 2 (c) 0 (d) -2
2. If $\frac{X-1}{X+3}$ is a rational number , then $X \neq$ ....
   (a) -3 (b) -1 (c) 1 (d) 3
3. The number $\frac{3X-12}{2X-4}$ is not rational , if $X =$ ....
   (a) -2 (b) 0 (c) 2 (d) 4

Solution

1. (c)

   The reason :

   $\frac{5}{2X} \in \mathbb{Q}$ if $2X \neq 0$ i.e. $X \neq 0$

2. (a)

   The reason :

   since $\frac{X-1}{X+3}$ is a rational number

   , then $X + 3 \neq 0$ i.e. $X \neq -3$

3 (c)

The reason :

$\frac{3x-12}{2x-4} \notin \mathbb{Q}$ if $2x-4=0$

i.e. $2x=4$ i.e. $x=\frac{4}{2}=2$

TRY by yourself 2 *Complete the following table :*

The number	$\frac{5}{x-3}$	$\frac{3}{4-x}$	$\frac{7}{8x}$	$\frac{6x}{x}$
Expresses a rational number if $x \neq$	....	....	....	....

Remark

If the rational number $\frac{a}{b}=0$, then $a=0$

Example 3 **If the rational number $\frac{x-3}{x+3}=0$, find the value of x**

Solution

Since $\frac{x-3}{x+3}=0$

, therefore $x-3=0$ i.e. $x=3$

TRY by yourself 3 *Complete the following table :*

The rational number	$\frac{x-2}{x-1}$	$\frac{6-x}{x-4}$	$\frac{2x}{x+5}$	$\frac{2x-4}{x+3}$
Equals zero if $x=$	....	....	....	....

Positive and negative rational number

The rational number $\frac{a}{b}$ is

positive	equal to zero	negative
if the product of its terms is positive i.e. $a \times b > 0$ and a, b have the same sign.	if its numerator is zero i.e. $a = 0$ notice that zero is not positive nor negative.	if the product of its terms is negative i.e. $a \times b < 0$ and a , b have different signs.
Examples for positive rational numbers : • $\frac{3}{5}$ • $\frac{-2}{-3}$	Examples for rational numbers equal to zero : • $\frac{0}{4}$ • $\frac{0}{-2}$	Examples for negative rational numbers : • $\frac{-3}{4}$ • $\frac{2}{-7}$ • $-\frac{4}{5}$

i.e. The set of rational numbers $\mathbb{Q} = \mathbb{Q}_+ \cup \{0\} \cup \mathbb{Q}_-$

Where $\mathbb{Q}_+$ is the set of positive rational numbers , $\mathbb{Q}_-$ is the set of negative rational numbers.

Note that : $\mathbb{Q}_+ \cap \mathbb{Q}_- = \varnothing$

TRY by yourself 4

Show which of the following numbers is positive , which is negative and which is zero :

$\frac{3}{4}$, $\frac{-2}{-9}$, $\frac{\text{zero}}{-5}$, $\left|-\frac{1}{2}\right|$, $\frac{-7}{11}$, $(-5)^2$

Different forms of a rational number

The rational number $\frac{a}{b}$ can be written in the form of another rational number $\frac{c}{d}$ equal to it by applying the following property :

Property

The value of the rational number $\frac{a}{b}$ does not change if its two terms are multiplied or divided by an integer $\neq$ zero.

For example:

- $\frac{3}{7} = \frac{3 \times \mathbf{2}}{7 \times \mathbf{2}} = \frac{6}{14}$, $\frac{3}{7} = \frac{3 \times \mathbf{3}}{7 \times \mathbf{3}} = \frac{9}{21}$

$\frac{3}{7} = \frac{6}{14} = \frac{9}{21}$

i.e. $\frac{3}{7}$, $\frac{6}{14}$, $\frac{9}{21}$ are different forms which represent **the same number**.

- $\frac{24}{36} = \frac{24 \div \mathbf{2}}{36 \div \mathbf{2}} = \frac{12}{18}$, $\frac{24}{36} = \frac{24 \div \mathbf{4}}{36 \div \mathbf{4}} = \frac{6}{9}$

$\frac{24}{36} = \frac{12}{18} = \frac{6}{9}$

i.e. $\frac{24}{36}$, $\frac{12}{18}$, $\frac{6}{9}$ are different forms which represent **the same number**.

TRY by yourself 5 ***Write in three other forms each of the following rational numbers :***

1. $\frac{2}{3}$
2. $\frac{16}{64}$

Writing a rational number $\frac{a}{b}$ in its simplest form

For any rational number expressed as $\frac{a}{b}$, we say that this rational number is in its simplest form if each of its terms has the smallest possible value.

For example:

- The simplest form of the rational number $\frac{16}{32}$ is $\frac{1}{2}$ and note that : $\frac{16}{32}$ and $\frac{1}{2}$ represent the same rational number.
- The rational number $\frac{3}{14}$ is in its simplest form and can not be simplified to more simple form.

$\frac{16}{32} = \frac{1}{2}$

So, they represent the same rational number.

"*To put a rational number $\frac{a}{b}$ in its simplest form ,*
divide each of its terms by the highest common factor (H.C.F.) between them."

Example 4 **Put each of the following numbers in its simplest form :**

1. $\frac{8}{12}$
2. $-\frac{12}{36}$

Solution

1. The (H.C.F.) of 8 and 12 is 4

   Dividing the two terms of $\frac{8}{12}$ by 4, we get : $\frac{8}{12} = \frac{2}{3}$

2. The (H.C.F.) of 12 and 36 is 12

   Dividing the two terms of $-\frac{12}{36}$ by 12, we get : $-\frac{12}{36} = -\frac{1}{3}$

TRY by yourself 6 *Complete the following table :*

The number	$\frac{5}{25}$	$-\frac{6}{9}$	$\frac{27}{45}$	$-\frac{12}{30}$
Its simplest form	....	....	....	....

Writing the rational number in the form of percentage

To write the rational number in the form of percentage we express it as $\frac{a}{100}$ which means a %

Example 5 Write each of the following numbers in the form of percentage :

1. $\frac{9}{20}$
2. $\frac{5}{16}$
3. $\frac{17}{1000}$
4. $5\frac{12}{125}$
5. 3.2

Solution

1. $\frac{9}{20} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100} = 45\ \%$

Another solution : $\frac{9}{20} = \frac{\frac{9}{20} \times 100}{100} = \frac{45}{100} = 45\ \%$

2. $\frac{5}{16} = \frac{\frac{5}{16} \times 100}{100} = \frac{31.25}{100} = 31.25\ \%$

3. $\frac{17}{1000} = \frac{\frac{17}{1000} \times 100}{100} = \frac{1.7}{100} = 1.7\ \%$

4. $5\frac{12}{125} = \frac{637}{125} = \frac{\frac{637}{125} \times 100}{100} = \frac{509.6}{100} = 509.6\ \%$

5. $3.2 = \frac{32}{10} = \frac{32 \times 10}{10 \times 10} = \frac{320}{100} = 320\ \%$

TRY by yourself 7 *Write each of the following numbers in the form of percentage :*

1. $\frac{4}{5}$
2. $\frac{3}{1000}$
3. 2.5

Changing a rational number from the form $\frac{a}{b}$ to a decimal form

Some rational numbers could be changed from the form $\frac{a}{b}$ into a terminating decimal.

For example:
- The rational number $\frac{3}{5}$ can be changed into 0.6
- The rational number $\frac{3}{2}$ can be changed into 1.5

"To write a rational number in the form of a terminating decimal , make its denominator equal to 10 , 100 , 1000 or ..."

$\frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$

$\frac{3 \times 5}{2 \times 5} = \frac{15}{10} = 1.5$

Example 6 **Write each of the following numbers in the form of a terminating decimal :**

1 $\frac{2}{5}$ **2** $\left|-\frac{3}{8}\right|$ **3** $-2\frac{7}{25}$

Solution

1 $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$

2 $\left|-\frac{3}{8}\right| = \frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$

3 $-2\frac{7}{25} = -2\frac{7 \times 4}{25 \times 4} = -2\frac{28}{100} = -2.28$

Check your answer using calculator

TRY by yourself 8 ***Write each of the following rational numbers in the form of a terminating decimal :***

1 $\frac{3}{4}$ **2** $\frac{11}{20}$

! Remark

Some rational numbers could not be changed into terminating decimal as the rational number $\frac{1}{3}$, then using calculator , you find that : $\frac{1}{3} = 0.333333 ...$

We express that as $(0.\dot{3})$ and read it as the infinite repeating decimal 0.3 (the recurring decimal 0.3) where the dot above the digit 3 means the digit is repeating (recurring).

Example 7 **Using a calculator , write each of the following rational numbers in the form of a recurring decimal :**

1 $\frac{2}{3}$ **2** $\frac{2}{11}$ **3** $5\frac{71}{333}$

Solution

1 Using the calculator , we get that :

$\frac{2}{3} = 0.6666666667$ **i.e.** $\frac{2}{3} = 0.\dot{6}$

2 Using the calculator , we get that :

$\frac{2}{11} = 0.1818181818$ **i.e.** $\frac{2}{11} = 0.\dot{1}\dot{8}$

3 Using the calculator , we get that :

$\frac{71}{333} = 0.2132132132$ i.e. $5\frac{71}{333} = 5.\dot{2}1\dot{3}$

Notice that :

Putting dots above the first and last digits means repeating all digits (first , last and between them)

$$5.\dot{2}1\dot{3}$$

TRY by yourself 9

Write each of the following rational numbers in the form of a recurring decimal :

1 $\frac{3}{11}$ 2 $\frac{41}{333}$

Remark

It is possible to write the recurring decimal in the form of $\frac{a}{b}$ by using scientific calculators of type CASIO fX-95ES plus or a different type.

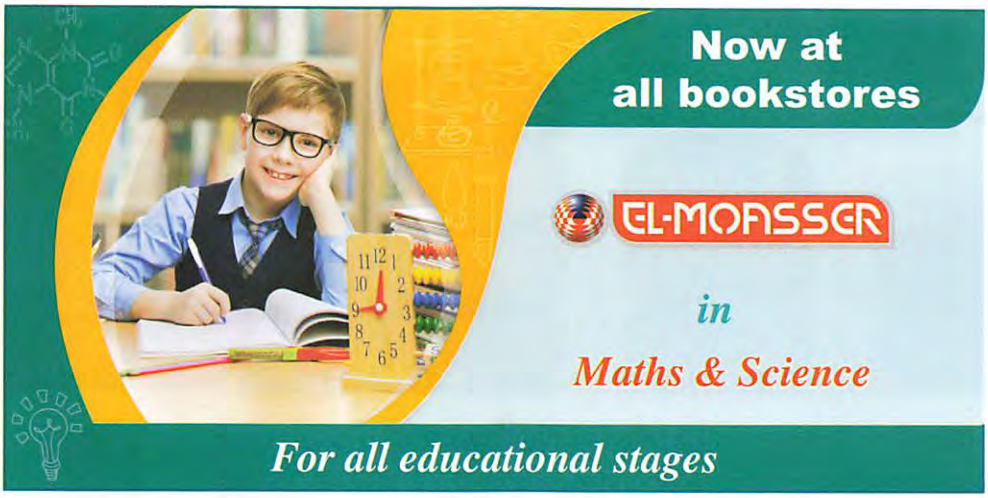
Notice that some scientific calculators can not be able to solve this problem.

For example:

- To write the number $0.\dot{2}\dot{1}$ in the form of $\frac{a}{b}$, insert the following numbers by the calculator till fill the screen : 0.21212121212121 , then press = you will get the rational number $\frac{7}{33}$
- To write the number $0.1\dot{3}\dot{6}$ in the form of $\frac{a}{b}$, insert the following numbers by the calculator till fill the screen : 0.1363636363636 , then press = you will get the rational number $\frac{3}{22}$

TRY by yourself 10 *Use the calculator to write each of the following in the form $\frac{a}{b}$:*

1 $0.\dot{1}\dot{5}$ 2 $0.1\dot{4}\dot{5}$



Answers of try by yourself

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

1 Because each of these numbers can be written in the form $\frac{a}{b}$ where a , b are integers , $b \neq 0$

$\left(\frac{5}{3} , \frac{307}{100} , \frac{-51}{1} , \frac{30}{100} , \frac{102}{100} \right)$

2 3 , 4 , 0 , 0

3 2 , 6 , 0 , 2

4 $\frac{3}{4} , \frac{-2}{-9} , \left|-\frac{1}{2}\right| , (-5)^2$ are positive , $\frac{-7}{11}$ is negative , $\frac{zero}{-5}$ is zero

5 1 $\frac{4}{6} , \frac{12}{18} , \frac{10}{15}$ "There are other solutions"

2 $\frac{8}{32} , \frac{4}{16} , \frac{1}{4}$ "There are other solutions"

6 $\frac{1}{5} , -\frac{2}{3} , \frac{3}{5} , -\frac{2}{5}$

7 1 80% 2 0.3% 3 250%

8 1 0.75 2 0.55

9 1 $0.\dot{2}\dot{7}$ 2 $0.\dot{1}2\dot{3}$

10 1 $\frac{5}{33}$ 2 $\frac{8}{55}$

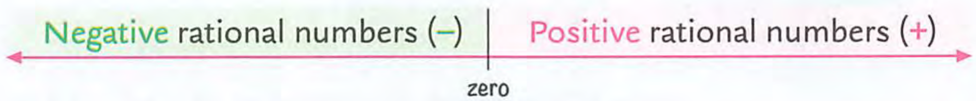
LESSON 2 Comparing and Ordering Rational Numbers

We will study how to represent a rational number on the number line before studying of comparing and ordering the rational numbers.

WATCH VIDEO WATCH VIDEO WATCH VIDEO

Representing the rational numbers on the number line

- Each rational number can be represented by a unique point on the number line.
- The positive rational numbers are represented on the number line by points lying on the right side of the point which represents the number zero and the negative rational numbers are represented by points lying on the left side of the point which represents the number zero.
  and the number zero neither positive nor negative



The following examples show how to represent the rational numbers on the number line :

Example 1 **Represent the rational number $\frac{3}{4}$ on the number line.**

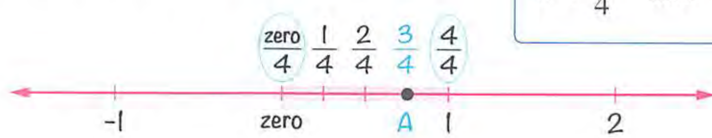
Solution

- Since the rational number $\frac{3}{4}$ lies between the two integers zero and 1 , then the point which represents the number $\frac{3}{4}$ lies between the two points which represent the two numbers zero and 1

- Divide the distance between the point representing the number zero and the point representing the number 1 to 4 **equal parts** as follows :

Notice that :

$0 = \frac{0}{4}$ and $1 = \frac{4}{4}$



- The point A represents the rational number $\frac{3}{4}$

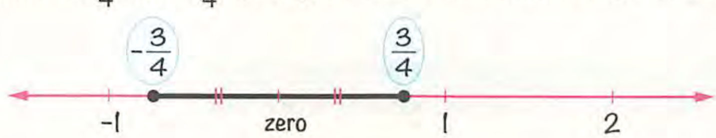
Remark

The two rational numbers A and – A are represented on the number line by two points equidistant from the point representing the number zero and on two different sides of it.

The rational number – A is the opposite of the rational number A and vice versa.

For example:

The two rational numbers $\frac{3}{4}$ and $-\frac{3}{4}$ are represented on the number line as the following figure :



The rational number $-\frac{3}{4}$ is the opposite of the rational number $\frac{3}{4}$ and vice versa.

Example 2

Represent on the number line each of the following rational numbers :

1 $\frac{7}{5}$ 2 $-\frac{24}{9}$

Solution

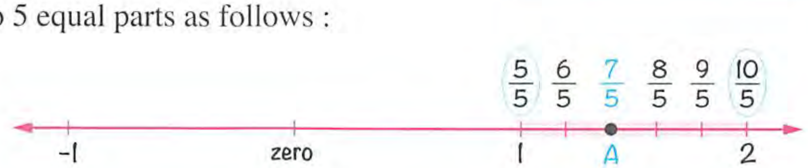
1 Since $\frac{7}{5} = 1\frac{2}{5}$

, then $\frac{7}{5}$ lies between the two integers 1 and 2 therefore , divide the distance between the point representing the number 1 and the point representing the number

2 into 5 equal parts as follows :

Notice that :

$1 = \frac{5}{5}$ and $2 = \frac{10}{5}$



The point A represents the rational number $\frac{7}{5}$

2 Before representing the rational number on the number line it is better to put the number in its simplest form.

$-\frac{24}{9} = -\frac{24 \div 3}{9 \div 3} = -\frac{8}{3}$ since $-\frac{8}{3} = -2\frac{2}{3}$, then $-\frac{24}{9} = -2\frac{2}{3}$

i.e. It lies between the two integers -2 , -3

Therefore , divide the distance between the point representing the number -2 and the point representing the number -3 into 3 equal parts as follows :

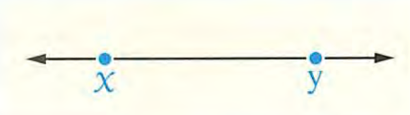
Notice that : $-2 = -\frac{6}{3}$ and $-3 = -\frac{9}{3}$

The point B represents the rational number $-\frac{24}{9}$

TRY by yourself 1 ***Represent on the number line the rational number $\frac{18}{8}$***

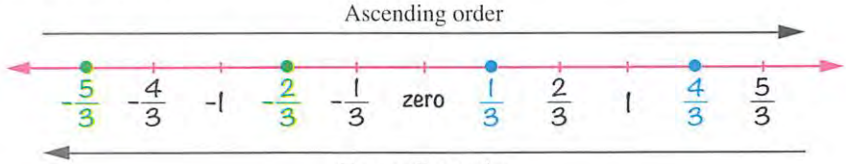
Comparing and ordering the rational numbers

If the point which represents the number x lies on the left of the point which represents the number y on the number line as shown in the opposite figure , then : $x < y$ or $y > x$



For example:

In the following figure we find that :



$-\frac{5}{3} < -\frac{2}{3}$ or $-\frac{2}{3} > -\frac{5}{3}$

Because :

the point which represents the number $-\frac{5}{3}$ lies on the left of the point which represents the number $-\frac{2}{3}$

$\frac{1}{3} < \frac{4}{3}$ or $\frac{4}{3} > \frac{1}{3}$

Because :

the point which represents the number $\frac{1}{3}$ lies on the left of the point which represents the number $\frac{4}{3}$

Example 3 Represent the following rational numbers on the number line, then arrange them ascendingly : $\frac{7}{5}$, zero , $\frac{9}{5}$, 2 , – 1

Solution

According to the positions of the numbers on the number line shown above you find that the ascending order is : – 1 , zero , $\frac{7}{5}$, $\frac{9}{5}$, 2

TRY by yourself 2

Represent the following rational numbers on the number line, then arrange them descendingly : 2 , $-\frac{5}{2}$, $\frac{7}{2}$, zero , – 1

Comparing two rational numbers

	Examples :
1 If the two numbers have different signs , then the positive number is greater than the negative number.	$0.05 > -\frac{15}{2}$
2 If one of the two numbers is greater than a certain number X and the other number is less than this number X , then the first number is greater than the second.	$\frac{65}{63} > \frac{57}{59}$ (Because : $\frac{65}{63} > 1$, $\frac{57}{59} < 1$)
3 If the two numbers are in the form $\frac{a}{b}$ and have the same positive denominator , then the number having the greater numerator will be the greater.	$\frac{7}{13} > \frac{5}{13}$ (Because : $7 > 5$)
4 If the two numbers are in the form $\frac{a}{b}$ and have the same positive numerator , then the number having the greater denominator will be the smaller.	$\frac{2}{5} > \frac{2}{9}$ (Because : $9 > 5$)
5 If the two numbers are in the form $\frac{a}{b}$ and different in numerator and denominator , convert the two numbers in order to have a common denominator taking care of the denominator should be positive , and then compare their numerators.	$\frac{2}{3} > \frac{8}{15}$ (Because : $\frac{2}{3} = \frac{10}{15}$, $\frac{10}{15} > \frac{8}{15}$)

Example 4 **Compare the two numbers in each of the following :**

1. $\frac{5}{12}$, $\frac{7}{12}$
2. $\frac{1}{4}$, $-\frac{5}{6}$
3. $\frac{11}{12}$, $\frac{11}{15}$
4. $\frac{6}{12}$, $\frac{2}{3}$
5. 3.2 , $\frac{11}{2}$
6. 23 % , $\frac{3}{8}$

Solution

1. $\frac{5}{12} < \frac{7}{12}$ (Because the two numbers have the same denominator , 5 < 7)
2. $\frac{1}{4} > -\frac{5}{6}$ (Because $\frac{1}{4}$ is positive , $-\frac{5}{6}$ is negative)
3. $\frac{11}{12} > \frac{11}{15}$ (Because the two numbers have the same numerator , 12 < 15)
4. $\frac{6}{12}$, $\frac{2}{3}$ are different in the numerator and the denominator ,
we convert the two numbers in order to have a common denominator.
Since the L.C.M. of the denominators = 12
Therefore $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ and since 8 > 6
Therefore $\frac{6}{12} < \frac{8}{12}$ i.e. $\frac{6}{12} < \frac{2}{3}$
5. $3.2 = 3\frac{2}{10}$, $\frac{11}{2} = 5\frac{1}{2}$
Therefore $3.2 < 5\frac{1}{2}$ (Because : 3 < 5) i.e. $3.2 < \frac{11}{2}$
Another solution : $3.2 < \frac{11}{2}$ (Because : $\frac{11}{2} = 5.5$, 3.2 < 5.5)
6. $23\ \% = \frac{23}{100}$ i.e. The two numbers are $\frac{23}{100}$ and $\frac{3}{8}$
Since the L.C.M. of the denominators = 200
Therefore $\frac{23}{100} = \frac{46}{200}$, $\frac{3}{8} = \frac{75}{200}$ and since 46 < 75
Therefore $\frac{46}{200} < \frac{75}{200}$ i.e. $23\ \% < \frac{3}{8}$
Another solution : $23\ \% < \frac{1}{4}$ (Because : $\frac{1}{4} = 25\ \%$)
, $\frac{3}{8} > \frac{1}{4}$ (Because : $\frac{1}{4} = \frac{2}{8}$) i.e. $23\ \% < \frac{3}{8}$

Example 5 **Arrange the following rational numbers ascendingly :**

$-\frac{2}{3}$, $\frac{3}{4}$, $-\frac{7}{12}$, $\frac{5}{6}$, -1

Solution Since L.C.M. of the denominators = 12

, then $-\frac{2}{3} = -\frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{5}{6} = \frac{10}{12}$, $-1 = -\frac{12}{12}$

, then the numbers after converting their denominators are :

$-\frac{8}{12}$, $\frac{9}{12}$, $-\frac{7}{12}$, $\frac{10}{12}$, $-\frac{12}{12}$

Since $-12 < -8 < -7 < 9 < 10$, then $-\frac{12}{12} < -\frac{8}{12} < -\frac{7}{12} < \frac{9}{12} < \frac{10}{12}$

i.e. $-1 < -\frac{2}{3} < -\frac{7}{12} < \frac{3}{4} < \frac{5}{6}$

TRY by yourself 3 ***Complete each of the following using the suitable sign*** (< , > or =) :

1. $\frac{7}{5}$ .... $\frac{4}{5}$
2. $-\frac{3}{4}$ .... $-\frac{2}{4}$
3. $\frac{1}{5}$ .... $\frac{1}{6}$
4. $\frac{3}{6}$ .... $\frac{2}{3}$
5. $\frac{4}{10}$ .... $\frac{14}{35}$
6. $\left|-\frac{10}{15}\right|$ .... $\frac{2}{3}$

The density of the rational numbers

Between every two different rational numbers there are an infinite number of rational numbers.

To illustrate this :

If we have two rational numbers as $\frac{1}{3}$ and $\frac{2}{3}$, we can deduce that there are other rational numbers between them as follows :

1. If we multiply both terms of the two numbers $\frac{1}{3}$ and $\frac{2}{3}$ by 2 ,
   we get the two rational numbers : $\frac{2}{6}$ and $\frac{4}{6}$ which are equal to them.
   It is clear that $\frac{3}{6}$ lies between $\frac{2}{6}$ and $\frac{4}{6}$
   i.e. **The rational number $\frac{3}{6}$ $(= \frac{1}{2})$ is lying between the two numbers $\frac{1}{3}$ and $\frac{2}{3}$**

2. If we multiply both terms of the two numbers : $\frac{1}{3}$ and $\frac{2}{3}$ by 3 ,
   then we get the two rational numbers $\frac{3}{9}$, $\frac{6}{9}$ which are equal to them.
   It is clear that $\frac{4}{9}$ and $\frac{5}{9}$ are lying between $\frac{3}{9}$ and $\frac{6}{9}$
   i.e. **The two rational numbers $\frac{4}{9}$ and $\frac{5}{9}$ are lying between the two numbers $\frac{1}{3}$ and $\frac{2}{3}$**

So, we can deduce that :

between the two rational numbers $\frac{1}{3}$ and $\frac{2}{3}$ there are an infinite number of rational numbers.

! Remarks

- There is no integer between any two consecutive integers , it means that the set of integers which is an infinite set , but doesn't have the property of density.
- For any integer , we can determine the integer which lies just after or just before it.
- For any rational number we cannot determine the rational number which lies just after or just before it.

Example 6

Find four rational numbers lying between : $\frac{1}{2}$ and $\frac{5}{7}$

Solution

Since L.C.M. of the denominators is 14

, then $\frac{1}{2} = \frac{1 \times 7}{2 \times 7} = \frac{7}{14}$

, $\frac{5}{7} = \frac{5 \times 2}{7 \times 2} = \frac{10}{14}$

Since $\frac{7}{14} < \frac{8}{14} < \frac{9}{14} < \frac{10}{14}$

, then $\frac{8}{14}$, $\frac{9}{14}$ are two rational numbers lying between : $\frac{1}{2}$, $\frac{5}{7}$

but the required is 4 numbers not two only.

Therefore, we multiply the two terms of each of $\frac{7}{14}$ and $\frac{10}{14}$ by 2

, then $\frac{7}{14} = \frac{7 \times 2}{14 \times 2} = \frac{14}{28}$

, $\frac{10}{14} = \frac{10 \times 2}{14 \times 2} = \frac{20}{28}$

Since $\frac{14}{28} < \frac{15}{28} < \frac{16}{28} < \frac{17}{28} < \frac{18}{28} < \frac{19}{28} < \frac{20}{28}$

i.e. $\frac{1}{2} < \frac{15}{28} < \frac{4}{7} < \frac{17}{28} < \frac{9}{14} < \frac{19}{28} < \frac{5}{7}$

, then $\frac{15}{28}$, $\frac{4}{7}$, $\frac{17}{28}$, $\frac{9}{14}$, $\frac{19}{28}$ are five rational numbers between $\frac{1}{2}$ and $\frac{5}{7}$

From the five numbers choose any 4 numbers only as required.

Notice that :
We should convert their denominators to have common denominator at first.

Notice that :
For facilitating the solution, we multiply each of the numerator and the denominator of the two numbers by 10

TRY by yourself 4

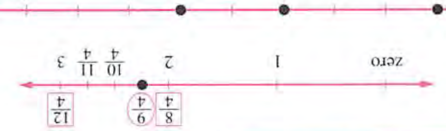
1. ***Find three rational numbers lying between :*** $\frac{1}{3}$ and $\frac{1}{2}$
2. ***Choose :***

   The number of the rational numbers lying between $\frac{2}{5}$, $\frac{4}{5}$ is ....

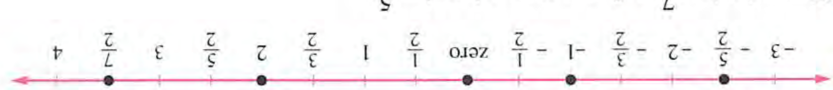
   (a) 0 (b) 1 (c) 2 (d) an infinite number.

Answers *of try by yourself*

1 $\frac{18}{8} = \frac{9}{4}$



2



The order is : $\frac{7}{2}$, 2 , zero , -1 and $-\frac{5}{2}$

3 1 $>$ 2 $<$ 3 $>$ 4 $<$ 5 $=$ 6 $=$

4 1 $\frac{3}{8}$, $\frac{5}{12}$, $\frac{11}{24}$ "There are other solutions". 2 (d)

LESSON 3 | Adding and Subtracting Rational Numbers

First Addition operation

Prelude

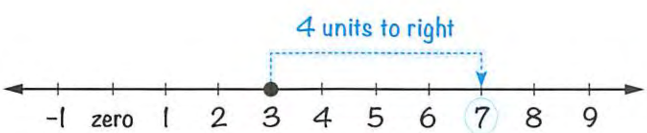
We will use the number line to explain the concept of addition in $\mathbb{Q}$ as follows :

To find the sum of a + b on the number line :

1. Determine the point that represents the number a on the number line.
2. Move to right or left according to the sign of b and with its units till you reach the point that represents (a + b).

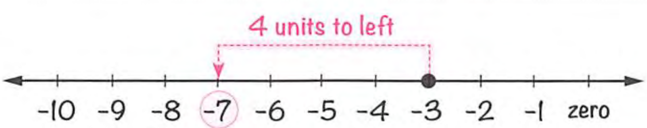
For example:

3 + 4
Start — Move right



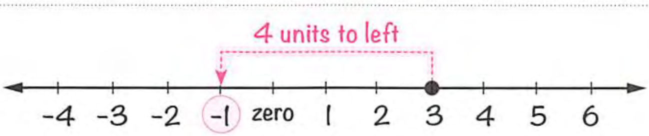
$3 + 4 = 7$

(– 3) + (– 4)
Start — Move left



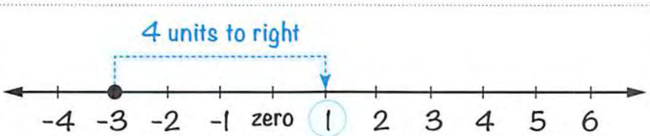
$(-3) + (-4) = -7$

3 + (– 4)
Start — Move left



$3 + (-4) = -1$

(– 3) + 4
Start — Move right



$(-3) + 4 = 1$

From the previous, notice that

For example :

1. The sum of two positive numbers is positive. — $2 + 3 = 5$
2. The sum of two negative numbers is negative. — $(-4) + (-5) = -9$
3. The sum of two numbers different in sign can be positive , negative or zero. — $5 + (-3) = 2$, $4 + (-7) = -3$, $2 + (-2) = 0$

Adding two rational numbers expressed as $\frac{a}{b}$

WATCH VIDEO

WATCH VIDEO

1 Adding two rational numbers having the same denominator :

If $\frac{a}{b}$ and $\frac{c}{b}$ are two rational numbers , then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

For example:

- $\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$
- $\frac{3}{5} + \left(-\frac{1}{5}\right) = \frac{3+(-1)}{5} = \frac{2}{5}$

2 Adding two rational numbers with different denominators :

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

For example:

$\frac{2}{5} + \frac{1}{7} = \frac{2 \times 7 + 1 \times 5}{5 \times 7} = \frac{14+5}{35} = \frac{19}{35}$

Example 1 Add :

1. $\frac{3}{8} + \frac{1}{4}$
2. $\frac{4}{12} + \left(-\frac{10}{15}\right)$
3. $\frac{2}{5} + 3$
4. $3\frac{1}{4} + \left(-2\frac{1}{5}\right)$

Solution

1. $\frac{3}{8} + \frac{1}{4} = \frac{3 \times 4 + 1 \times 8}{8 \times 4}$

$= \frac{12+8}{32} = \frac{20}{32} = \frac{5}{8}$

Notice that :
After carrying out the operation of addition , the result should be put in its simplest form.

Another solution «by finding the common denominator» :

Since L.C.M. of the denominators 8 and 4 is 8

Therefore $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$ i.e. $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{2}{8} = \frac{3+2}{8} = \frac{5}{8}$

2 Since $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$,

$-\frac{10}{15} = -\frac{10 \div 5}{15 \div 5} = -\frac{2}{3}$

Therefore $\frac{4}{12} + (-\frac{10}{15}) = \frac{1}{3} + (-\frac{2}{3})$

$= \frac{1 + (-2)}{3} = -\frac{1}{3}$

Notice that :
Before adding the two rational numbers it is better to write them in the simplest form as shown in the opposite solution.

3 Since $3 = \frac{15}{5}$

Therefore $\frac{2}{5} + 3 = \frac{2}{5} + \frac{15}{5} = \frac{17}{5}$

Another solution :

$\frac{2}{5} + 3 = 3\frac{2}{5}$, then convert the mixed number into an improper fraction ,

we find : $3\frac{2}{5} = \frac{17}{5}$

4 Since $3\frac{1}{4} = \frac{13}{4}$, $-2\frac{1}{5} = -\frac{11}{5}$ therefore $3\frac{1}{4} + (-2\frac{1}{5}) = \frac{13}{4} + (-\frac{11}{5})$

Since L.C.M. of the denominators 4 and 5 is 20

Therefore $\frac{13}{4} + (-\frac{11}{5}) = \frac{65}{20} + (-\frac{44}{20}) = \frac{21}{20} = 1\frac{1}{20}$

Another solution :

Since L.C.M. of the denominators 4 and 5 is 20 , then :

$3\frac{1}{4} + (-2\frac{1}{5}) = 3\frac{5}{20} + (-2\frac{4}{20}) = 1\frac{1}{20}$

TRY by yourself 1

Add each of the following :

1 $\frac{1}{5} + \frac{2}{5}$

2 $\frac{2}{5} + \frac{1}{3}$

3 $\frac{1}{2} + (-\frac{5}{6})$

4 $-\frac{3}{4} + \frac{1}{5}$

5 $\frac{8}{12} + (-\frac{15}{18})$

UNIT 1

Properties of the addition operation in $\mathbb{Q}$

1 Closure property

The sum of any two rational numbers is a rational number.

i.e. $\mathbb{Q}$ is closed under addition operation.

For example:

The sum of the two rational numbers $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{5}{6}$ which is a rational number too.

2 Commutative property :

If a and b are two rational numbers , then $a + b = b + a$

For example:

$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20}$, $\frac{2}{5} + \frac{3}{4} = \frac{8}{20} + \frac{15}{20} = \frac{23}{20}$

i.e. $\frac{3}{4} + \frac{2}{5} = \frac{2}{5} + \frac{3}{4}$

3 Associative property :

If a , b and c are three rational numbers , then $(a + b) + c = a + (b + c)$

For example:

$(\frac{3}{7} + \frac{2}{7}) + \frac{1}{7} = \frac{5}{7} + \frac{1}{7} = \frac{6}{7}$, $\frac{3}{7} + (\frac{2}{7} + \frac{1}{7}) = \frac{3}{7} + \frac{3}{7} = \frac{6}{7}$

i.e. $(\frac{3}{7} + \frac{2}{7}) + \frac{1}{7} = \frac{3}{7} + (\frac{2}{7} + \frac{1}{7})$

4 The existence of identity element (Neutral element) property in addition :

If a is a rational number , then $a + 0 = 0 + a = a$

i.e. When we add zero to any rational number the value of this number does not change.

Then we say : zero is the identity element in addition operation in $\mathbb{Q}$

For example: $\frac{1}{2} + 0 = 0 + \frac{1}{2} = \frac{1}{2}$

5 The existence of additive inverse property :

For every rational number a there exist an additive inverse to it that is – a where a + (– a) = zero (the identity element in addition)

For example:

The additive inverse of the number $\frac{3}{4}$ is $-\frac{3}{4}$ and vice versa the additive inverse of $-\frac{3}{4}$ is $\frac{3}{4}$ because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4}$ = zero (the identity element in addition).

Notice that : Zero is its own additive inverse.

Example 2

Choose the correct answer from the given ones :

1 The additive inverse of $(-\frac{3}{5})^{zero}$ is ....

(a) $\frac{3}{5}$ (b) 1 (c) $-\frac{3}{5}$ (d) – 1

2 The additive inverse of $-|-\frac{2}{7}|$ is ....

(a) $-\frac{2}{7}$ (b) zero (c) $\frac{2}{7}$ (d) $\frac{7}{2}$

3 $\frac{1}{3}$ + zero = $\frac{1}{3}$ (.......... property)

(a) Commutative (b) Associative

(c) Additive inverse (d) Additive identity element

4 $-\frac{2}{9} + \frac{2}{9}$ = zero (.......... property)

(a) Commutative (b) Associative

(c) Additive inverse (d) Additive identity element

5 $\frac{5}{7} + (-\frac{1}{2}) = -\frac{1}{2} + \frac{5}{7}$ (.......... property)

(a) Commutative (b) Associative

(c) Additive inverse (d) Additive identity element

6 If $\frac{2}{3} + x = \frac{2}{3}$, then x = ...

(a) $-\frac{2}{3}$ (b) zero (c) 1 (d) $\frac{2}{3}$

Solution

1 (d) The reason : Since $(-\frac{3}{5})^{zero} = 1$ and hence the additive inverse of 1 is – 1 , then the additive inverse of $(-\frac{3}{5})^{zero}$ is – 1

2 (c) The reason : Since $-|-\frac{2}{7}| = -\frac{2}{7}$, then additive inverse of $-|-\frac{2}{7}|$ is $\frac{2}{7}$

3 (d) 4 (c) 5 (a) 6 (b)

Example 3 **Use the addition properties in $\mathbb{Q}$ to find the result of :**

$$\frac{6}{35} + (-\frac{5}{11}) + \frac{19}{35} + \frac{10}{22}$$

Solution

Since $\frac{10}{22} = \frac{10 \div 2}{22 \div 2} = \frac{5}{11}$

Therefore $\frac{6}{35} + (-\frac{5}{11}) + \frac{19}{35} + \frac{10}{22} = \frac{6}{35} + (-\frac{5}{11}) + \frac{19}{35} + \frac{5}{11}$

$= (\frac{6}{35} + \frac{19}{35}) + (-\frac{5}{11} + \frac{5}{11})$ (commutative and associative properties)

$= \frac{25}{35}$ + zero (the additive inverse)

$= \frac{25}{35}$ (the identity element)

$= \frac{5}{7}$ (the result in its simplest form)

TRY by yourself 2 *Use the addition properties in $\mathbb{Q}$ to find the result of :* $\frac{4}{5} + (-\frac{3}{7}) + \frac{1}{5} + \frac{3}{7}$

Second Subtraction operation

Since each rational number has an additive inverse , then the subtraction operation is always possible in $\mathbb{Q}$ and it is defined as follows :

Definition

If a and b are two rational numbers , then $a - b = a + (-b)$

i.e. The subtraction operation in $\mathbb{Q}$ is defined as adding the minuend (a) to the additive inverse of the subtrahend (b)

Example 4 **Find the result of each of the following in its simplest form :**

1. $\frac{5}{8} - \frac{3}{8}$
2. $\frac{3}{4} - \frac{5}{6}$
3. $\frac{5}{7} - 1$
4. $-\frac{2}{5} - \frac{3}{5}$
5. $7\frac{2}{5} - 3\frac{1}{4}$

Solution

1 $\frac{5}{8} - \frac{3}{8} = \frac{5}{8} + (-\frac{3}{8}) = \frac{2}{8} = \frac{1}{4}$

2 Since L.C.M. of denominators is 12 , therefore

$$\frac{3}{4} - \frac{5}{6} = \frac{3 \times 3}{4 \times 3} + (-\frac{5 \times 2}{6 \times 2})$$
$$= \frac{9}{12} + (-\frac{10}{12}) = -\frac{1}{12}$$

3 $\frac{5}{7} - 1 = \frac{5}{7} + (-1) = \frac{5}{7} + (-\frac{7}{7})$
$= -\frac{2}{7}$

4 $-\frac{2}{5} - \frac{3}{5} = -\frac{2}{5} + (-\frac{3}{5}) = -\frac{5}{5} = -1$

5 Since : $7\frac{2}{5} = \frac{37}{5}$ and $3\frac{1}{4} = \frac{13}{4}$ and since :

L.C.M. of denominators is 20 , then :

$$7\frac{2}{5} - 3\frac{1}{4} = \frac{37 \times 4}{5 \times 4} + (-\frac{13 \times 5}{4 \times 5}) = \frac{148}{20} + (-\frac{65}{20}) = \frac{83}{20}$$

Another solution : Since L.C.M. of denominators is 20

, therefore $7\frac{2}{5} - 3\frac{1}{4} = 7\frac{2 \times 4}{5 \times 4} + (-3\frac{1 \times 5}{4 \times 5}) = 7\frac{8}{20} + (-3\frac{5}{20}) = 4\frac{3}{20}$

Notice that :
We can do without the step of converting subtraction operation into addition operation as follows :

- $\frac{5}{8} - \frac{3}{8} = \frac{5-3}{8}$
$= \frac{2}{8} = \frac{1}{4}$
- $\frac{3}{4} - \frac{5}{6} = \frac{9}{12} - \frac{10}{12}$
$= \frac{9-10}{12} = -\frac{1}{12}$

TRY by yourself 3

Find each of the following in its simplest form :

1 $\frac{3}{5} - \frac{2}{5}$

2 $\frac{7}{9} - \frac{4}{9}$

3 $\frac{3}{4} - \frac{2}{3}$

4 $4\frac{1}{5} - 3\frac{1}{8}$

! Remarks

- $\mathbb{Q}$ is closed under subtraction operation.

  i.e. The result of subtracting any two rational numbers is a rational number.
- The subtraction operation in $\mathbb{Q}$ is not commutative and not associative.
- There is no identity element with respect to subtraction in $\mathbb{Q}$ and hence there are no inverses for the numbers with respect to subtraction in $\mathbb{Q}$

Example 5 If $a = \frac{3}{4}$, $b = -\frac{5}{2}$ and $c = \frac{1}{2}$, find the numerical value of each of the following :

1. $a - b$
2. $(a + b) - c$

Solution

1. $a - b = \frac{3}{4} - (-\frac{5}{2}) = \frac{3}{4} + \frac{5}{2}$ «from the subtraction operation definition»

$= \frac{3}{4} + \frac{10}{4} = \frac{13}{4}$

2. $(a + b) - c = [\frac{3}{4} + (-\frac{5}{2})] - \frac{1}{2} = [\frac{3}{4} + (-\frac{10}{4})] - \frac{1}{2} = -\frac{7}{4} - \frac{1}{2}$

$= -\frac{7}{4} - \frac{2}{4} = -\frac{9}{4}$

Wonders of numbers

From wonders of the **number 7** is that if one of its multiples up to 63 is multiplied by **15873** , every time the product is a number consisting of the same digits.

- $7 \times 15873 = 111\,111$
- $14 \times 15873 = 222\,222$
- $21 \times 15873 = 333\,333$

Try other multiples

Answers of try by yourself

1 1. $\frac{3}{5}$ 2. $\frac{11}{15}$ 3. $-\frac{1}{3}$ 4. $-\frac{11}{20}$ 5. $-\frac{1}{6}$

2 1

3 1. $\frac{1}{5}$ 2. $\frac{1}{3}$ 3. $\frac{1}{12}$ 4. $\frac{43}{40}$

LESSON 4 | Multiplying and Dividing Rational Numbers

Prelude Before studying the concept of multiplication and division operations in $\mathbb{Q}$ we have to remember the sign's rule :

Sign's rule in multiplication

$(+) \times (+) = (+)$, $(-) \times (-) = (+)$

$(+) \times (-) = (-)$, $(-) \times (+) = (-)$

Sign's rule in division

$(+) \div (+) = (+)$, $(-) \div (-) = (+)$

$(+) \div (-) = (-)$, $(-) \div (+) = (-)$

For example:

- $3 \times 4 = 12$
- $2 \times (-5) = -10$
- $(-2) \times (-3) = 6$
- $(-4) \times 2 = -8$
- $8 \div 2 = 4$
- $14 \div (-7) = -2$
- $(-50) \div (-5) = 10$
- $(-20) \div 4 = -5$

First Multiplication operation

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

WATCH VIDEO

i.e. To multiply two rational numbers , multiply their numerators to get the numerator of the product and multiply their denominators to get the denominator of the product.

For example: • $\frac{3}{4} \times \frac{1}{5} = \frac{3 \times 1}{4 \times 5} = \frac{3}{20}$ • $-\frac{2}{3} \times \frac{5}{7} = -\frac{2 \times 5}{3 \times 7} = -\frac{10}{21}$

Example 1 Find the result of each of the following in its simplest form :

1. $\frac{3}{6} \times \frac{2}{5}$
2. $-\frac{3}{4} \times \frac{2}{9}$
3. $\frac{2}{4} \times (-2)$
4. $-4\frac{2}{7} \times (-3\frac{1}{6})$

Solution

1. $\frac{3}{6} \times \frac{2}{5} = \frac{3 \times 2}{6 \times 5} = \frac{6}{30} = \frac{1}{5}$

Notice that : After multiplying the numbers we should put the product in its simplest form.

2. $-\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}} = \frac{-1 \times 1}{2 \times 3}$

$= -\frac{1}{6}$

Notice that : When we multiply we can reduce the numerator of one of the numbers with the denominator of the other.

3. $\frac{\cancel{2}^{1}}{\cancel{4}_{2}} \times (-2) = \frac{1}{2} \times (-2) = \frac{1}{_{1}\cancel{2}} \times (-\frac{\cancel{2}^{1}}{1})$

$= -1$

Notice that : It is better to put the rational numbers in the simplest form to make the operation easier.

4. $-4\frac{2}{7} \times (-3\frac{1}{6}) = -\frac{^{5}\cancel{30}}{7} \times (-\frac{19}{\cancel{6}_{1}})$

$= \frac{-5 \times (-19)}{7 \times 1}$

$= \frac{95}{7}$

Notice that : We should convert the mixed number to an improper fraction before carrying out the multiplication operation.

TRY by yourself 1

Find the result of each of the following in its simplest form :

1. $\frac{3}{2} \times \frac{5}{9}$
2. $\frac{8}{5} \times (-\frac{4}{9})$
3. $-5 \times \frac{3}{10}$
4. $-4\frac{1}{2} \times (-\frac{5}{9})$

Properties of the multiplication operation in $\mathbb{Q}$

1 Closure property :

The product of any two rational numbers is a rational number.

i.e. $\mathbb{Q}$ is closed under multiplication operation.

For example:

The product of the two rational numbers $\frac{3}{5}$ and $\frac{1}{4}$ is $\frac{3}{20}$ which is a rational number too.

2 Commutative property :

If a and b are two rational numbers , then : $a \times b = b \times a$

For example:

$\frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$, $\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$

i.e. $\frac{2}{7} \times \frac{3}{5} = \frac{3}{5} \times \frac{2}{7}$

3 Associative property :

If a , b and c are three rational numbers , then : $(a \times b) \times c = a \times (b \times c)$

For example:

$\left(\frac{1}{2} \times \frac{1}{3}\right) \times \frac{7}{5} = \frac{1}{6} \times \frac{7}{5} = \frac{7}{30}$, $\frac{1}{2} \times \left(\frac{1}{3} \times \frac{7}{5}\right) = \frac{1}{2} \times \frac{7}{15} = \frac{7}{30}$

i.e. $\left(\frac{1}{2} \times \frac{1}{3}\right) \times \frac{7}{5} = \frac{1}{2} \times \left(\frac{1}{3} \times \frac{7}{5}\right)$

4 The existence of multiplicative identity (neutral) element property :

If a is a rational number , then : $a \times 1 = 1 \times a = a$

i.e. As multiplying any rational number by 1 , the value of this number does not change.

Then we say : the number 1 is the multiplicative identity (neutral) in $\mathbb{Q}$

For example:

- $\frac{2}{3} \times 1 = 1 \times \frac{2}{3} = \frac{2}{3}$
- $-\frac{3}{7} \times 1 = 1 \times -\frac{3}{7} = -\frac{3}{7}$

5 The existence of multiplicative inverse of the rational number property :

For every rational number $\frac{a}{b}$ except zero there is a multiplicative inverse that is the rational number $\frac{b}{a}$ where $\frac{a}{b} \times \frac{b}{a} = 1$ (the multiplicative identity)

For example:
- The multiplicative inverse of the number $\frac{3}{2}$ is $\frac{2}{3}$ and vice versa the multiplicative inverse of the number $\frac{2}{3}$ is $\frac{3}{2}$
- The multiplicative inverse of the number $-\frac{3}{4}$ is $-\frac{4}{3}$ and vice versa the multiplicative inverse of the number $-\frac{4}{3}$ is $-\frac{3}{4}$
- The multiplicative inverse of the number $\frac{1}{5}$ is 5 and vice versa the multiplicative inverse of the number 5 is $\frac{1}{5}$

Remarks

- The multiplicative inverse of the rational number is called the reciprocal of the rational number.
- Zero has no multiplicative inverse because : $\frac{1}{\text{zero}}$ is meaningless i.e. (undefined)
- The multiplicative inverse of the number 1 is itself and the multiplicative inverse of the number – 1 is itself also.
- Multiplying any rational number by zero equals zero.

For example: $0 \times \frac{1}{2} = 0$, $-\frac{5}{8} \times 0 = 0$

6 Property of distributing multiplication over addition and subtraction :

If a, b and c are three rational numbers, then :

1. $a \times (b + c) = a \times b + a \times c$, $(b + c) \times a = b \times a + c \times a$

   i.e. Multiplication is distributed over addition in $\mathbb{Q}$ from right and from left.

2. $a \times (b - c) = a \times b - a \times c$, $(b - c) \times a = b \times a - c \times a$

   i.e. Multiplication is distributed over subtraction in $\mathbb{Q}$ from right and from left.

Example 2 **Choose the correct answer from the given ones :**

1 The multiplicative inverse of $2\frac{1}{3}$ is ....

(a) $-\frac{7}{3}$ (b) $\frac{3}{7}$ (c) 3 (d) $\frac{7}{3}$

2 $-\frac{2}{7} \times .... = -\frac{2}{7}$

(a) -1 (b) zero (c) 1 (d) 2

3 If $\frac{2}{9} \times X = \frac{2}{9}$, then $9 \times X =$ ....

(a) 1 (b) 2 (c) 9 (d) 18

4 If $\frac{2}{5} \times X = -\frac{3}{5} \times \frac{2}{5}$, then $X =$ ....

(a) -1 (b) $\frac{3}{5}$ (c) $-\frac{2}{5}$ (d) $-\frac{3}{5}$

5 $\frac{1}{3} \times \left(2 - \frac{2}{3}\right) = \frac{1}{3} \times 2 - \frac{1}{3} \times$ ....

(a) $-\frac{1}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

Solution

1 (b) The reason : $2\frac{1}{3} = \frac{7}{3}$, since the multiplicative inverse of $\frac{7}{3}$ is $\frac{3}{7}$

, then the multiplicative inverse of $2\frac{1}{3}$ is $\frac{3}{7}$

2 (c) The reason : Because 1 is the multiplicative identity in $\mathbb{Q}$

3 (c) The reason : Since $\frac{2}{9} \times X = \frac{2}{9}$, then $X = 1$

(multiplicative identity) , then $9 \times X = 9 \times 1 = 9$

4 (d) The reason : Commutative property in multiplication.

5 (d) The reason : Distributing multiplication over subtraction in $\mathbb{Q}$

Example 3 **Use the distributing property to find the value of each of the following :**

1 $\frac{5}{11} \times \frac{6}{7} + \frac{5}{11} \times \frac{1}{7}$

2 $\frac{9}{17} \times 21 - \frac{9}{17} \times 4$

3 $\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25}$

4 $\frac{7}{12} \times 5 + \frac{49}{12} - \frac{7}{12} \times 11$

Solution

1. $\frac{5}{11} \times \frac{6}{7} + \frac{5}{11} \times \frac{1}{7} = \frac{5}{11}\left(\frac{6}{7} + \frac{1}{7}\right)$ (Distributing multiplication over addition)

$= \frac{5}{11} \times \frac{7}{7} = \frac{5}{11} \times 1 = \frac{5}{11}$

2. $\frac{9}{17} \times 21 - \frac{9}{17} \times 4 = \frac{9}{17}(21 - 4) = \frac{9}{17} \times 17 = 9$

3. $\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25} = \frac{22}{25}\left(\frac{6}{11} + \frac{5}{11} - 1\right)$

$= \frac{22}{25}\left(\frac{11}{11} - 1\right) = \frac{22}{25} \times (1 - 1)$

$= \frac{22}{25} \times \text{zero} = \text{zero}$

4. $\frac{7}{12} \times 5 + \frac{49}{12} - \frac{7}{12} \times 11 = \frac{7}{12} \times 5 - \frac{7}{12} \times 11 + \frac{49}{12}$ (Commutative property)

$= \frac{7}{12}(5 - 11) + \frac{49}{12} = \frac{7}{12}(-6) + \frac{49}{12}$

$= -\frac{42}{12} + \frac{49}{12} = \frac{7}{12}$

Another solution :

$\frac{7}{12} \times 5 + \frac{49}{12} - \frac{7}{12} \times 11 = \frac{7}{12} \times 5 + \frac{7}{12} \times 7 - \frac{7}{12} \times 11$

$= \frac{7}{12}(5 + 7 - 11) = \frac{7}{12} \times 1 = \frac{7}{12}$

TRY by yourself 2

Use the distributing property to find the value of each of the following :

1. $\frac{5}{7} \times \frac{2}{3} + \frac{5}{7} \times \frac{1}{3}$

2. $11 \times \frac{3}{10} - \frac{3}{10}$

Second Division operation

Since every rational number (except zero) has a multiplicative inverse , then we can define the division operation in $\mathbb{Q}$ as follows :

Definition

If $\frac{a}{b}$, $\frac{c}{d}$ are two rational numbers , $\frac{c}{d} \neq$ zero , then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

For example: • $\frac{2}{3} \div \frac{7}{5} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$ • $-\frac{2}{5} \div \frac{6}{5} = -\frac{\cancel{2}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{5}^{1}}{\cancel{6}_{3}} = -\frac{1}{3}$

! Remarks

- Since division by zero is impossible in $\mathbb{Q}$, therefore $\mathbb{Q}$ is not closed with respect to division operation.
- Division operation in $\mathbb{Q}$ is not commutative and not associative.
- There is no identity element in division operation in $\mathbb{Q}$ and hence there are no inverses numbers with respect to division operation in $\mathbb{Q}$

Example 4

Find the result of each of the following in its simplest form :

1. $-\frac{2}{3} \div \frac{5}{3}$
2. $\frac{3}{7} \div (-8)$
3. $2\frac{1}{5} \div 5\frac{1}{2}$
4. $0.2 \div \frac{1}{5}$
5. $\left(\frac{2}{7} + \frac{3}{7}\right) \div \frac{10}{7}$
6. $\left(\frac{5}{6} - \frac{3}{4}\right) \div \left(\frac{7}{12} - \frac{5}{9}\right)$

Solution

1. $-\frac{2}{3} \div \frac{5}{3} = -\frac{2}{\cancel{3}_1} \times \frac{\cancel{3}^1}{5} = -\frac{2}{5}$
2. $\frac{3}{7} \div (-8) = \frac{3}{7} \times \left(-\frac{1}{8}\right) = -\frac{3}{56}$
3. $2\frac{1}{5} \div 5\frac{1}{2} = \frac{11}{5} \div \frac{11}{2} = \frac{\overset{1}{\cancel{11}}}{5} \times \frac{2}{\underset{1}{\cancel{11}}} = \frac{2}{5}$
4. $0.2 \div \frac{1}{5} = \frac{2}{10} \div \frac{1}{5} = \frac{2}{10} \times \frac{5}{1} = \frac{10}{10} = 1$
5. $\left(\frac{2}{7} + \frac{3}{7}\right) \div \frac{10}{7} = \frac{5}{7} \div \frac{10}{7} = \frac{\cancel{5}^1}{\cancel{7}_1} \times \frac{\cancel{7}^1}{\cancel{10}_2} = \frac{1}{2}$
6. $\left(\frac{5}{6} - \frac{3}{4}\right) \div \left(\frac{7}{12} - \frac{5}{9}\right) = \left(\frac{10}{12} - \frac{9}{12}\right) \div \left(\frac{21}{36} - \frac{20}{36}\right)$

$= \frac{1}{12} \div \frac{1}{36} = \frac{1}{\cancel{12}_1} \times \frac{\cancel{36}^3}{1} = 3$

TRY by yourself 3

Find the result of each of the following in its simplest form :

1. $\frac{3}{7} \div \frac{9}{14}$
2. $\frac{3}{4} \div \left(-\frac{15}{2}\right)$
3. $2\frac{1}{3} \div \left(-\frac{7}{3}\right)$
4. $-\frac{5}{6} \div 10$

Example 5 If $x = -\frac{1}{3}$, $y = \frac{3}{4}$ and $z = -3$, find the numerical value of each of the following :

1 $\frac{y}{z}$ 2 $\frac{xy}{z}$ 3 $\frac{x}{y} - \frac{y}{z}$

Solution

1 $\frac{y}{z} = \frac{3}{4} \div (-3) = \frac{\cancel{3}^{1}}{4} \times (-\frac{1}{\cancel{3}_{1}}) = -\frac{1}{4}$

2 $\frac{xy}{z} = (-\frac{1}{\cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{4}) \div (-3) = -\frac{1}{4} \div (-3)$

$= -\frac{1}{4} \times (-\frac{1}{3}) = \frac{1}{12}$

Notice that : $xy = x \times y$

3 $\frac{x}{y} = -\frac{1}{3} \div \frac{3}{4} = -\frac{1}{3} \times \frac{4}{3} = -\frac{4}{9}$, $\frac{y}{z} = -\frac{1}{4}$

$\frac{x}{y} - \frac{y}{z} = -\frac{4}{9} - (-\frac{1}{4}) = -\frac{16}{36} + \frac{9}{36} = -\frac{7}{36}$

Wonders of numbers

- Choose an integer between 100 , 1000
- Multiply it by 7 , then multiply the product by 11 and multiply the product by 13
- Do it using different numbers and notice the product each time !

Answers *of try by yourself*

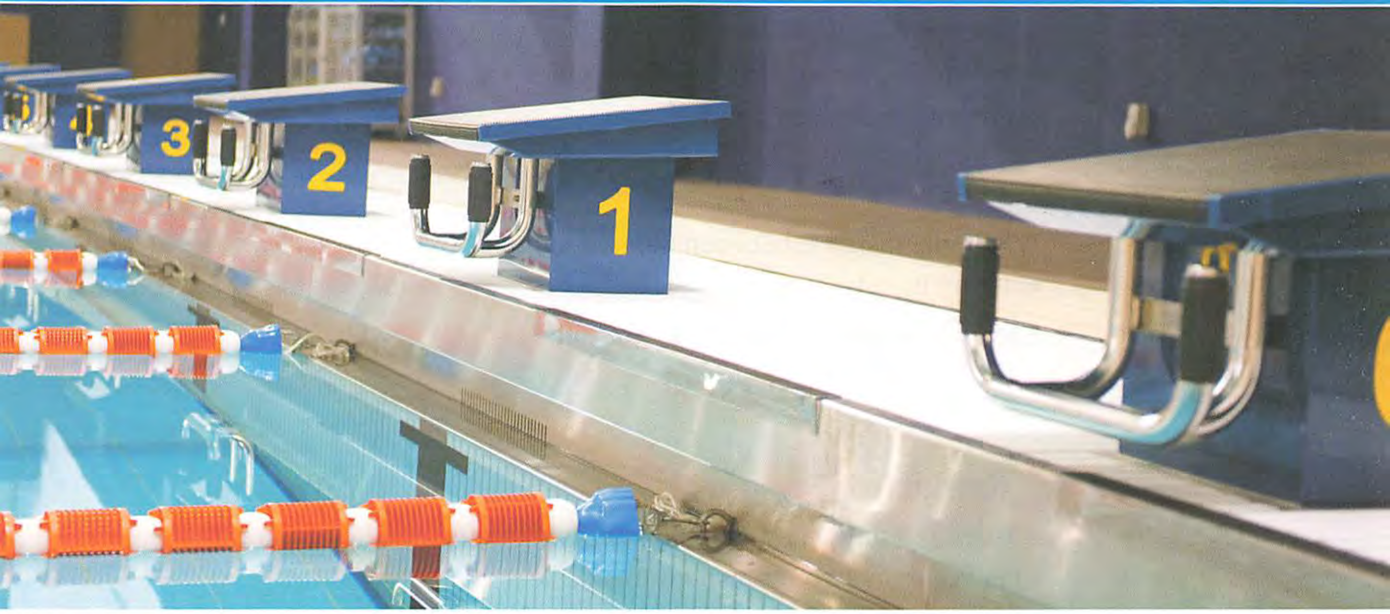
1 (1) $\frac{5}{6}$ (2) $-\frac{32}{45}$ (3) $-\frac{3}{2}$ (4) $\frac{5}{2}$

2 (1) $\frac{5}{7}$ (2) 3

3 (1) $\frac{2}{3}$ (2) $-\frac{1}{10}$ (3) -1 (4) $-\frac{1}{12}$

LESSON **5**

Applications on Rational Numbers



The distance between two numbers

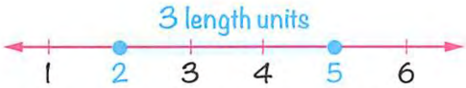
It is possible to express the distance between the two numbers x , y on the number line by using the absolute value as follows :

$|x - y|$ or $|y - x|$

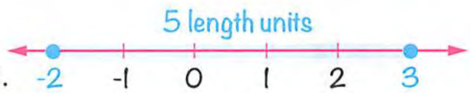
Notice that :
$|x - y| = |y - x|$

For example:

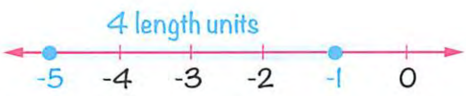
- The distance between 2 and 5 $= |2 - 5| = |-3|$
  $=$ **3 length units**.



- The distance between -2 and 3 $= |-2 - 3|$
  $= |-5| =$ **5 length units**.



- The distance between -1 and -5 $= |-1 - (-5)|$
  $= |-1 + 5|$
  $= |4| =$ **4 length units**.



Example 1 **Find a rational number lying at the middle of the way between 3 and 7**

Solution

- If we notice the number line in the opposite figure we find :

The number that lies at the midpoint of the distance between 3 and 7 is 5 , from that we can deduce the following rule :

The number that lies at the **midpoint of the way** between any two numbers	=	The **smaller** number + $\frac{1}{2}$ the **distance** between the two numbers
		or
		The **greater** number − $\frac{1}{2}$ the **distance** between the two numbers

Where the distance between the two numbers 3 and 7 is $|3-7| = |-4|$ $= 4$ length units.

i.e. The required number is :

$3 + \frac{1}{\cancel{2}_1} \times \cancel{4}^{2} = 3 + 2 = 5$ or $7 - \frac{1}{\cancel{2}_1} \times \cancel{4}^{2} = 7 - 2 = 5$

! Remark

There is a unique rational number lying at the middle of the way between any two rational numbers.

Example 2 **Find a rational number in half-way between $\frac{2}{5}$ and $\frac{3}{7}$**

Solution Since L.C.M. of the denominators = 35

Therefore , $\frac{2}{5} = \frac{2 \times 7}{5 \times 7} = \frac{14}{35}$, $\frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35}$

and since $\frac{14}{35} < \frac{15}{35}$

Therefore , the required number is :

$\frac{14}{35} + \frac{1}{2} \left| \frac{15}{35} - \frac{14}{35} \right| = \frac{14}{35} + \frac{1}{2} \times \frac{1}{35} = \frac{14}{35} + \frac{1}{70} = \frac{28}{70} + \frac{1}{70} = \frac{29}{70}$

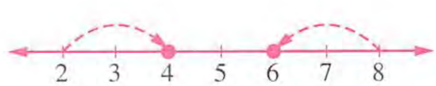
TRY by yourself 1 ***Find a rational number in half-way between $\frac{5}{6}$ and $\frac{3}{8}$***

Example 3 **Find a rational number lying at one third of the way between 2 and 8**

1 From the side of the smaller number.

2 From the side of the greater number.

Solution By observing the opposite number line we find the following :



The number that lies at one third of the way between two numbers

From the side of **the smaller number** = The **smaller** number + $\frac{1}{3}$ the distance between the two numbers.

The number that lies at one third of the way between two numbers

From the side of **the greater number**	=	The **greater** number	−	$\frac{1}{3}$ the distance between the two numbers.

Therefore

1. The number that lies at one third of the way between 2 and 8 from the side of 2

$$= 2 + \frac{1}{3}|8 - 2| = 2 + \frac{1}{\cancel{3}_{1}} \times \cancel{6}^{2} = 4$$

2. The number that lies at one third of the way between 2 and 8 from the side of 8

$$= 8 - \frac{1}{3}|8 - 2| = 8 - \frac{1}{\cancel{3}_{1}} \times \cancel{6}^{2} = 6$$

Example 4 **Find a rational number lying at one fourth of the way between :**

$-\frac{1}{6}$ and $-\frac{1}{3}$ from the side of the smaller number.

Solution Since L.C.M. of the denominators is 6

$-\frac{1}{3} = -\frac{2}{6}$

The greater number $= -\frac{1}{6}$

The smaller number $= -\frac{2}{6}$

The distance between the two numbers $= \left|-\frac{1}{6} - \left(-\frac{2}{6}\right)\right| = \left|\frac{1}{6}\right| = \frac{1}{6}$

The required number is :

The smaller number + $\frac{1}{4}$ the distance between the two numbers

$$= -\frac{2}{6} + \frac{1}{4} \times \frac{1}{6} = -\frac{2}{6} + \frac{1}{24} = -\frac{8}{24} + \frac{1}{24} = -\frac{7}{24}$$

TRY by yourself 2 ***Find a rational number lying at one fifth of the way between :***

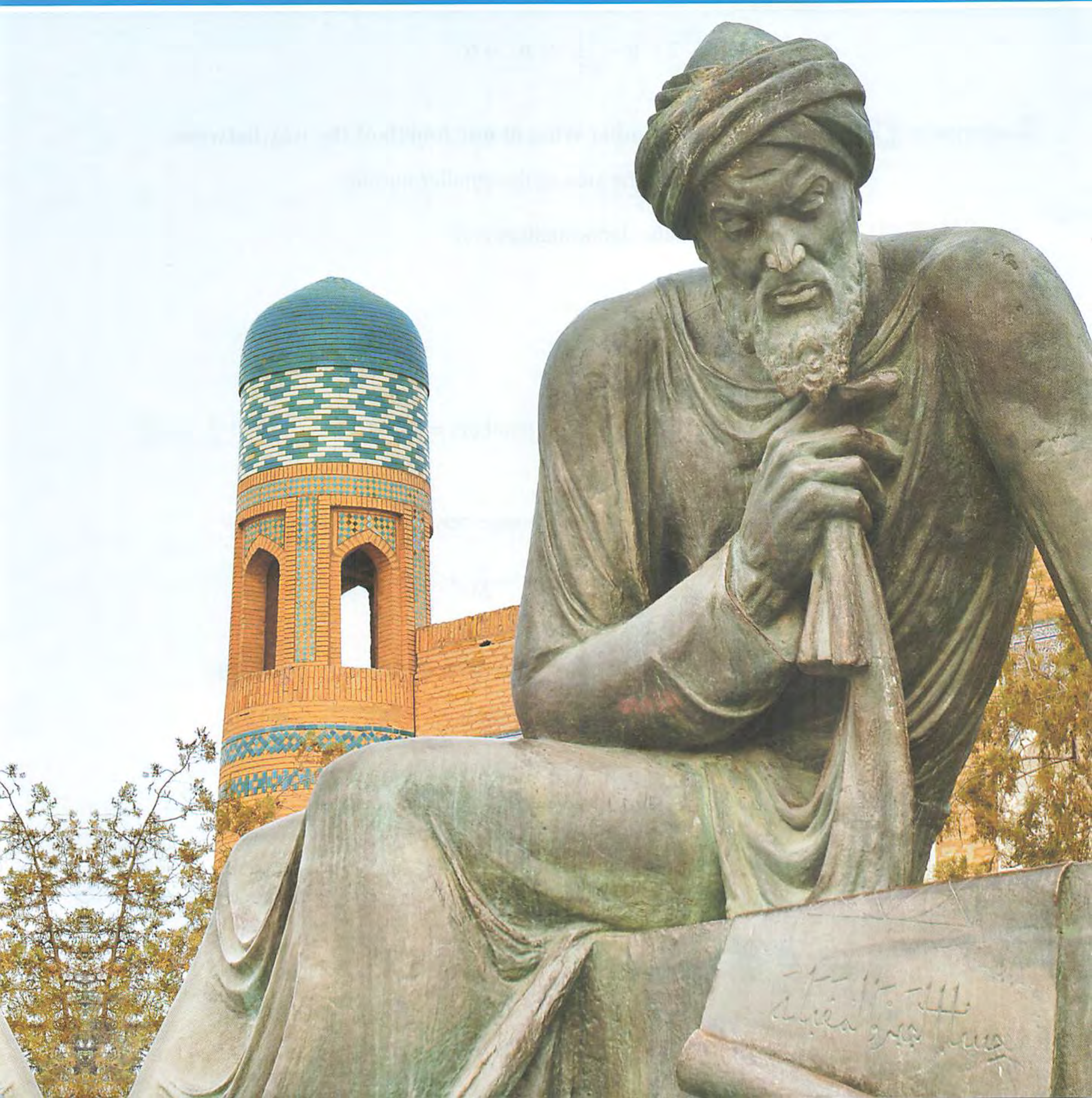
$\frac{2}{5}$ and $\frac{4}{7}$ from the side of the greater number.

Answers *of try by yourself*

1. $\frac{29}{48}$ 2. $\frac{94}{175}$

UNIT 2

Algebra



Lessons of the unit :

1. Algebraic terms and algebraic expressions.
2. Like algebraic terms.
3. Adding and subtracting algebraic expressions.
4. Multiplying and dividing algebraic terms.
5. Multiplying a monomial by an algebraic expression.
6. Multiplying a binomial by an algebraic expression.
7. Dividing an algebraic expression by a monomial.
8. Dividing an algebraic expression by another one.
9. Factorization by identifying the highest common factor (H.C.F.).

▶ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the algebraic term and the algebraic expression and their degrees.
- perform the operations on the like algebraic terms.
- reduce the algebraic expression.
- multiply a monomial by an algebraic expression.
- perform the operations on the algebraic expressions.
- multiply two binomials by inspection.
- divide an algebraic expression by a monomial.
- divide an algebraic expression by another one.
- factorize the algebraic expression by identifying the highest common factor.
- solve different problems on the operations on the algebraic terms and the algebraic expressions.
- appreciate the role of mathematics in solving the real life problems.

Al Khowarezmy

(the father of algebra)

A muslim Iraqi scientist

(781 A.D. - 847 A.D.)

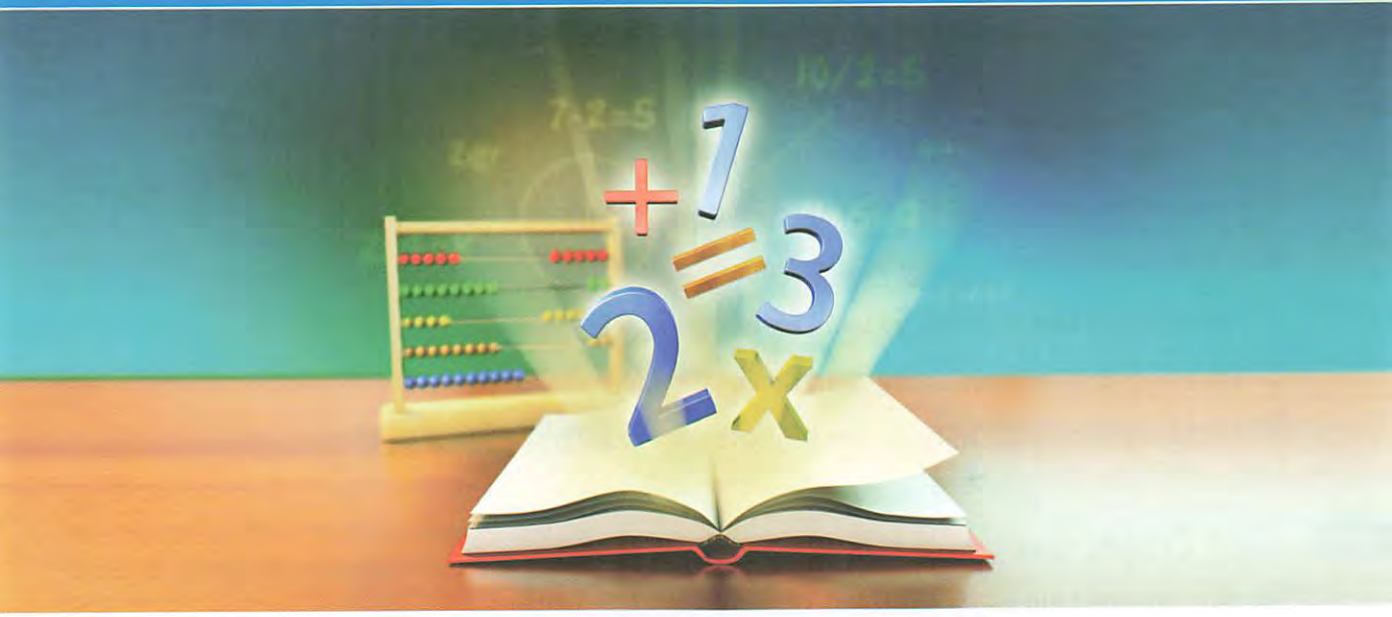
He is the first one to use algebra.

Thanks to Al Khowarezmy , the world knew the use of the Arabian digits which changed our concept of numbers. He also introduced the concept of zero.

Mohamed Ibn Moussa
Al Khowarezmy
(781 A.D. - 847 A.D.)

LESSON 1

Algebraic Terms and Algebraic Expressions



Introduction : variable and constant

The variable is a letter as : x or y or n or ... , which represents any number in a specified set of numbers.

For example:

We can write 7 n to represent the multiples of 7

In this case , the letter n represents any number in the set of integers.

If we replace n by 5 , we obtain : $7 n = 7 \times 5 = 35$ which is a multiple of 7

and if we replace n by 100

, we obtain : $7 n = 7 \times 100 = 700$ which is a multiple of 7 , and so on.

The constant is a number or letter represents only one number.

Algebraic terms and algebraic expressions

WATCH VIDEO

First Algebraic term

The algebraic term is a number , a variable or the product of numbers and variables.

i.e. The algebraic term consists of the product of two factors or more.

* **In the previous example :**

  7 n is an algebraic term which consists of two factors : 7 and n

  7 is called numerical factor (coefficient) and n is called algebraic factor.

* Also , $-5 x y$ is an algebraic term which consists of the factors : -5 , x and y

  -5 (numerical factor) , x (algebraic factor) and y (algebraic factor)

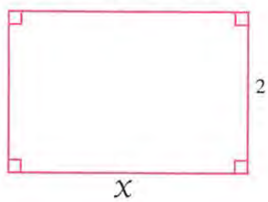
Second Algebraic expression

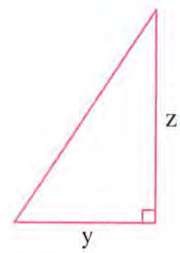
The algebraic expression consists of one or more terms connected by the sign + or –

For example:

- $5a + 3b$ is an algebraic expression which consists of the two terms : $5a$ and $3b$ "binomial"
- $5y^2 + 2xy - 3x$ is an algebraic expression which consists of three terms. "trinomial"
- $3x$ is an algebraic expression which consists of one term.

Example 1 **Write the algebraic term that represents the area of each of the following shapes :**

1 

2 

Solution

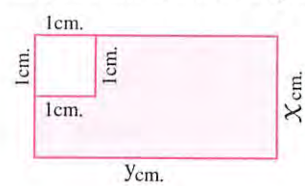
1 The area of the rectangle = length × width = $2x$

2 The area of the triangle = $\frac{1}{2}$ base length × height = $\frac{1}{2}yz$

Example 2 **Write the algebraic expression that expresses each of the following :**

1 The length of $\overline{AB}$

2 The area of the coloured part



Solution

1 The length of $\overline{AB}$ = AC + CB   i.e. The length of $\overline{AB} = x + y$

It is an algebraic expression consisting of two terms.

2 The area of the coloured part = the area of the rectangle – the area of the square

$$= (x \times y) - (1 \times 1)$$

i.e. The area of the coloured part = $(xy - 1)$ cm^2.

It is an algebraic expression consisting of two terms.

Remark

The algebraic term that has no algebraic factors is called the absolute term as the algebraic term 3 in the algebraic expression : $y^2 - 2y + 3$

The degree of the algebraic term

The degree of the algebraic term is the sum of the indices of the algebraic factors in this term.

For example:

$7m^{2}n$

$2+1=3$

3^{rd} degree term

- The term 2 a is of the 1^{st} degree because the index of a is 1
- The term $-7x^2$ is of the 2^{nd} degree because the index of x is 2
- The term $-5xy$ is of the 2^{nd} degree because the sum of indices of the two symbols x and y is 2
- The term $7m^2n$ is of the 3^{rd} degree because the sum of indices of the two symbols m and n is 3

Remark

Any number is an algebraic term of zero degree.

For example:

The number -2 is an algebraic term of zero degree because it can be written in the form : $-2 \times x^0$ "where : $x^0 = 1$"

Example 3

Choose the correct answer from the given ones :

1 The algebraic term $5xy^2$ is of the .... degree.
(a) first (b) second (c) third (d) fifth

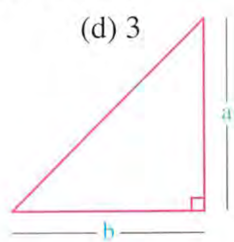
2 The degree of the algebraic term $2^2 a^2 b^2$ equals the degree of the algebraic term ....
(a) $a^3 b^3$ (b) $4xy^3$ (c) $4a^3 b^2$ (d) $5xyz$

3 The coefficient of the algebraic term $5^3 x$ is ....
(a) 3 (b) 5 (c) 25 (d) 125

4 The coefficient of the algebraic term $-a^2 b^3$ is ....
(a) -1 (b) zero. (c) 1 (d) 5

5 If the algebraic term $3x^2 y^n$ is of the sixth degree , then n = ...
(a) 6 (b) 5 (c) 4 (d) 3

6 The algebraic term representing the area of the opposite figure is of the .... degree.



(a) first (b) second
(c) third (d) fourth

Solution

1 (c) The reason : The sum of indices of the symbols x and y is 3

2 (b) The reason : The algebraic term $2^2\ a^2\ b^2$ is of the 4^{th} degree because the sum of indices of a and b is 4 , also the algebraic term $4\ x\ y^3$ is of the 4^{th} degree because the sum of indices of x and y is 4

3 (d) The reason : The coefficient of the term $5^3\ x$ is 5^3 which equals 125

4 (a)

5 (c) The reason : Since the algebraic term $3\ x^2\ y^n$ is of the sixth degree , then $2 + n = 6$, then $n = 4$

6 (b) The reason : The area of the triangle $= \frac{1}{2}$ base length × height
$= \frac{1}{2}$ a b (of the second degree)

TRY by yourself 1

Complete the following table :

The algebraic term	$5\ x$	$3\ x\ y$	$-5\ a^2$	$4\ x^2\ y$	$-2\ a^2\ b^2$	$15\ a^3\ b$	x	-4	$(-3)^2$
Its coefficient	....	....	....	....	....	....	....	....	....
Its degree	....	....	....	....	....	....	....	....	....

The degree of the algebraic expression

The degree of the algebraic expression is the highest degree of the terms forming it.

For example:

- The algebraic expression : $5\ x - 3$ is of the 1^{st} degree because $5\ x$ is the term of the highest degree that is 1
- The algebraic expression : $7\ x^2 - 3\ x + 1$ is of the 2^{nd} degree because $7\ x^2$ is the term of the highest degree that is 2
- The algebraic expression : $5\ a\ b - 2\ a^2\ b - b^2$ is of the 3^{rd} degree because $-2\ a^2\ b$ is the term of the highest degree that is 3

$7x^2 - 3x + 1$

$7x^2$ is the term of the highest degree

so, the algebraic expression is of the 2^{nd} degree

Example 4 **Arrange the algebraic expression : $5x + 2x^3 - 4 - x^2$:**

1 According to the descending order of the indices of x

2 According to the ascending order of the indices of x

Solution

1 According to the descending order of the indices of x

The expression $= 2x^3 - x^2 + 5x - 4$

2 According to the ascending order of the indices of x

The expression $= -4 + 5x - x^2 + 2x^3$

Example 5 **State the degree of the algebraic expression : $2a^3b^2 - 7ab^3 + 5a^2b$, then arrange it :**

1 According to the descending order of the indices of a

2 According to the ascending order of the indices of b

Solution The expression is of the fifth degree because the term $2a^3b^2$ is the term of the highest degree that is 5

1 According to the descending order of the indices of a

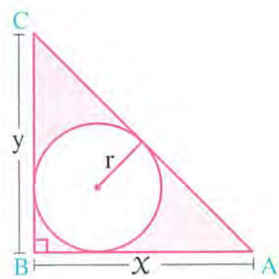
The expression $= 2a^3b^2 + 5a^2b - 7ab^3$

2 According to the ascending order of the indices of b

The expression $= 5a^2b + 2a^3b^2 - 7ab^3$

Example 6 **From the opposite figure :**

Write the algebraic expression which represents the area of the coloured part , then state its degree (given that the area of the circle $= \pi r^2$)



Solution The area of the coloured part = the area of Δ ABC – the area of the circle

$$= \frac{1}{2}x \times y - \pi r^2$$

Therefore the algebraic expression which represents the area of the coloured part

$= \frac{1}{2}xy - \pi r^2$

and it is of the 2nd degree.

Remember that

π expresses a number has an approximated value but does not express an algebraic symbol.

TRY by yourself 2 ***Complete the following table :***

The algebraic expression	The number of its terms	Its name	Its degree
$-2a^2b^3$	....	....	....
$a^3-5a^2b^2+3b^2$	....	....	....
$\frac{1}{2}a+\frac{1}{4}b-5$	....	....	....
$2x^2y+5xy+4y$	....	....	....
$1-7x^2y$	....	....	....
$3^2x^2+2^4x$	....	....	....

Wonders of numbers

The two digits 8 , 5

- $8 \times 5 = 40$
- $88 \times 5 = 440$
- $888 \times 5 = 4440$
- $8888 \times 5 = 44440$

Try it yourself !

Answers *of try by yourself*

1 Coefficient of the algebraic term : 5 , 3 , −5 , 4 , −2 , 15 , 1 , −4 , 9

Degree of the algebraic term : 1 , 2 , 2 , 3 , 4 , 4 , 1 , zero , zero

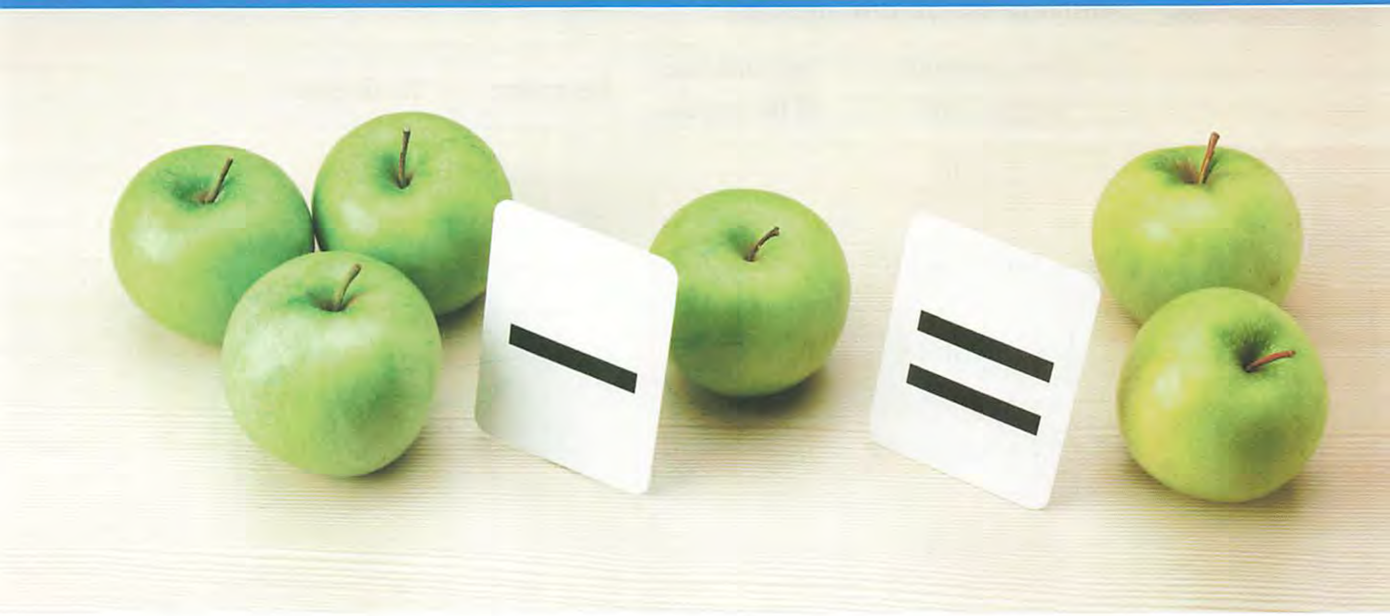
2 Number of terms of the algebraic expression : 1 , 3 , 3 , 3 , 2 , 2

Name of the algebraic expression : monomial , trinomial , trinomial , trinomial , binomial , binomial.

Degree of the algebraic expression : 5 , 4 , 1 , 3 , 3 , 2

LESSON **2**

Like Algebraic Terms



The algebraic terms are said to be like if the algebraic symbols forming their factors are like and the indices of these symbols are equal.

WATCH VIDEO

Examples for like algebraic terms :

- $2a$, a and $-5a$
- $2x^2y$, $4yx^2$ and $-\frac{1}{2}x^2y$

(**Notice that :** $x^2y = yx^2$ «commutative property»)

Examples for unlike algebraic terms :

- $2x$, $-3x^2$ and $7x^3$ are unlike algebraic terms **because their indices are different**.
- $4x^2$, $5xy$ and $-y^2$ are unlike algebraic terms **because their symbols are different**.

Adding and subtracting like terms

Adding or subtracting operation is performed as the following :

1. Add or subtract the numerical coefficients.
2. Use the sum or the difference as the coefficient of the result algebraic term.

Example 1 **Add :**

1. $5a$, $3a$, a , $6a$
2. $7ab^2$, $-2b^2a$, $-4b^2a$, ab^2

Solution

1. $5a + 3a + a + 6a = (5 + 3 + 1 + 6)a = 15a$
2. $7ab^2 + (-2b^2a) + (-4b^2a) + ab^2 = [7 + (-2) + (-4) + 1]ab^2 = 2ab^2$

Example 2 **Subtract :**

1. $5\,x\,y$ from $7\,x\,y$
2. $2\,x^2\,y$ from $-5\,x^2\,y$
3. $-3\,a^2\,b^2$ from $5\,a^2\,b^2$
4. $-3\,x^3\,y$ from $-2\,y\,x^3$

Solution

1. $7\,x\,y - 5\,x\,y = (7-5)\,x\,y = 2\,x\,y$
2. $-5\,x^2\,y - 2\,x^2\,y = (-5-2)\,x^2\,y = -7\,x^2\,y$
3. $5\,a^2\,b^2 - (-3\,a^2\,b^2) = 5\,a^2\,b^2 + 3\,a^2\,b^2 = 8\,a^2\,b^2$
4. $-2\,y\,x^3 - (-3\,x^3\,y) = -2\,x^3\,y + 3\,x^3\,y = x^3\,y$

Example 3 **Choose the correct answer from the given ones :**

1. $5\,x - (-x) = ...$

   (a) $4\,x$ (b) $6\,x$ (c) $-4\,x$ (d) $-6\,x$

2. $3\,a\,b - 3\,b\,a = ...$

   (a) $a\,b$ (b) $3\,a\,b$ (c) $6\,a\,b$ (d) zero

3. $3\,a^2\,c$ is more than $-5\,a^2\,c$ by ....

   (a) $8\,a^2\,c$ (b) $-2\,a^2\,c$ (c) $2\,a^2\,c$ (d) $-8\,a^2\,c$

4. $-7\,y$ is less than $-10\,y$ by ....

   (a) $-17\,y$ (b) $-3\,y$ (c) $3\,y$ (d) $17\,y$

5. The result of subtracting $7\,a$ from $-7\,a$ equals ....

   (a) $-14\,a$ (b) $-a$ (c) zero (d) $14\,a$

Solution

1. (b) The reason : $5\,x - (-x) = 5\,x + x = 6\,x$
2. (d)
3. (a) The reason : $3\,a^2\,c - (-5\,a^2\,c) = 3\,a^2\,c + 5\,a^2\,c = 8\,a^2\,c$
4. (b) The reason : $-10\,y - (-7\,y) = -10\,y + 7\,y = -3\,y$
5. (a) The reason : The result $= -7\,a - 7\,a = -14\,a$

TRY by yourself 1 *Put the suitable term in each space :*

1. $4x + 5x = \boxed{}$
2. $2x - 4x + x = \boxed{}$
3. $3x^2 + \boxed{} = 5x^2$
4. $7a^3 - \boxed{} = 2a^3$
5. $2b^4 + \boxed{} = b^4$
6. $3y^5 - \boxed{} = 5y^5$
7. $4x$ is less than $7x$ by $\boxed{}$
8. $7y$ is more than $-2y$ by $\boxed{}$

Reducing the algebraic expression

The algebraic expression is said to be in its simplest form if all its terms are unlike.

WATCH VIDEO

For example:

- The expression : $9x^2 - 3x + 1$ is in its simplest form

because there are not like terms among its terms.

- The expression : $6x + 7y + 4x + 3y$ is **not** in its simplest form

because there are like terms among its terms which are :

$6x, 4x$ and $7y, 3y$

Reducing the algebraic expression means putting it in the simplest form. This will be carried out by adding like terms using the commutative and associative properties.

Example 4 **Reduce to the simplest form :**

1. $6x + 7y + 4x - 3y$
2. $6x^2 - 7x - 4x^2 + 5x - 3x + x^2$

Solution

1. $6x + 7y + 4x - 3y$

$= 6x + 4x + 7y - 3y$ (commutative property)

$= (6x + 4x) + (7y - 3y)$ (associative property)

$= 10x + 4y$

Notice that :
We can not add or subtract unlike terms.
For example:
$10x + 4y \neq 14xy$

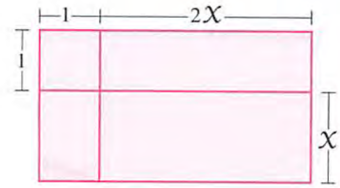
2. The expression $= (6x^2 - 4x^2 + x^2) + (-7x + 5x - 3x)$

(commutative and associative properties)

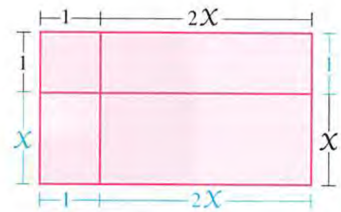
$= 3x^2 + (-5x) = 3x^2 - 5x$

Example 5 **In the opposite figure :**

Write the algebraic expression that expresses the perimeter of the coloured figure.



Solution We can deduce the left lengths of the figure as in the opposite figure , then :



The perimeter of the figure

$= 2X + 1 + 1 + X + 1 + 2X + X + 1$

$= (2X + X + 2X + X) + (1 + 1 + 1 + 1)$

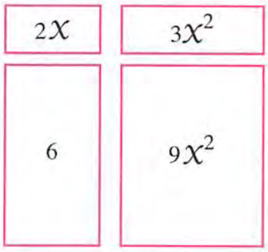
$= (6X + 4)$ length unit.

TRY by yourself 2

1 ***Reduce the following expression to its simplest form :***

$a^2 + 3a - 4 + 4a^2 - 5a + 1$

2 Write the algebraic expression which expresses the sum of the areas of the rectangles which are shown in the opposite figure.



Wonders of numbers

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

Try it yourself !

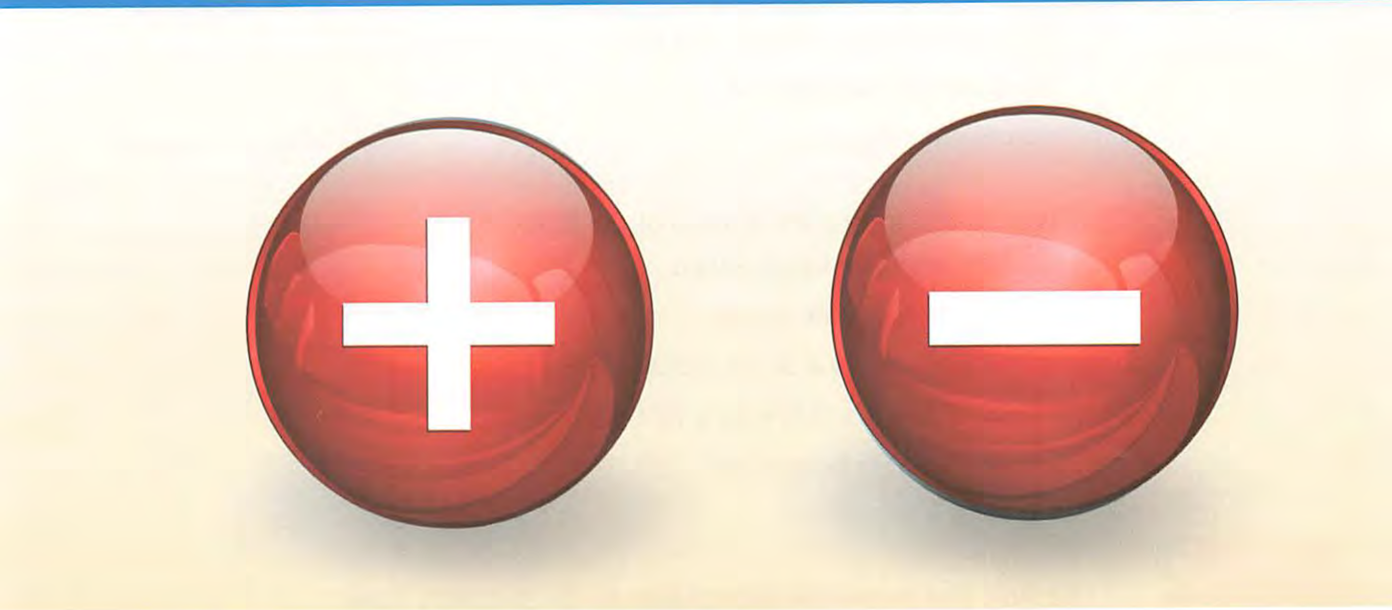
Answers of try by yourself

1 (1) $9X$ (2) $-X$ (3) $2X^2$ (4) $5a^3$ (5) $-b^4$ (6) $-2y^5$ (7) $3X$ (8) $9y$

2 (1) $5a^2 - 2a - 3$ (2) $12X^2 + 2X + 6$

LESSON 3

Adding and Subtracting Algebraic Expressions



First Adding algebraic expressions

There are two methods for adding algebraic expressions as shown in the following example.

Example 1 **Add the two expressions :** $5a - 7b + 3$ **and** $2b - 1 - a$

Solution

The horizontal method :

In this method , we use the commutative and associative properties :

$$\begin{aligned}\text{The sum} &= (5a - 7b + 3) + (2b - 1 - a)\\ &= (5a - a) + (-7b + 2b) + (3 - 1)\\ &\quad \text{(commutative and associative properties)}\\ &= 4a - 5b + 2\end{aligned}$$

The vertical method :

In this method , we use the commutative property to arrange the two expressions such that the like terms lie under each other as follows :

The first expression	:	$5a - 7b + 3$
The second expression	:	$-a + 2b - 1$
The sum		$= 4a - 5b + 2$

Example 2 **Add the following expressions :**

$3x^3-4x^2+2x-1$, $5x^2-2x^3+3$ **and** $2-3x+x^2$

Solution It is better to rearrange each expression ascendingly or descendingly according to the indices of the symbol x such that we leave a space under the terms that have no like terms to them.

The first expression	:	$3x^3-4x^2+2x-1$
The second expression	:	$-2x^3+5x^2\qquad+3$
The third expression	:	$+\ x^2-3x+2$
The sum	=	$x^3+2x^2-\ x+4$

Example 3 **Add :** $4x^2-3xy+y^2$ **and** $3xy-3x^2+2y^2$

, then find the numerical value of the result when : $x=-2$ **and** $y=1$

Solution

$$\begin{array}{r} 4x^2-3xy+y^2 \\ -3x^2+3xy+2y^2 \\ \hline \end{array}$$

The sum = $x^2\ +\text{zero}\ +3y^2=x^2+3y^2$

The numerical value $=(-2)^2+3\times1^2=4+3=7$

TRY by yourself 1

Add : $3x^2-5+2x$, $x+5x^2+7$ ***and*** $-4x^2-3$

, then find the numerical value of the result when : $x=2$

The additive inverse of the algebraic expression

- The additive inverse of the algebraic expression is that whose terms are the additive inverses of the terms of the first expression.
- The sum of any algebraic expression and its additive inverse is equal to zero.

For example:

The algebraic expression :	$+x^2$	$-2x$	$+3$	
	change	change	change	
Its additive inverse :	$-x^2$	$+2x$	-3	
The sum	$=0$	$+0$	$+0$	$=0$

Second Subtracting algebraic expressions

We have two methods to subtract algebraic expressions as we studied before in addition. That will be shown in the following example.

Example 4 **Subtract :** $5x - 3y + 2z$ **from** $2y - z + 7x$

Solution **The horizontal method :**

In this method , we put the subtraction operation in the form :

The remainder = (the minuend) – (the subtrahend)

After removing brackets , we reduce the like terms.

Therefore , the remainder

$= (2y - z + 7x) - (+5x - 3y + 2z)$

change change change

$= 2y - z + 7x - 5x + 3y - 2z$

$= (7x - 5x) + (2y + 3y) + (-z - 2z)$

$= 2x + 5y - 3z$

The vertical method :

In this method , we rearrange the terms of the subtrahend down the terms of the minuend , then we add the minuend and the additive inverse of the subtrahend.

The minuend :	$+2y - z + 7x$
	$+ \quad - \quad -$
The subtrahend	$-3y + 2z + 5x$
The remainder	$= 5y - 3z + 2x$

Notice that : We change the signs of the subtrahend terms to get its additive inverse.

Remember that

- Subtract a from b , means b – a
- What is the increase of a than b ? means a – b
- What is the decrease of a than b ? means b – a
- What is the expression which should be added to a to get b ? means b – a
- What is the expression which should be subtracted from a to get b ? means a – b

Example 5 **What is the expression that should be added to $8 - 3a^2 + 2a^3$ to get $5 + 4a^3 - 7a$?**

Solution

The minuend	:	$4a^3$		$-7a + 5$
The subtrahend	:	$\ominus\,+2a^3$	$\oplus\,-3a^2$	$\ominus\,+8$
The remainder	=	$2a^3$	$+3a^2$	$-7a - 3$

Notice that :

We rearranged the terms of the minuend and the terms of the subtrahend descendingly according to the indices of a , and we left spaces up and down the terms which have no like terms.

Example 6 **What is the increase of $3a^2 - 4b^2 + 2ab$ than the sum of $2a^2 - 3ab + b^2$ and $2b^2 + a^2 + ab$?**

Solution

$$\begin{array}{rrr} 2a^2 & -3ab & +b^2 \\ a^2 & +ab & +2b^2 \\ \hline \end{array}$$

The sum $= 3a^2 - 2ab + 3b^2$

To get the increase , subtract the resulting sum from the given expression :

$$\begin{array}{rrr} 3a^2 & +2ab & -4b^2 \\ \ominus\; 3a^2 & \oplus\; -2ab & \ominus\; +3b^2 \\ \hline \end{array}$$

The increase $= 4ab - 7b^2$

TRY by yourself 2

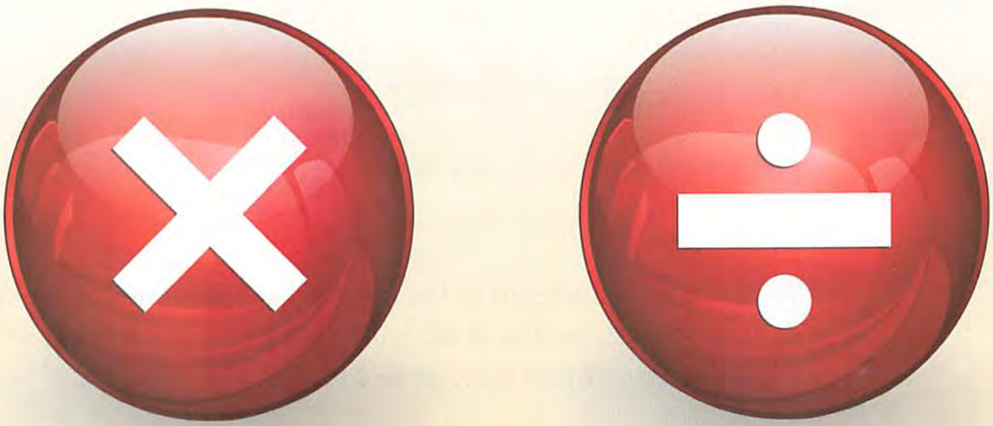
1. ***What is the expression which should be subtracted from :*** $-x^2 + 2x - 1$ to get $3x^2 - 5$?
2. ***What is the decrease of*** $7 - 5a + a^2$ than $3a^2 - 5a - 2$?

Answers *of try by yourself*

1 $4x^2 + 3x - 1$, 21

2 [1] $-4x^2 + 2x + 4$ [2] $2a^2 - 9$

LESSON 4 Multiplying and Dividing Algebraic Terms



Multiplying the like bases

- **We know that :**

$2^3 = 2 \times 2 \times 2 = 8$, $2^2 = 2 \times 2 = 4$, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

- And we know that : $8 \times 4 = 32$ i.e. $2^3 \times 2^2 = 2^5$ "Note the addition of the indices"

Generally

When multiplying the like bases , add their indices.

i.e. If a is a rational number , m and n are two positive integers

, then : $a^m \times a^n = a^{m+n}$

Dividing the like bases

- **We know that :**

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, $2^3 = 2 \times 2 \times 2 = 8$, $2^2 = 2 \times 2 = 4$

- And we know that : $\frac{32}{8} = 4$ i.e. $\frac{2^5}{2^3} = 2^2$ "Note the subtraction of the indices"

Generally

When dividing the like bases , subtract their indices.

i.e. If a is a rational number not equal to zero , m and n are two positive integers where $m \geq n$

, then : $a^m \div a^n = a^{m-n}$

First Multiplying the algebraic terms

When multiplying the algebraic terms , follow the following :

1. Multiply the coefficients using the signs rule.
2. Multiply the symbols by adding the indices of symbols which have like bases.

For example:

- $2a \times 5b = (2 \times 5) \times (a \times b) = 10ab$
- $5x^2 \times 3x = (5 \times 3) \times (x^2 \times x) = 15x^3$

Remark

After training , you can give the result directly as follows :

Adding the powers

$$5x^{2} \times 3x^{1} = 15x^{3}$$

Multiplying the coefficients

Example 1

Find the result of each of the following :

1. $5a^3b \times 3ab$
2. $\frac{3}{4}a^2 \times \frac{4}{3}a$
3. $\frac{2}{5}x^2 \times (-15x^3)$

Solution

1. $5a^3b \times 3ab = 15a^4b^2$
2. $\frac{3}{4}a^2 \times \frac{4}{3}a = a^3$
3. $\frac{2}{5}x^2 \times (-15x^3) = -6x^5$

TRY by yourself 1

Complete each of the following :

1. $2a \times (-3ab) =$ ....
2. $-2x^2y \times 3xy^2 =$ ....
3. $-4\ell m^2 \times \frac{1}{2}\ell^2 m^2 =$ ....
4. $\frac{2}{3}m^2n \times \frac{9}{4}n =$ ....

Second Dividing the algebraic terms

When dividing an algebraic term by another algebraic term , follow the following :

1. Divide the coefficients using the signs rule.
2. Divide the symbols taking care that the indices of like bases should be subtracted. (subtracting the indices of the divisor from the indices of the dividend)

Example 2

Find the quotient of each of the following :

1. $12\,a^3$ **by** $3\,a$
2. $21\,x$ **by** -3
3. $-15\,x^2 y^3$ **by** $5\,x\,y^2$
4. $\dfrac{-24\,a^5 b^3 c^2}{-8\,a^2 b}$

Solution

1. $12\,a^3 \div 3\,a = (12 \div 3) \times (a^3 \div a)$
   $= 4\,a^{3-1} = 4\,a^2$
2. $21\,x \div (-3) = -7\,x$
3. $-15\,x^2 y^3 \div 5\,x\,y^2 = -3\,x^{2-1}\,y^{3-2}$
   $= -3\,x\,y$
4. $\dfrac{-24\,a^5 b^3 c^2}{-8\,a^2 b} = 3\,a^{5-2}\,b^{3-1}\,c^2$
   $= 3\,a^3 b^2 c^2$

You can write the answer directly as follows :

Subtracting the powers

$$12a^{3} \div 3a^{1} = 4a^{2}$$

Dividing the coefficients

! Remarks

1. The quotient of two equal factors is 1
   Hence , we can cancel the equal factors in division operation.
   For example: $\dfrac{-15\,\cancel{a^5}\,\cancel{b^3}\,c^2}{3\,\cancel{a^5}\,\cancel{b^3}\,c} = -5\,c$
2. Dividing any term by zero is meaningless. Thus all the problems that we will deal with which contains symbols , the divisor is not equal to zero.

TRY by yourself 2

Choose the correct answer from the given ones :

1. $5\,a^6 \div 5\,a =$ .... where $a \neq 0$
   (a) $25\,a^6$ (b) a^6 (c) $5\,a^5$ (d) a^5
2. $\dfrac{6\,a^2 b^3}{-3\,a\,b^2} =$ ....
   (a) $-2\,a\,b$ (b) $2\,a^2\,b$ (c) $-18\,a^3 b^5$ (d) $-2\,a^3 b^5$

3 $\frac{-12 x^3 y^2}{4 x^2 y^2} = \dots\dots\dots$

(a) $-3 x y$ (b) $3 x^2 y$ (c) $-3 x$ (d) $3 x^2 y^2$

4 $-8 x^5 y^3 \div (-y^3 x^4) = \dots\dots\dots$

(a) $8 x$ (b) $-8 x^2 y$ (c) $8 x^2 y$ (d) $-8 x y$

5 $10 x^5 \div \dots\dots\dots = 2 x^3$

(a) $20 x^8$ (b) $5 x^8$ (c) $5 x^2$ (d) $2 x^3$

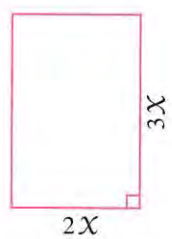
6 $48 a^4 b^7 = 12 a^2 b^2 \times \dots\dots\dots$

(a) $576 a^6 b^9$ (b) $4 a^6 b^9$ (c) $3 a b^2$ (d) $4 a^2 b^5$

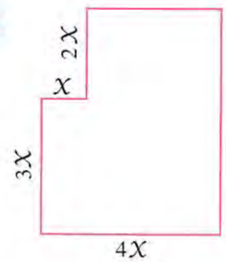
Applications on multiplying and dividing algebraic terms

Example 3 **Calculate the perimeter and the area of each figure of the following :**

1



2



Solution

1 • The perimeter of the rectangle = (length + width) × 2

$= (3 x + 2 x) \times 2$

$= 5 x \times 2 = 10 x$

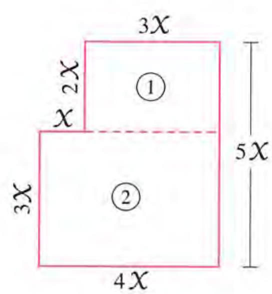
• The area of the rectangle = length × width

$= (3 x) \times (2 x) = 6 x^2$

2 • The perimeter of the figure

$= 5 x + 4 x + 3 x + x + 2 x + 3 x = 18 x$

• To find the area of the figure , we can divide it into two parts as shown in the figure , then : we find the sum of areas of the two parts.



Therefore ,

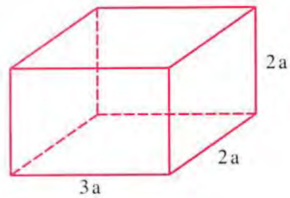
the area of the figure = the area of part (1) + the area of part (2)

$= (3 x \times 2 x) + (4 x \times 3 x)$

$= 6 x^2 + 12 x^2 = 18 x^2$

Try to solve number 2 using a different way of dividing the figure.

Example 4 **Calculate the total surface area and the volume of the opposite solid.**



Solution

- The total surface area of the cuboid

$= $ its lateral area $+$ ($2 \times$ the area of the base)

$= 2\,(2\,a + 3\,a) \times 2\,a + 2 \times 2\,a \times 3\,a$

$= 10\,a \times 2\,a + 12\,a^2 = 20\,a^2 + 12\,a^2$

$= 32\,a^2$

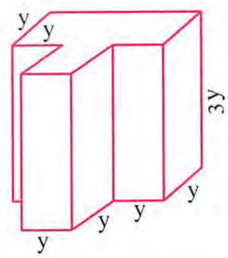
Notice that :
The lateral area of the cuboid
= perimeter of base × height

- The volume of the cuboid = length × width × height

$= 3\,a \times 2\,a \times 2\,a$

$= 12\,a^3$

Example 5 **Find the volume of the opposite solid.**



Solution

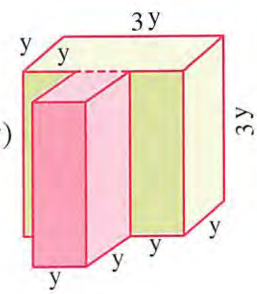
The solid is formed from two cuboids.

Therefore ,

the volume of the solid $= (y \times y \times 3\,y) + (3\,y \times y \times 3\,y)$

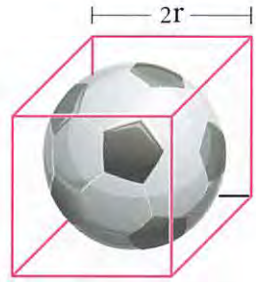
$= 3\,y^3 + 9\,y^3$

$= 12\,y^3$



Example 6 A sphere is put inside a cube as shown in the opposite figure to touch all its six faces internally. Find the ratio between the volume of the sphere and that of the cube

(Consider : $\pi \simeq \frac{22}{7}$)



(The volume of the sphere = $\frac{4}{3}\pi r^3$)

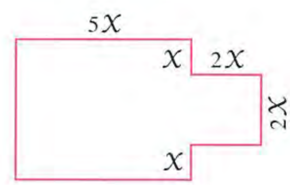
Solution The diameter length of the sphere = the edge length of the cube = 2 r

Therefore , the ratio between the sphere volume and the cube volume

$$= \frac{\text{The volume of the sphere}}{\text{The volume of the cube}} = \frac{\frac{4}{3}\pi r^3}{2r \times 2r \times 2r} = \frac{\frac{4}{3}\pi \cancel{r^3}}{8\cancel{r^3}}$$

$$= \left(\frac{\overset{1}{\cancel{4}}}{3} \times \frac{1}{\underset{2}{\cancel{8}}}\right)\pi = \frac{1}{6}\pi \simeq \frac{1}{6} \times \frac{22}{7} \simeq \frac{11}{21}$$

TRY by yourself 3 ***Calculate the perimeter and the area of the opposite figure.***



Answers *of try by yourself*

1 (1) $-6a^2b$ (2) $-6x^3y^3$ (3) $-2\ell^3m^4$ (4) $\frac{3}{2}m^2n^2$

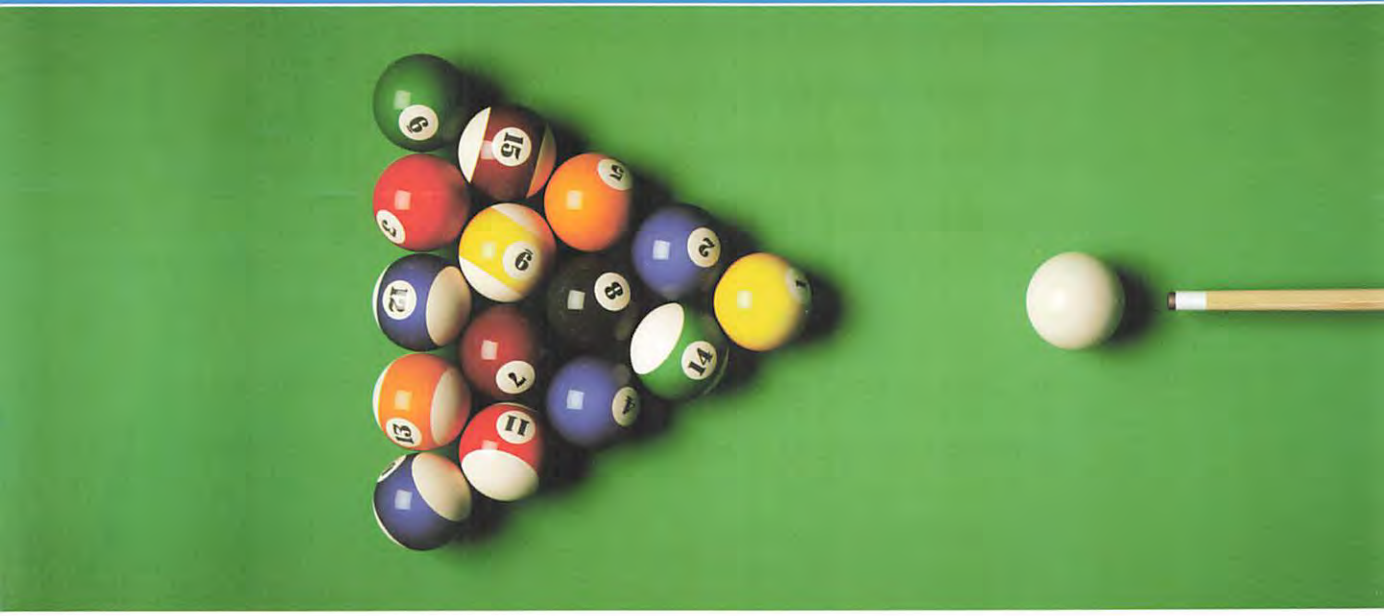
2 (1) (d) (2) (a) (3) (c) (4) (a)

(5) (c) because $\frac{10x^5}{2x^3} = 5x^2$ (6) (d) because $\frac{48a^4b^7}{12a^2b^2} = 4a^2b^5$

3 The perimeter = $22x$, the area = $24x^2$

LESSON **5**

Multiplying a Monomial by an Algebraic Expression



To multiply a monomial by an algebraic expression , we have to multiply this monomial by each term of the algebraic expression using the distribution property.

For example: $2x(3x + 5y) = (2x \times 3x) + (2x \times 5y)$ (distribution property)

$= 6x^2 + 10xy$

We can find the product by using the vertical method as follows :

$$\begin{array}{r} 3x + 5y \\ \times\ 2x \\ \hline \end{array}$$

The product $= 6x^2 + 10xy$

Example 1 **Find the product of each of the following :**

1. $b(-2a + a^2 b)$
2. $-3ab(5a - 2b + 3)$
3. $(a^2 - ab - 2b^2) \times 4ab$

Solution

1. $b(-2a + a^2 b) = -2ab + a^2 b^2$
2. $-3ab(5a - 2b + 3) = -15a^2 b + 6ab^2 - 9ab$
3. $(a^2 - ab - 2b^2) \times 4ab = 4a^3 b - 4a^2 b^2 - 8ab^3$

Try to solve the example by the vertical method

Example 2 **Choose the correct answer from the given ones :**

1 $x(x^2 + 3x) = \dots\dots\dots + 3x^2$

(a) x (b) x^2 (c) x^3 (d) $3x^2$

2 $-3ab(\dots\dots\dots + 4a^2b) = -6a^2b^2 - 12a^3b^2$

(a) $2ab$ (b) $-6ab$ (c) ab (d) $2a$

3 $2y(5x + 3xy) = 6xy^2 + \dots\dots\dots$

(a) $6xy^2$ (b) $10xy$ (c) $16xy^2$ (d) $6x^2$

4 If $x + 2y = 7$ and $z = 5$, then $x + 2(y - z) = \dots\dots\dots$

(a) 12 (b) 17 (c) -3 (d) -2

Solution

1 (c) 2 (a) 3 (b)

4 (c)

The reason :

$x + 2(y - z) = x + 2y - 2z = 7 - 2 \times 5 = 7 - 10 = -3$

Example 3 **Simplify :**

$2a(a + 4b) - 3b(a - 3b) - (2a^2 + 8b^2)$

, then find the numerical value of the result when : $a = 1$ and $b = -2$

Solution

The expression $= 2a^2 + 8ab - 3ab + 9b^2 - 2a^2 - 8b^2$

$= b^2 + 5ab$

The numerical value $= (-2)^2 + 5 \times 1 \times (-2)$

$= 4 - 10 = -6$

TRY by yourself

1 ***Find the product of each of the following :***

1 $3a(2a - 4b)$ 2 $-2x(3xy - 5x)$

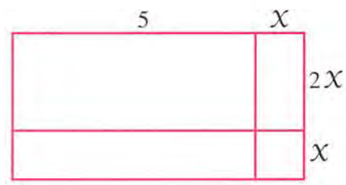
2 ***Simplify the following :*** $2x(3x - 2) + 3x(x + 1)$

Example 4

In the opposite figure :

A rectangle divided into three rectangles and a square.

Find the area of the whole figure.



Solution

The length of the whole rectangle = $x + 5$

, its width = $2x + x = 3x$

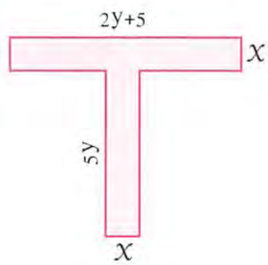
Therefore , its area = length × width

$$= (x + 5) \times 3x = 3x^2 + 15x$$

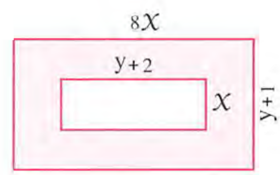
Example 5

Find the area of the coloured part in each of the following figures :

1



2



Solution

1 The area of the coloured part = The area of the horizontal rectangle + the area of the vertical rectangle

$$= x(2y + 5) + x \times 5y$$

$$= 2xy + 5x + 5xy$$

$$= 7xy + 5x$$

2 The area of the coloured part = The area of the external rectangle − the area of the internal rectangle

$$= 8x(y + 1) - x(y + 2)$$

$$= 8xy + 8x - xy - 2x$$

$$= 7xy + 6x$$

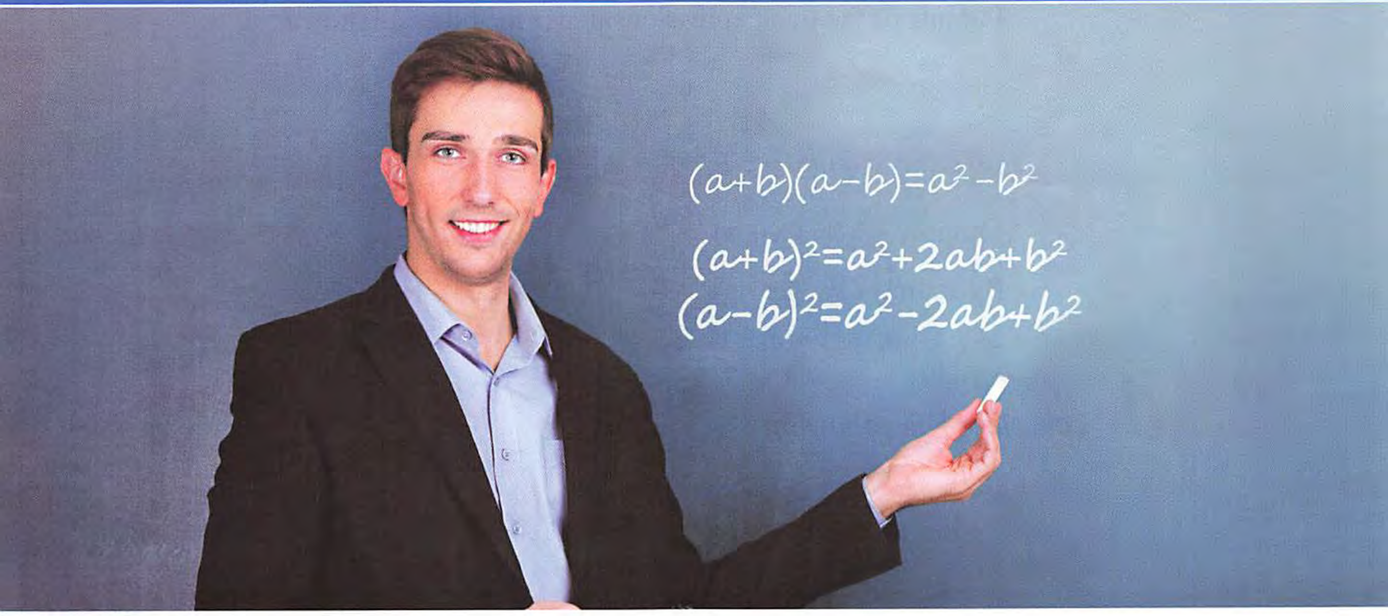
Answers of try by yourself

1 [1] $6a^2 - 12ab$ [2] $-6x^2y + 10x^2$

2 $9x^2 - x$

LESSON 6

Multiplying a Binomial by an Algebraic Expression



Multiplying two binomials

$$(a + b)(c + d) = (a \times c) + (a \times d) + (b \times c) + (b \times d)$$

first × first + first × second + second × first + second × second

We can find the product of two binomials using one of the two explained methods in the next example.

Example 1 **Find the product of :** $(X + 5)(2X - 3)$

Solution **The horizontal method :**

$$(X + 5)(2X - 3) = X(2X - 3) + 5(2X - 3)$$

$$= 2X^2 \boxed{- 3X + 10X} - 15$$

$$= 2X^2 + 7X - 15$$

Notice that : Simplifying the product to the simplest form by adding the two like terms $-3X$ and $10X$

The vertical method :

- Put one of the two expressions under the other as shown :

$$\begin{array}{r} x+5 \\ 2x-3 \\ \hline \end{array}$$

- Multiply $2x$ by $(x+5)$ ⟶ $2x^2+10x$
- Multiply -3 by $(x+5)$ ⟶ $-3x-15$
- By adding , we get ⟶ $2x^2+7x-15$

$-3x$ must be under $10x$ because they are like terms.

TRY by yourself 1

Complete the following :

1. $(3x+7)(2x-3) = \cdots\cdots - 9x + \cdots\cdots - 21$

   $= \cdots\cdots + 5x - \cdots\cdots$

2.
$$\begin{array}{r} 2x+3y \\ \times \quad x-y \\ \hline 2x^2+\cdots\cdots \quad\quad \\ -\cdots\cdots-3y^2 \\ \hline \cdots\cdots+\cdots\cdots-\cdots\cdots \end{array}$$

Multiplying by inspection

In the previous example , we found that :

$$(x+5)(2x-3) = 2x^2+7x-15$$

Noticing the product, we get :

- **The first** term $(2x^2)$ = the 1st term in the 1st expression (x) × the 1st term in the 2nd expression $(2x)$
- **The third** term (-15) = the 2nd term in the 1st expression (5) × the 2nd term in the 2nd expression (-3)
- **The middle** term $(7x)$ = the product of means $(10x)$ + the product of extremes $(-3x)$

Notice that :

- The two terms 5 and $2x$ are called the means.
- The two terms x and -3 are called the extremes.

Example 2 **Find by inspection the product of each of the following :**

1 $(2a + 3)(5a + 1)$

2 $(3X + 4)(2X - 5)$

3 $(5a - 2b)(7a - 3b)$

4 $(4X - 3y)(3y + X)$

Solution

1 $(2a + 3)(5a + 1) = \underset{\text{The first}}{\overset{\text{The first}}{\times}} + \underset{\text{means}}{\overset{\text{Product}}{\text{of}}} + \underset{\text{extremes}}{\overset{\text{Product}}{\text{of}}} + \underset{\text{The second}}{\overset{\text{The second}}{\times}}$

$$= (2a \times 5a) + (3 \times 5a + 2a \times 1) + (3 \times 1)$$

$$= 10a^2 + (15a + 2a) + 3$$

$$= 10a^2 + 17a + 3$$

By more training , we will not write the previous steps.

2 $(3X + 4)(2X - 5) = 6X^2 - 7X - 20$

3 $(5a - 2b)(7a - 3b) = 35a^2 - 29ab + 6b^2$

4 $(4X - 3y)(3y + X) = (4X - 3y)(X + 3y)$ (Rearranging terms)

$$= 4X^2 + 9Xy - 9y^2$$

TRY by yourself 2 ***Complete the missing terms in each of the following :***

1 $(2a + 1)(5a + 3) = 10a^2 +$ .... + ...

2 $(3X + 4)(2X - 1) =$ .... + ... $- 4$

UNIT 2

Two special cases

I Expanding the square of a binomial

1 $(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

$(x+y)^2 \rightarrow x^2 + 2xy + y^2$

Generally

The square of a binomial consisting of the sum of two terms

= The square of the first + 2 × The first × The second + The square of the second.

2 $(x-y)^2 = (x-y)(x-y) = x^2 - 2xy + y^2$

$(x-y)^2 \rightarrow x^2 - 2xy + y^2$

Generally

The square of a binomial consisting of the difference between two terms

= The square of the first − 2 × The first × The second + The square of the second.

Example 3 Find the expansion of each of the following :

1 $(3a+5)^2$ 2 $(2x-3y)^2$

Solution

1 $(3a+5)^2 = (3a)^2 + (2 \times 3a \times 5) + (5)^2$

$= 9a^2 + 30a + 25$

2 $(2x-3y)^2 = (2x)^2 - (2 \times 2x \times 3y) + (3y)^2$

$= 4x^2 - 12xy + 9y^2$

TRY by yourself 3 *Find the expansion of each of the following :*

1 $(3m+2)^2$ 2 $(5x-7y)^2$

II The product of the sum of two terms and the difference between them

$(a + b)(a - b) = a^2 - a b + a b - b^2 = a^2 - b^2$

Generally

The product of the sum of two terms and their difference
= the square of the first – the square of the second.

$(a + b)(a - b)$
$a^2 - b^2$

Example 4

Find the product of each of the following :

1. $(2\ell - 5)(2\ell + 5)$
2. $(5x + 3y)(5x - 3y)$
3. $(a^2 + 2b)(a^2 - 2b)$
4. $\left(\frac{1}{3}a - \frac{2}{5}b\right)\left(\frac{1}{3}a + \frac{2}{5}b\right)$

Solution

1. $(2\ell - 5)(2\ell + 5) = (2\ell)^2 - (5)^2 = 4\ell^2 - 25$
2. $(5x + 3y)(5x - 3y) = (5x)^2 - (3y)^2 = 25x^2 - 9y^2$
3. $(a^2 + 2b)(a^2 - 2b) = (a^2)^2 - (2b)^2 = a^4 - 4b^2$
4. $\left(\frac{1}{3}a - \frac{2}{5}b\right)\left(\frac{1}{3}a + \frac{2}{5}b\right) = \left(\frac{1}{3}a\right)^2 - \left(\frac{2}{5}b\right)^2 = \frac{1}{9}a^2 - \frac{4}{25}b^2$

Example 5

Put each of the following in the simplest form :

1. $(x + 4)^2 - (x + 2)(x + 6)$
2. $(x + 5)(x - 5) + (x - 5)^2$

Solution

1. $(x + 4)^2 - (x + 2)(x + 6) = (x^2 + 8x + 16) - (x^2 + 8x + 12)$
$= \cancel{x^2} + \cancel{8x} + 16 - \cancel{x^2} - \cancel{8x} - 12 = 4$
2. $(x + 5)(x - 5) + (x - 5)^2 = (x^2 - 25) + (x^2 - 10x + 25)$
$= x^2 - \cancel{25} + x^2 - 10x + \cancel{25} = 2x^2 - 10x$

TRY by yourself 4

1 *Choose the correct answer from the given ones :*

1. If $(x - 5)(x + 5) = x^2 - k$, then k = ...
(a) 5 (b) 25 (c) – 25 (d) 10

2. If $(x - 7)(x + 7) = x^2 + k$, then k = ...
(a) 7 (b) – 7 (c) 49 (d) – 49

3. The middle term in the expansion of $(3x - 4y)^2$ is ....
(a) $12xy$ (b) $-12xy$ (c) $-24xy$ (d) $24x^2y^2$

4 $(x + 4)(2x - 3) = 2x^2$ .... $- 12$

(a) $+ 5x$ (b) $- 5x$ (c) $+ 6x$ (d) $- 3x$

2 ***Put in the simplest form :*** $(3x - 2)(3x + 2) - 6$

, then find the numerical value of the result when : $x = -2$

Multiplying a binomial by an expression formed from more than two terms

As we studied before how to multiply two binomials , the operation of multiplication can be performed by one of two methods as shown in the following example and it is better to arrange the terms descendingly with respect to the indices of one of the given symbols.

Example 6 **Find the product of :** $(x - 3)(4x + x^2 - 7)$

Solution **The horizontal method :**

$$(x - 3)(x^2 + 4x - 7) = x\,(x^2 + 4x - 7) - 3\,(x^2 + 4x - 7)$$
$$= x^3 + 4x^2 - 7x - 3x^2 - 12x + 21$$
$$= x^3 + x^2 - 19x + 21$$

The vertical method :

Multiplicand ⟶ $x^2 + 4x - 7$

Multiplier ⟶ $x - 3$

Multiply (x) by multiplicand ⟶ $x^3 + 4x^2 - 7x$

Multiply (– 3) by multiplicand ⟶ $- 3x^2 - 12x + 21$

By adding , the product is ⟶ $x^3 + x^2 - 19x + 21$

In the vertical method , notice that :

- Putting the expression $x^2 + 4x - 7$ firstly because it consists of more terms than the other.
- Putting the like terms under each other during the performing of multiplying operation.

! Remark

It is better to use the vertical method for multiplying algebraic expressions forming from more than two terms.

Example 7 **Find the product of :** $3a^3 + a^2 - 4$ **by** $2a + 3$

Solution

$$\begin{array}{lllll}
3a^3 & + a^2 & - 4 & & \\
2a & + 3 & & & \\
\hline
6a^4 + & 2a^3 & & -8a & \\
 & 9a^3 + & 3a^2 & & -12 \\
\hline
6a^4 + & 11a^3 + & 3a^2 & - 8a & - 12
\end{array}$$

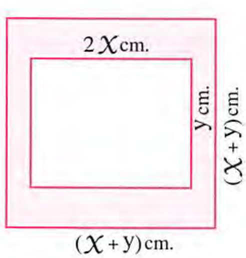
Notice that :
We leave spaces up and down the terms which have no like terms to them.

TRY by yourself 5 ***Find the product of :*** $(-3x + x^2 + 3)(x - 2)$

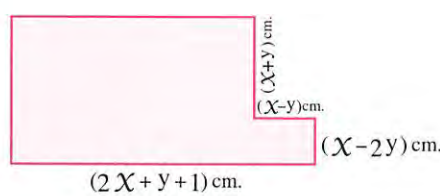
Applications on multiplying the algebraic expressions

Example 8 **Find the expression which expresses the area of the coloured part in each of the following figures :**

1



2



Solution

1 The area of the coloured part $= (x + y)^2 - 2xy$

$= x^2 + 2xy + y^2 - 2xy$

$= (x^2 + y^2)$ cm^2.

2 The area of the coloured part

$= (x + y + x - 2y)(2x + y + 1) - (x + y)(x - y)$

$= (2x - y)(2x + y + 1) - x^2 + y^2$

$= 4x^2 + 2xy + 2x - 2xy - y^2 - y - x^2 + y^2$

$= (3x^2 + 2x - y)$ cm^2.

Example 9 **Use the multiplication by inspection to find the value of each of the following easily :**

1 $(52)^2$ 2 $(195)^2$ 3 502×498

Solution

1 $(52)^2 = (2 + 50)^2 = 4 + 200 + 2500 = 2704$

2 $(195)^2 = (200 - 5)^2 = 40000 - 2000 + 25 = 38025$

3 $502 \times 498 = (500 + 2)(500 - 2) = (500)^2 - (2)^2 = 250000 - 4 = 249996$

TRY by yourself 6 ***Complete the following :***

1 $(31)^2 = (30 + \cdots\cdots)^2 = 900 + \cdots\cdots + \cdots\cdots = \cdots\cdots$

2 $(89)^2 = (\cdots\cdots - 1)^2 = \cdots\cdots - \cdots\cdots + 1 = \cdots\cdots$

3 $42 \times 38 = (40 + \cdots\cdots)(\cdots\cdots - \cdots\cdots) = \cdots\cdots - \cdots\cdots = \cdots\cdots$

Answers *of try by yourself*

1 (1) $(3x + 7)(2x - 3) = 6x^2 - 9x + 14x - 21 = \mathbf{6x^2 + 5x - 21}$

(2)
$$\begin{array}{r} 2x + 3y \\ \times \quad x - y \\ \hline 2x^2 + 3xy \\ - 2xy - 3y^2 \\ \hline 2x^2 + xy - 3y^2 \end{array}$$

2 (1) $10a^2 + 11a + 3$ (2) $6x^2 + 5x - 4$

3 (1) $9m^2 + 12m + 4$ (2) $25x^2 - 70xy + 49y^2$

4 (1) 1. (b) 2. (d) 3. (c) 4. (a)

(2) $9x^2 - 10$, the numerical value = 26

5 $x^3 - 5x^2 + 9x - 6$

6 (1) $(31)^2 = (30 + 1)^2 = 900 + 60 + 1 = 961$

(2) $(89)^2 = (90 - 1)^2 = 8100 - 180 + 1 = 7921$

(3) $42 \times 38 = (40 + 2)(40 - 2) = (40)^2 - (2)^2 = 1596$

LESSON **7**

Dividing an Algebraic Expression by a Monomial

- We know that in fractions , we can consider : $\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9}$ and also $\frac{2+5}{9} = \frac{2}{9} + \frac{5}{9}$
- We can do the same in dividing an algebraic expression by a monomial if it is not equal to zero

we can write : $\frac{6x^2 + 2xy}{2x} = \frac{6x^2}{2x} + \frac{2xy}{2x}$

which equals $= 3x + y$

Generally

When we divide an algebraic expression by a monomial , we divide each term of the expression by this monomial.

Example 1

Find the quotient in each of the following where $x \neq 0$ and $y \neq 0$:

1. $\frac{21x^2 + 14x}{7x}$
2. $(16x^3y + 8x^2y^3 - 12x^2y)$ by $(-4x^2y)$

Solution

1. $\frac{21x^2 + 14x}{7x} = \frac{21x^2}{7x} + \frac{14x}{7x} = 3x + 2$
2. $(16x^3y + 8x^2y^3 - 12x^2y) \div (-4x^2y)$

$$= \frac{16x^3y}{-4x^2y} + \frac{8x^2y^3}{-4x^2y} + \frac{-12x^2y}{-4x^2y} = -4x - 2y^2 + 3$$

Notice that :
We can check the answer by multiplying the divisor by the quotient to obtain the dividend.

Example 2 **Divide :** $\dfrac{3\,a\,b^2\,c - 5\,a^2\,b\,c + 2\,a\,b\,c^2}{a\,b\,c}$ where $a\,b\,c \neq$ zero

, then find the absolute value of the result when : $a = 1$, $b = -2$ and $c = 3$

Solution

$$\frac{3\,a\,b^2\,c - 5\,a^2\,b\,c + 2\,a\,b\,c^2}{a\,b\,c} = 3\,b - 5\,a + 2\,c$$

The absolute value $= |3 \times (-2) - 5 \times 1 + 2 \times 3| = |-6 - 5 + 6| = |-5| = 5$

TRY *by yourself*

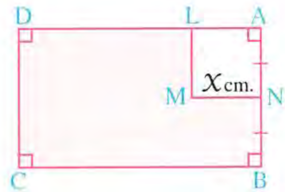
Find the quotient of each of the following where the symbols represent integers not equal to zero :

1. $(12\,x^4 + 8\,x^2) \div 4\,x$
2. $(14\,x^3 - 21\,x^2 + 7\,x) \div (-7\,x)$
3. $\dfrac{10\,a^6\,b^4 - 8\,a^5\,b^3 + 2\,a^4\,b^2}{2\,a^4\,b}$

Example 3 **In the opposite figure :**

ABCD is a rectangle , ANML is a square , N is the midpoint of $\overline{AB}$ and NM $= x$ cm. If the area of the coloured part is $(x^2 + 10\,x)$ cm^2.

, find the length of $\overline{LD}$



Solution

The area of the square $= x \times x = x^2$ cm^2.

The area of the rectangle = the area of the square + the area of the coloured part

$= x^2 + x^2 + 10\,x = (2\,x^2 + 10\,x)$ cm^2.

Since , the width of the rectangle = twice of the length of $\overline{AN} = 2\,x$ cm.

Therefore , the length of the rectangle (AD) $= \dfrac{\text{the area of the rectangle}}{\text{the width of the rectangle}}$

$= (2\,x^2 + 10\,x) \div 2\,x$

$= (x + 5)$ cm.

Therefore , LD = AD – AL $= x + 5 - x = 5$ cm.

Answers *of try by yourself*

1. $3\,x^3 + 2\,x$
2. $-2\,x^2 + 3\,x - 1$
3. $5\,a^2\,b^3 - 4\,a\,b^2 + b$

LESSON 8 Dividing an Algebraic Expression by Another One

Illustrative example

Divide : $x^2 + x - 12$ by $x + 4$ where $x \neq -4$

To operate the division , we do the following steps :

1. Divide x^2 by x , then the result is x
2. Multiply x by $(x + 4)$, then we get $x^2 + 4x$
3. Subtract $x^2 + 4x$ from $x^2 + x - 12$, then we get $-3x - 12$
4. Repeat the previous steps (in order) till the difference will be equal to zero. Then the operation of division will be finished and the quotient $= x - 3$

$x + 4$	$x^2 + x \quad - 12$
$x - 3$	$\ominus \quad \ominus$
	$x^2 + 4x$
	$-3x - 12$
	$\oplus \quad \oplus$
	$-3x - 12$
	$0 \qquad 0$

Notice that :

The like terms should be written down in one column.

! Remark

It is necessary to arrange each of the dividend and the divisor either in a descending order or in an ascending order according to the powers of the given symbol (x)

(It is preferable to arrange in a descending order)

Example 1 **Find the quotient of dividing :**

$5a - 10a^2 + 6a^3 + 3$ **by** $3 + 2a^2 - 4a$ where the divisor $\neq 0$

Solution

$$
\begin{array}{l|l}
2a^2 - 4a + 3 & 6a^3 - 10a^2 + 5a + 3 \\
\hline
3a + 1 & \quad \ominus \quad \oplus \quad \ominus \\
 & 6a^3 - 12a^2 + 9a \\
\cline{2-2}
 & \qquad 2a^2 - 4a + 3 \\
 & \qquad \ominus \quad \oplus \quad \ominus \\
 & \qquad 2a^2 - 4a + 3 \\
\cline{2-2}
 & \qquad 00 \quad 00 \quad 00
\end{array}
$$

Notice that : Each of the dividend and the divisor is in a descending order according to the powers of "a".

i.e. The quotient $= 3a + 1$

Example 2 **Find the quotient of dividing :** $x^3 + x + 10$ **by** $x + 2$ **where** $x \neq -2$

Solution

$$
\begin{array}{l|l}
x + 2 & x^3 + \qquad\qquad x + 10 \\
\hline
x^2 - 2x + 5 & \ominus \quad \ominus \\
 & x^3 + 2x^2 \\
\cline{2-2}
 & \quad -2x^2 + \quad x + 10 \\
 & \quad \oplus \quad \oplus \\
 & \quad -2x^2 - 4x \\
\cline{2-2}
 & \qquad\qquad 5x + 10 \\
 & \qquad\qquad \ominus \quad \ominus \\
 & \qquad\qquad 5x + 10 \\
\cline{2-2}
 & \qquad\qquad 00 \quad 00
\end{array}
$$

Notice that : There is no term with x^2 in the dividend , so we leave its place empty.

i.e. The quotient $= x^2 - 2x + 5$

Example 3 **If $(x - 1)$ is one of the factors of $(x^2 + 5x - 6)$, then find the other factor.**

Solution

The other factor is the quotient of dividing $x^2 + 5x - 6$ by $(x - 1)$

$$
\begin{array}{l|l}
x - 1 & x^2 + 5x - 6 \\
\hline
x + 6 & \ominus \quad \oplus \\
 & x^2 - x \\
\cline{2-2}
 & \quad 6x - 6 \\
 & \quad \ominus \quad \oplus \\
 & \quad 6x - 6 \\
\cline{2-2}
 & \quad 00 \quad 00
\end{array}
$$

i.e. The other factor is $(x + 6)$

Example 4

If the expression $(2x^3 + 11x^2 + 12x + m)$ is divisible by $(x + 3)$, find the value of m

Solution

Where the dividend is divisible by $(x + 3)$
Then, the remainder should vanish

$$
\begin{array}{r|l}
x + 3 & 2x^3 + 11x^2 + 12x + m \\
\hline
2x^2 + 5x - 3 & \underline{2x^3 + 6x^2} \\
 & \quad 5x^2 + 12x + m \\
 & \quad \underline{5x^2 + 15x} \\
 & \qquad -3x + m \\
 & \qquad \underline{-3x - 9} \\
 & \qquad m + 9
\end{array}
$$

i.e. $m + 9 = 0$
So, $m = -9$

Example 5

A rectangle whose area is $(8x^2 + 6xy - 9y^2)$ cm^2. , if its width is $(4x - 3y)$ cm. , then find its length , and calculate its perimeter when $x = 2$ and $y = 1$

Solution

The length of the rectangle = its area ÷ its width
$= (8x^2 + 6xy - 9y^2) \div (4x - 3y)$

$$
\begin{array}{r|l}
4x - 3y & 8x^2 + 6xy - 9y^2 \\
\hline
2x + 3y & \underline{8x^2 - 6xy} \\
 & \quad 12xy - 9y^2 \\
 & \quad \underline{12xy - 9y^2} \\
 & \quad 00 \qquad 00
\end{array}
$$

i.e. The length of the rectangle $= (2x + 3y)$ cm.
when $x = 2$, $y = 1$, then :
The length of the rectangle $= 2x + 3y = 2 \times 2 + 3 \times 1 = 7$ cm.
, the width of the rectangle $= 4x - 3y = 4 \times 2 - 3 \times 1 = 5$ cm.
, so the perimeter of the rectangle = (length + width) $\times 2 = (7 + 5) \times 2 = 24$ cm.

TRY by yourself

Find the quotient of dividing the following expressions , where the divisors $\neq 0$:

1. $14x^2 + 25x + 6$ ***by*** $2x + 3$
2. $2x^3 + x^2 - 19x + 10$ ***by*** $2x - 5$

Answers *of try by yourself*

1. $7x + 2$
2. $x^2 + 3x - 2$

LESSON 9

Factorization by Identifying the Highest Common Factor (H.C.F.)

Meaning of the factorization

Factorization of a number is to write it as a product of two factors or more.

For example:

- The number 24 can be factorized as the following :

$24 = 2 \times 12$ or $24 = 3 \times 8$ or $24 = 3 \times 2 \times 2 \times 2$ or ...

- Also the number 36 can be factorized as the following :

$36 = 3 \times 12$ or $36 = 6 \times 6$ or $36 = 2 \times 2 \times 3 \times 3$ or ...

Also factorization of an algebraic term is to write it as a product of two factors or more.

For example:

- The algebraic term $4x$ can be factorized as the following :

$4x = 4 \times x$ or $4x = 2 \times 2x$ or ...

- Also the algebraic term $6x^2$ can be factorized as the following :

$6x^2 = 6 \times x^2$ or $6x^2 = 2x \times 3x$ or ...

Meaning of the common factor

The common factor of two numbers is the number that divides each of the two numbers.

For example:

3 is a common factor for the two numbers 24 and 36 because it divides each of them.

$\left(\frac{24}{3} = 8 , \frac{36}{3} = 12\right)$

Also the common factor of two algebraic terms is the algebraic term that divides each of the two terms.

For example:

2 is a common factor for the two algebraic terms $4x$ and $6x^2$ $\left(\frac{4x}{2} = 2x , \frac{6x^2}{2} = 3x^2\right)$

, $2x$ is a common factor for the two algebraic terms $4x$ and $6x^2$ $\left(\frac{4x}{2x} = 2 , \frac{6x^2}{2x} = 3x\right)$

Meaning of the highest common factor

The highest common factor for two numbers is the greatest number divides each of the two numbers , and denoted by (H.C.F.)

For example:

12 is the highest common factor for the two numbers 24 and 36

The highest common factor for two algebraic terms is the greatest term divides each of the two terms , and also denoted by (H.C.F.)

For example:

$2x$ is the highest common factor for the two terms $4x$ and $6x^2$

To find the highest common factor (H.C.F.) for some algebraic terms :

1. Find the highest common factor of the numerical coefficients of these terms.
2. Take each repeated symbol in all terms with the smallest index.

For example:

The H.C.F. of the algebraic terms : $6x^2y$, $-8xy^3$, $4xyz$ is $2xy$

The method of factorization by identifying the highest common factor (H.C.F.) :

1. Find H.C.F. of the algebraic terms of the expression.
2. Put H.C.F. outside two brackets.
3. Divide each term of the algebraic expression by the H.C.F. and write the quotients inside the two brackets.

Example 1 **Factorize each of the following by identifying the highest common factor :**

1. $5a + 15b$
2. $10xy - 8xz$
3. $12x^2 - 4xy$
4. $21a^3b^2 - 7a^2b^2 - 35a^2b^3$

Solution

1. Since H.C.F. = 5 Then $5a + 15b = 5(a + 3b)$

2. Since H.C.F. = $2x$ Then $10xy - 8xz = 2x(5y - 4z)$

3. Since H.C.F. = $4x$ Then $12x^2 - 4xy = 4x(3x - y)$

4. Since H.C.F. = $7a^2b^2$

Then $21a^3b^2 - 7a^2b^2 - 35a^2b^3 = 7a^2b^2(3a - 1 - 5b)$

Example 2 **Choose the correct answer from the given ones :**

1. $8x^3 + 24y^2 = 8(\cdots\cdots\cdots)$

(a) x^3y^2 (b) $x^3 + 24y^2$ (c) $x^3 + 3y^2$ (d) $x + y$

2. $x^2y + xy^2 = \cdots\cdots\cdots (x + y)$

(a) x (b) y (c) xy (d) x^2y^2

3. If $3x + 4y = 7$, then $9x + 12y = \cdots\cdots\cdots$

(a) 3 (b) 7 (c) 10 (d) 21

4. If $x + y = 4$, $x^2 + xy = 24$, then $x = \cdots\cdots\cdots$

(a) 4 (b) 6 (c) 8 (d) 24

Solution

1 (c) 2 (c)

3 (d) The reason : $9\,x + 12\,y = 3\,(3\,x + 4\,y) = 3 \times 7 = 21$

4 (b) The reason : Since $x^2 + x\,y = 24$ i.e. $x\,(x + y) = 24$

, therefore $x \times 4 = 24$, then $x = \frac{24}{4} = 6$

TRY by yourself 1

Factorize each of the following by identifying the highest common factor :

1 $3\,x + 21\,y$

2 $2\,a^3 + 6\,a^2 - 4\,a$

3 $3\,x^2 + 15\,x\,z + 21\,x\,y^2$

4 $4\,a^2\,b^2 - 2\,a\,b + 6\,a\,b^2$

! Remark

Sometimes the H.C.F. is an algebraic expression consists of more than one algebraic term.

Example 3 **Factorize each of the following by identifying the highest common factor :**

1 $(x - y)\,(x + 3\,y) + 2\,x\,(x - y)$

2 $3\,a\,(c - d) + 4\,b\,(d - c)$

Solution

1 Since H.C.F. $= (x - y)$

Then $(x - y)\,(x + 3\,y) + 2\,x\,(x - y) = (x + 3\,y + 2\,x)\,(x - y)$

$= (3\,x + 3\,y)\,(x - y)$

$= 3\,(x + y)\,(x - y)$

2 Since $d - c = -c + d = -(c - d)$

Then $3\,a\,(c - d) + 4\,b\,(d - c) = 3\,a\,(c - d) - 4\,b\,(c - d)$

Since H.C.F. $= (c - d)$

Then $3\,a\,(c - d) - 4\,b\,(c - d) = (3\,a - 4\,b)\,(c - d)$

Example 4 **If $m - 2n = 10$, find using factorization by identifying H.C.F. the numerical value of the expression :** $3m(m - 2n) - 6n(m - 2n)$

Solution Since H.C.F. $= 3(m - 2n)$

Then $3m(m - 2n) - 6n(m - 2n) = 3(m - 2n)(m - 2n)$

$= 3 \times 10 \times 10 = 300$

Another solution :

Since $m - 2n = 10$

Then $3m(m - 2n) - 6n(m - 2n) = 3m \times 10 - 6n \times 10$

$= 30m - 60n = 30(m - 2n)$

$= 30 \times 10 = 300$

TRY by yourself 2 ***Choose the correct answer from the given ones :***

1. $3x(a + b) - 7(a + b) = \cdots\cdots$

(a) $(3x + 7)(a + b)$ (b) $(3x - 7)(a - b)$

(c) $(3x - 7)(a + b)$ (d) $(3x - 7)(-a - b)$

2. If $a + b = 7$ and $x - y = 5$, then $x(a + b) - y(a + b) = \cdots\cdots$

(a) -35 (b) 2 (c) 12 (d) 35

3. If $a - b = 3$, then $a(a - b) + b(b - a) = \cdots\cdots$

(a) -9 (b) -3 (c) 3 (d) 9

Example 5 **Use factorization by identifying the highest common factor to find the result of each of the following :**

1. $57 \times 43 - 57 \times 33$
2. $(153)^2 - 153 \times 53$
3. $4(10)^2 + 24 \times 10 - 28 \times 10$

Solution

1 Since H.C.F. = 57

Then $57 \times 43 - 57 \times 33 = 57\ (43 - 33) = 57 \times 10 = 570$

2 Since H.C.F. = 153

Then $(153)^2 - 153 \times 53 = 153\ (153 - 53) = 153 \times 100 = 15300$

3 Since H.C.F. = 4×10

Then $4\ (10)^2 + 24 \times 10 - 28 \times 10 = 4 \times 10\ (10 + 6 - 7) = 40 \times 9 = 360$

TRY by yourself 3

Use factorization by identifying the highest common factor to find the result of each of the following :

1 $47 \times 15 - 23 \times 15 + 76 \times 15$

2 $12 \times 75 + 13 \times 75 + (75)^2$

Wonders of numbers

From wonders of the **number 37** is that if it multiplied by **3** or one of its multiples up to **27**, every time you get number consisting of the same digits.

- $37 \times 3 = 111$
- $37 \times 6 = 222$
- $37 \times 9 = 333$

Try it yourself !

Answers *of try by yourself*

1 [1] $3\ (X + 7\ y)$ [2] $2\ a\ (a^2 + 3\ a - 2)$

[3] $3\ X\ (X + 5\ z + 7\ y^2)$ [4] $2\ a\ b\ (2\ a\ b - 1 + 3\ b)$

2 [1] (c) [2] (d) [3] (d)

3 [1] $15\ (47 - 23 + 76) = 15 \times 100 = 1500$ [2] $75\ (12 + 13 + 75) = 75 \times 100 = 7500$

UNIT

3 Statistics

Lessons of the unit :

1. The arithmetic mean.
2. The median.
3. The mode.

▶ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of the central tendency.
- recognize the concept of the arithmetic mean.
- calculate the arithmetic mean of a set of values.
- recognize the concept of the median.
- find the median of a set of values.
- recognize the concept of the mode.
- find the mode of a set of values.
- solve different problems on the arithmetic mean, the median and the mode.
- appreciate the role of statistics in the practical life.

Gauss

A German mathematician who developed the methods, theories and applications of statistics.

Carl Friedrich Gauss
(1777 A.D. - 1855 A.D.)

LESSON **1**

The Arithmetic Mean

Measures of central tendency

- At studying different phenomena , we find that the data of any phenomenon tend to the centering and accumulation around a certain value which is the mean of this phenomenon or measure of its tendency.

For example:

Heights of men accumulate around a certain number which is the mean of heights , also their weights and intelligence rates , and another different phenomena.

- Measures of central tendency (averages) are measures used to measure the position of centering of the data , and used to give an abbreviation description of the phenomenon which we study.

- There are many measures of central tendency , in this unit we will study three of these measures :

1. The arithmetic mean.
2. The median.
3. The mode.

WATCH VIDEO

The arithmetic mean

Definition

$$\text{The arithmetic mean of a set of values} = \frac{\text{Sum of these values}}{\text{Number of these values}}$$

Example 1 If the number of studying hours daily of a student in 6 days are :
6 , 5 , 6 , 4 , 7 and 2
, what is the arithmetic mean of number of studying hours daily of this student ?

Solution

$$\text{The arithmetic mean} = \frac{\text{Sum of number of studying hours}}{\text{Number of days}}$$

$$= \frac{6+5+6+4+7+2}{6} = \frac{30}{6} = 5 \text{ hours}$$

From the previous exmaple , notice the following :

- Number of studying hours of this student spent daily during the six days is not constant. In other words changes from a day to another , and its total number during the six days is 30 hours.
- This student can preserve the total number of studying hours during the six days (30 hours) where he spends it in a constant way daily which is 5 hours every day.
  $[5+5+5+5+5+5=6+5+6+4+7+2=30]$

i.e. The arithmetic mean of a set of values is the value if it replaced each value of the set of values , the sum of the new values equals the sum of the original values.

TRY by yourself 1 ***Find the arithmetic mean of the values :*** 3 , 8 , 11 , 4 and 9

Example 2 **If the arithmetic mean of the values : 5 , 7 , x and 9 is 6 , find the value of x**

Solution

$$\text{Since the arithmetic mean} = \frac{\text{Sum of values}}{\text{Number of values}}$$

, then $6 = \frac{5+7+x+9}{4}$, then $6 = \frac{21+x}{4}$, then $x = 3$

Example 3 **If the arithmetic mean of the values : 2 a , a + 3 , 3 a – 2 , 11 – a and 3 is 13 , find the value of a**

Solution

$$\text{Since the arithmetic mean} = \frac{\text{Sum of values}}{\text{Number of values}}$$

, then $13 = \frac{(2\,a) + (a+3) + (3\,a-2) + (11-a) + 3}{5}$

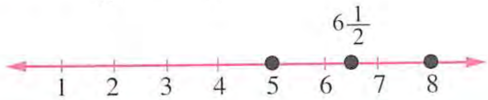
, then $13 = \frac{5\,a + 15}{5}$, then $13 = \frac{\cancel{5}\,(a+3)}{\cancel{5}}$, then $13 = a + 3$, then $a = 10$

TRY by yourself 2

If the arithmetic mean of the values : k , 3 k , 5 and 7 is 4 , then find the value of k

Example 4 **Find the arithmetic mean of the two numbers 5 and 8 , then represent the three numbers on the number line. What do you notice ?**

Solution The arithmetic mean $= \frac{5+8}{2} = 6\frac{1}{2}$



We notice that the number $6\frac{1}{2}$ lies at the middle of the distance between 5 and 8

Generally

The number that lies at the middle of the distance between two numbers is the number which represents the arithmetic mean of the two numbers.

TRY by yourself 3

Find the rational number that lies at the middle of the distance between the two numbers : $\frac{1}{6}$ and $\frac{5}{6}$

Answers of try by yourself

1 7 **2** 1 **3** $\frac{1}{2}$

LESSON 2

The Median

Definition

The median of a set of values is the value which lies exactly in the middle of the set if it is arranged ascendingly or descendingly.

i.e. The median is the value which divides the set of values into two parts such that the number of values which are greater than it is equal to the number of values which are smaller than it.

First Finding the median if the number of values is odd

If the number of values (n) is odd , then the median equals the value which lies in the middle of the values after arranging them , which is the value whose order is $\frac{n+1}{2}$

Example 1 The following are the lengths of 7 students from the students of first preparatory in centimetres : 142 , 150 , 160 , 155 , 140 , 145 and 158

What is the median length for these students ?

Solution

1 **Arrange the lengths ascendingly (or descendingly) as the following :**

140 , 142 , 145 , 150 , 155 , 158 , 160

arrange the lengths ascendingly

2 Determine the order of the median :

Where the number of values = 7 (odd number)

7 values (odd number)

140 , 142 , 145 , 150 , 155 , 158 , 160

then the order of the median = $\frac{7+1}{2} = 4$

3 Find the median :

The median length is the fourth value which equals 150 cm. (notice that there are three values smaller than it and three values greater than it)

140 , 142 , 145 , 150 , 155 , 158 , 160

median = 4^{th} value = 150

Second Finding the median if the number of values is even

If the number of values (n) is even , then the median equals the arithmetic mean of the two values which lie in the middle of the values after arranging them , and the orders of these values are $\frac{n}{2}$, $\frac{n}{2} + 1$

Example 2 The following are the marks of 8 students in an exam of mathematics :
44 , 47 , 50 , 39 , 48 , 46 , 37 and 41

What is the median mark for these students ?

Solution

1 Arrange the marks ascendingly (or descendingly) as the following :

50 , 48 , 47 , 46 , 44 , 41 , 39 , 37

arrange the marks descendingly

2 Determine the order of the median :

Where the number of values = 8 [even number]

8 values (even number)

50 , 48 , 47 , 46 , 44 , 41 , 39 , 37

then the order of the median is $\frac{8}{2}$, $\frac{8}{2} + 1$ i.e. 4 , 5

3 **Find the median :**

The median mark is the arithmetic mean of the fourth and the fifth marks which are 46 and 44 , and they are the two marks which lie in the middle of the set of marks where there are three marks greater than them and three marks smaller than them.

50 , 48 , 47 , 46 , 44 , 41 , 39 , 37

i.e. median mark = $\frac{46+44}{2} = 45$

Remarks

- The order of the median is always a positive integer.
- The value of the median may be a negative number or a fraction according to the given values.

Conclusion : **To get the median, do as follows :**

Arrange the values ascendingly or descendingly
then

If the number of values is odd

Then :
The median is the value which is in the middle exactly.

For example:

- **If the values are :**
  42 , 23 , 17 , 30 , 20
- **Then its ascending order is :**
  17 , 20 , $\boxed{23}$, 30 , 42

The median = 23

If the number of values is even

Then :
The median = $\frac{\text{The sum of the two middle values}}{2}$

For example:

- **If the values are :**
  27 , 13 , 23 , 24 , 13 , 21
- **Then its ascending order is :**
  13 , 13 , $\boxed{21 , 23}$, 24 , 27

The median = $\frac{21+23}{2} = 22$

Example 3 **Choose the correct answer from the given ones :**

1 The order of the median of the values 7 , 3 , 5 , 4 and 9 is ....

(a) the fifth. (b) the third.

(c) the fourth. (d) the second.

2 The order of the median of the values 3 , 2.4 , $5\frac{1}{2}$, 3.7 , 4.2 and 7.3 is ....

(a) the first and the second. (b) the third.

(c) the fourth. (d) the third and the fourth.

3 If the order of the median of a set of arranged values is the fifth , then the number of these values is ....

(a) 8 (b) 9 (c) 10 (d) 5

4 If the median of the values $X + 4$, $X + 1$ and $X + 5$ is 7 , then $X =$ ....

(a) 6 (b) 11 (c) 3 (d) 2

Solution

1 (b) The reason : Since the number of values is 5 , then the order of the median $= \frac{5+1}{2} = 3$

2 (d) The reason : Since the number of values is 6 , then the order of the median is $\frac{n}{2}$, $\frac{n}{2} + 1$ i.e. $\frac{6}{2} = 3$, $\frac{6}{2} + 1 = 4$

3 (b) The reason : Since the order of the median is 5 , then $\frac{n+1}{2} = 5$, then $n + 1 = 5 \times 2 = 10$, than $n = 10 - 1 = 9$

4 (c) The reason : The ascending order is $X + 1$, $X + 4$, $X + 5$, then the median is $X + 4 = 7$, then $X = 7 - 4 = 3$

TRY by yourself

1 ***Find the median of the values :*** 5 , 11 , 7 , 14 , 10

2 ***Find the median of the values :*** 2 , 6 , 1 , 8 , 4 , 10

Answers of try by yourself

1 10 2 5

LESSON 3

The Mode

Definition

The mode of a set of data is the most common data.

The mode is used as a measure of central tendency in the case of numerical data , also in the case of descriptive data.

Example 1 **Find the mode of each of the following :**

1. 5 , 8 , 7 , 5 , 6 , 8 , 5
2. very good , excellent , very good , pass , excellent , very good , pass , excellent , very good.

Solution

1. The most common value is 5

   5 , 8 , 7 , 5 , 6 , 8 , 5

   So, the mode = 5

2. The most common data is very good , then the mode is very good.

Example 2 **The following table shows the marks of 30 pupils in an examination :**

Mark	5	6	7	8	9	10
Number of pupils (frequency)	3	5	7	9	4	2

The opposite table is called simple frequency table.

Find the mode mark.

Solution From the table , we find that the greatest number of pupils obtained a mark is 9 pupils and they obtained the mark 8, then the mode = 8

Remarks

- If all of the data are different , then these data have not a mode.

  For example:

  The mode of the values : 25 , 19 , 26 , 7 , 10 , 32 and 15 is not exist because all of the values are different. In other words , there is not a value of these repeated more than the others.

- Some of data have more than one mode.

  For example:

  For the set of values : 15 , 10 , 24 , 7 , 10 , 31 and 7 , there are two values repeated more than the others and they are : 10 and 7 (each one of them repeated twice).

  i.e. This set of values has two modes which are : 10 and 7 and is called a set of two modes.

TRY by yourself

1 ***Complete :***

The mode of the values 6 , 8 , 8 , 5 , 6 , 8 is ....

2 ***The following table is the frequency table of ages in years of a group of friends :***

The age	9	10	11	12	13
Frequency	2	3	4	3	1

Find the mode.

Answers of try by yourself

1 8

2 11

Second Geometry

UNIT 4

Geometry and Measurement

Lessons of the unit :

1. Geometric concepts - The relations between the angles.
2. The relations between the angles "Follow".
3. Congruence.
4. Congruent triangles.
5. Parallelism.
6. Geometric constructions.

▶ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of each of : line segment – straight line – ray – angle.
- recognize the types of angles.
- recognize the complementary angles and the supplementary angles.
- recognize the relation between two vertically opposite angles.
- recognize the sum of measures of accumulative angles at a point.
- recognize the conditions of congruence of two polygons.
- recognize the cases of congruence of two triangles.
- solve different problems on congruence of two triangles.
- prove that two lines are parallel.
- construct a perpendicular to a straight line from a point does not belong to it.
- construct a perpendicular to a straight line from a point belongs to it.
- construct an axis of symmetry of a line segment.
- bisect an angle of a given measure.
- construct an angle to be congruent to a given angle.
- draw a straight line parallel to another straight line.

Euclid

Euclid is a Greek mathematician scientist. He lived in Alexandria.
He is considered the father of Geometry. He said that "What is made without evidence can be refused without evidence". He put some definitions such that :

- The point is what has no part.
- The straight line is length without width.

And from his axioms :

- A straight line can be drawn by joining any two points.
- A straight line segment can be extended infinitely in a straight line.
- All right angles are equal.

Euclid
(325 B.C. - 265 B.C.)

LESSON 1

Geometric Concepts - The Relations between the Angles

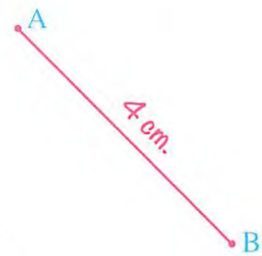
Geometric concepts

1 The line segment

WATCH VIDEO

The line segment is a set of points consisting of two distinct points and all points between them when we join them by a ruler.

- A line segment has two end points , and the symbol — on top of two letters is used to denote the line segment.
  The opposite figure represents the line segment whose end points are A and B , and is denoted by $\overline{AB}$ or $\overline{BA}$

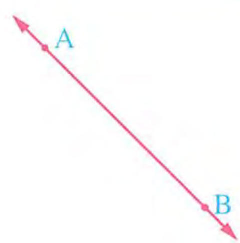


- A line segment has a length and this is the number which refers to the distance between its end points.
  If the length of the line segment whose end points are A and B is 4 cm.
  , then we write : the length of $\overline{AB}$ = 4 cm. or AB = 4 cm. or BA = 4 cm.

2 The straight line

The straight line is a line segment extended from both directions infinitely.

- A straight line does not have a starting point and an end point , and the symbol ⟷ on top of two letters is used to denote the straight line , where the two arrows show that the line can be extended without limit on both sides.
  The opposite figure represents the straight line which passes through the two points A and B , and is denoted by $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$



- The straight line is extended without limit on both sides , then it has no length.
- Through any two distinct points exactly one straight line can be drawn.

3 The ray

The ray is a line segment extended from only one of its terminals without limit.

- A ray has a starting point and it has no end point and the symbol $\rightarrow$ on top of the starting point and any other point on the ray is used to denote the ray.
- If the line segment $\overline{AB}$ is extended from its terminal B without limit in a straight line , we will get the ray $\overrightarrow{AB}$ which starts at A and passes through the point B , and it is denoted by the symbol $\overrightarrow{AB}$

- If the line segment $\overline{AB}$ is extended from its terminal A without limit in a straight line , we will get the ray $\overrightarrow{BA}$ which starts at B and passes through the point A , and it is denoted by the symbol $\overrightarrow{BA}$

- A ray extends from one of its terminals without limit , so it has no length.

Notice that :
$\overrightarrow{AB} \neq \overrightarrow{BA}$

Remarks

- Each of the line segment , the straight line and the ray is an infinite set of points.
- $\overline{AB} \subset \overrightarrow{AB}$, $\overrightarrow{AB} \subset \overleftrightarrow{AB}$ i.e. $\overline{AB} \subset \overrightarrow{AB} \subset \overleftrightarrow{AB}$

WATCH VIDEO

4 The angle

The angle is the union of two rays with the same starting point , and this point is called the vertex of the angle , and the two rays are called the two sides of the angle.

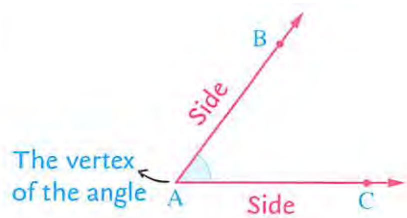
For example:

In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two rays having the same starting point A , then : $\overrightarrow{AB} \cup \overrightarrow{AC}$ = the angle CAB

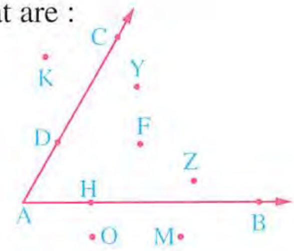
* A is the vertex of the angle CAB

* $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are the two sides of the angle CAB



- The symbol $\angle$ is used to denote an angle.
- An angle can be named by the letters of three points , one letter from one side , one letter from the other side , and the third is the vertex of the angle where the middle letter is the letter of the vertex , then we write : $\angle$ CAB or $\angle$ BAC , it can also be named by only of the vertex , then we write : $\angle$ A if no other angle shares the same vertex.

- The angle divides the plane in which it lies to three sets of points that are :

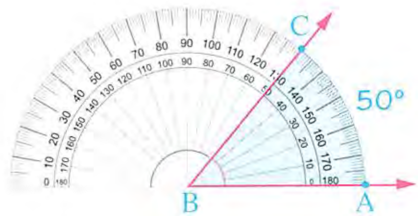


1. The set of "points of the angle" as : B , C , H , ...
2. The set of "interior points of the angle" as : F , Y , Z , ...
3. The set of "exterior points of the angle" as : M , K , O , ...

Measurement of the angle

The measurement of the angle is the number expressing the amount of happened divergence between its two sides.

- A protractor is used to measure an angle , and the angle is measured using degree unit which is denoted by (°) and the opposite figure represents an angle of measure 50° , then we write : m (∠ ABC) = 50°



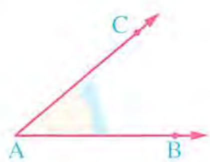
- A degree is divided into parts smaller than it , and they are minute (`) and second (``) where :
  - The degree equals 60 minutes ($1^\circ = 60'$)
  - The minute equals 60 seconds ($1' = 60''$)

The types of angles according to their measures

1 Zero angle

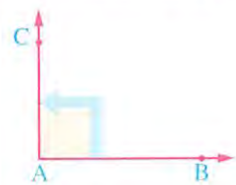
Its measure = 0°
Its sides are coincident.

2 Acute angle



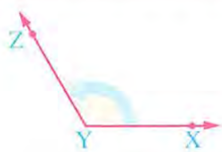
Its measure is more than 0° and less than 90°

3 Right angle



Its measure = 90°

4 Obtuse angle

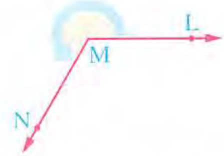


Its measure is more than 90° and less than 180°

5 Straight angle

Its measure = 180°
Its sides are forming one straight line.

6 Reflex angle



Its measure is more than 180° and less than 360°

! Remark

In the opposite figure :

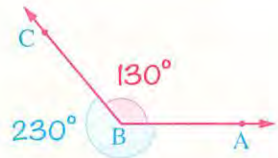
m (∠ ABC) + m (reflex ∠ ABC) = 360°

For example:

If m (∠ ABC) = 130°

, then m (reflex ∠ ABC) = 360° – 130° = 230°

m (∠ ABC) = 130°
m (reflex ∠ ABC) = ?
The sum = 360°



Example 1

Mention the type of each of the angles whose measures are as follows :

1 32°	2 90°	3 110°	4 180°
5 250°	6 $179^\circ\ 60'$	7 $180\frac{1}{4}^\circ$	8 $159\frac{3}{8}^\circ$

Solution

1 acute.	2 right.	3 obtuse.	4 straight.
5 reflex.	6 straight.	7 reflex.	8 obtuse.

Final answers of try by yourself questions are at the end of each lesson to check your answer.

TRY by yourself 1

Complete the following two tables :

1

m (∠ ABC)	45°	180°	200°	150°	90°	$94^\circ\ 10'$	$89^\circ\ 61'$
Its type	....	....	....	....	....	....	....

2

m (∠ ABC)	135°	58°	80°	100°	110°	$52^\circ\frac{1}{2}$	$89^\circ\ 60'$
m (reflex ∠ ABC)	....	....	....	....	....	....	....

Some relations between the angles

Adjacent angles

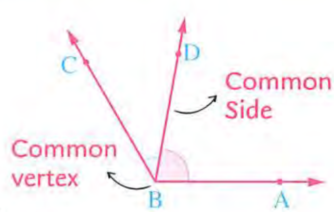
Two angles are said to be adjacent if they have a common vertex and a common side and the other two sides are on opposite sides of this common side.

For example:

In the opposite figure :

∠ ABD and ∠ DBC are two adjacent angles , for :

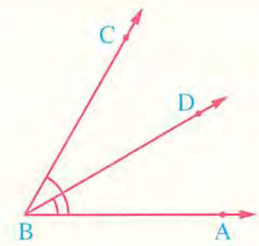
- They have a common vertex B and a common side $\overrightarrow{BD}$
- The two other sides $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are on two opposite sides of $\overrightarrow{BD}$



Remarks

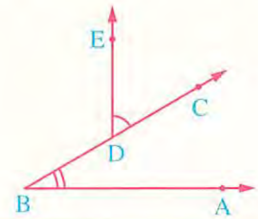
1. **In the opposite figure :**

   $\angle$ ABD and $\angle$ ABC are not two adjacent angles because the two sides $\overrightarrow{BD}$ and $\overrightarrow{BC}$ are on the same side of the common side $\overrightarrow{BA}$



2. **In the opposite figure :**

   $\angle$ ABC and $\angle$ CDE are not two adjacent angles because they have not a common vertex and also they have not a common side.



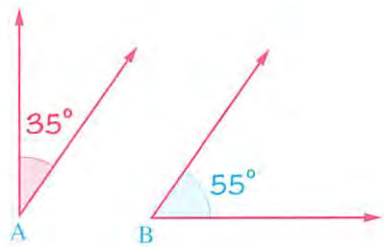
Complementary angles

Two angles are said to be complementary if the sum of their measures is 90°

For example:

The two angles whose measures are 55° and 35° are called two complementary angles because 55° + 35° = 90°

m ($\angle$ B) = 55°
m ($\angle$ A) = 35°
The sum = 90°



Remarks

1. The two complementary angles are either acute angles or one of them is zero angle and the other is a right angle.
2. The complements of the same angle (or the equal angles in measure) are equal in measure.

   i.e. If $\angle$ A complements $\angle$ B , $\angle$ C complements $\angle$ B , then m ($\angle$ A) = m ($\angle$ C)

Supplementary angles

Two angles are said to be supplementary if the sum of their measures is 180°

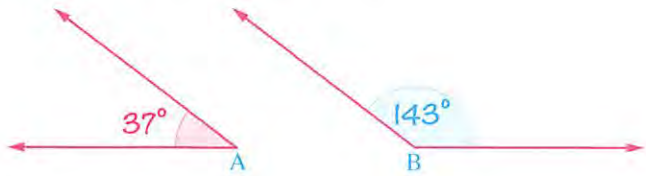
For example:

The two angles whose measures are 143° and 37°

are called two supplementary angles because 143° + 37° = 180°

m (∠ B) = 143°
m (∠ A) = 37°

The sum = 180°



Remarks

1. The two supplementary angles are either one of them is obtuse and the other is acute or each of them is a right angle or one of them is zero angle and the other is a straight angle.
2. The supplements of the same angle (or the equal angles in measure) are equal in measure.
   i.e. If ∠ A supplements ∠ B and ∠ C supplements ∠ B , then m (∠ A) = m (∠ C)

Example 2 **Choose the correct answer from the given ones :**

1. The angle whose measure is 55° complements an angle of measure ....
   (a) 35° (b) 125° (c) 110° (d) 305°
2. The angle whose measure is .... supplements an angle of measure 23°
   (a) 23° (b) 67° (c) 157° (d) 337°
3. If m (∠ X) = m (∠ Y) , ∠ X complements ∠ Y , then m (∠ Y) = ...
   (a) 90° (b) 45° (c) 180° (d) 360°
4. If ∠ A complements ∠ B , m (∠ A) = $\frac{3}{7}$ m (∠ B) , then m (∠ B) = ...
   (a) 10° (b) 27° (c) 63° (d) 126°
5. If ∠ A supplements ∠ B , m (∠ A) = 4 m (∠ B) , then m (∠ B) = ...
   (a) 36° (b) 18° (c) 144° (d) 72°

Solution

1 (a) The reason : $90° - 55° = 35°$

2 (c) The reason : $180° - 23° = 157°$

3 (b) The reason : Since $\angle X$ complements $\angle Y$

, then $m(\angle X) + m(\angle Y) = 90°$

Since $m(\angle X) = m(\angle Y)$, then $m(\angle Y) = \frac{90°}{2} = 45°$

4 (c) The reason : $m(\angle A) : m(\angle B)$: The sum

3	7	10
	?	90°

, then $m(\angle B) = \frac{7 \times 90°}{10°} = 63°$

5 (a) The reason : $m(\angle A) : m(\angle B)$: The sum

4	1	5
	?	180°

, then $m(\angle B) = \frac{1 \times 180°}{5} = 36°$

TRY by yourself 2

Complete the following :

1 The angle whose measure is 75° complements an angle of measure ....° and supplements an angle of measure ....°

2 The angle whose measure is ....° complements an angle of measure 67° and supplements an angle of measure ....°

3 The angle whose measure is ....° complements an angle of measure ....° and supplements an angle of measure 154°

The two adjacent supplementary angles

Two adjacent angles formed by a straight line and a ray with a starting point on this straight line , are supplementary.

i.e. **In the opposite figure :**

If $\overleftrightarrow{AB} \cap \overrightarrow{CD} = \{C\}$

Therefore

, $m(\angle ACD) + m(\angle DCB) = 180°$

"Straight angle"

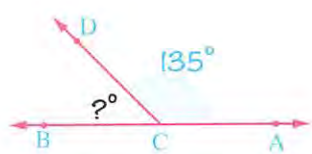
And if $m(\angle ACD) = 135°$

Then $m(\angle DCB) = 180° - 135° = 45°$

$m(\angle ACD) = 135°$

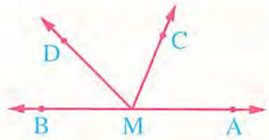
$m(\angle DCB) = ?°$

The sum $= 180°$



Remark

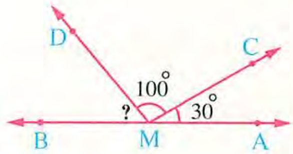
If $M \in \overleftrightarrow{AB}$, and $\overrightarrow{MC}$ and $\overrightarrow{MD}$ are drawn on one side of $\overleftrightarrow{AB}$,
then m (∠ AMC) + m (∠ CMD) + m (∠ DMB) = 180°



For example:

In the opposite figure :

If $M \in \overleftrightarrow{AB}$, m (∠ AMC) = 30°
, m (∠ CMD) = 100°
, then : m (∠ DMB) = 180° – (30° + 100°) = 180° – 130° = 50°

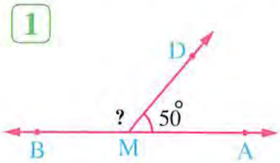


TRY by yourself 3

In each of the following figures :

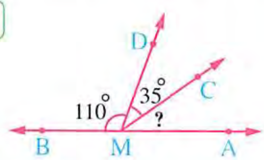
If $M \in \overleftrightarrow{AB}$, then find the measure of the angle marked by (?) :

1



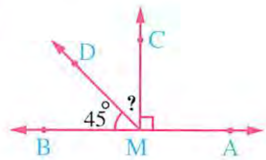
m (∠ DMB) = ...°

2



m (∠ AMC) = ...°

3



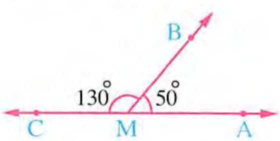
m (∠ DMC) = ...°

The two outer sides of two adjacent angles

If two adjacent angles are supplementary , then their outer sides are on the same straight line.

For example:

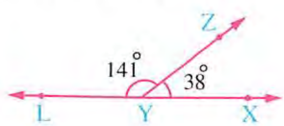
- **In the following figure :**



$\overrightarrow{MA}$ and $\overrightarrow{MC}$ are on the same straight line because :

m (∠ AMB) + m (∠ BMC) = 50° + 130°
= 180°

- **In the following figure :**



$\overrightarrow{YX}$ and $\overrightarrow{YL}$ are not on the same straight line because :

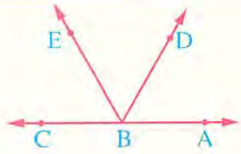
m (∠ XYZ) + m (∠ ZYL) = 38° + 141°
= 179° ≠ 180°

! Remark

In the opposite figure :

If m (∠ ABD) + m (∠ DBE) + m (∠ EBC) = 180°

, then $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are on the same straight line.



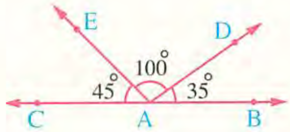
For example:

In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are on the same straight line.

because : m (∠ BAD) + m (∠ DAE) + m (∠ EAC)

= 35° + 100° + 45° = 180°



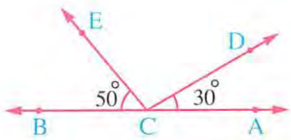
Example 3

In the opposite figure :

m (∠ ACD) = 30° , m (∠ ECB) = 50°

and m (∠ DCE) = 2 m (∠ ECB)

State with giving the reason if $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line or not.



Solution

$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.

The reason :

m (∠ DCE) = 2 × 50° = 100° because m (∠ DCE) = 2 m (∠ ECB)

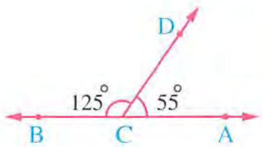
i.e. m (∠ ACD) + m (∠ DCE) + m (∠ ECB) = 30° + 100° + 50° = 180°

TRY by yourself 4

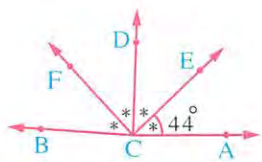
In each of the following figures ,

state if $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line or not , and why ?

1



2



Remark

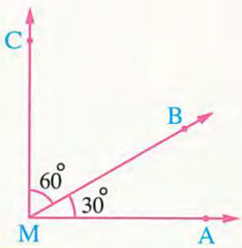
If the two adjacent angles are complementary angles , then their outer sides are perpendicular.

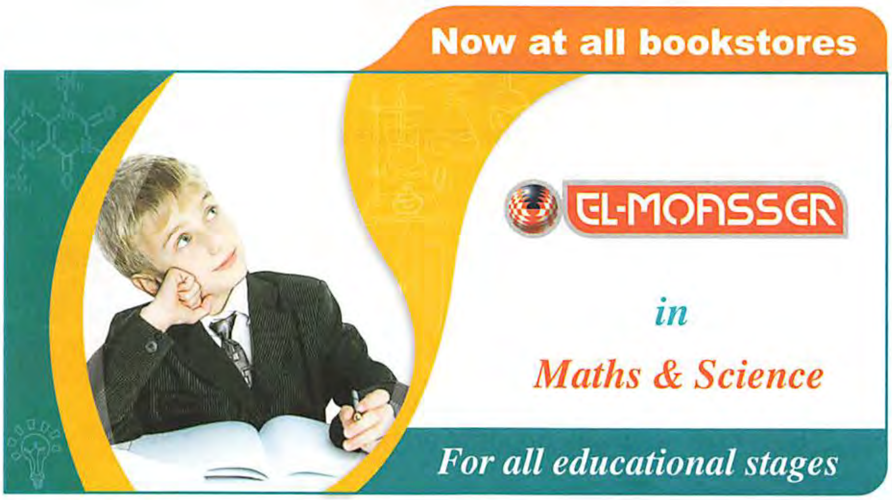
For example:

In the opposite figure :

$\overrightarrow{MA} \perp \overrightarrow{MC}$

Because : m ($\angle$ AMB) + m ($\angle$ BMC) = 30° + 60° = 90°





Answers of try by yourself

1 (1) acute , straight , reflex , obtuse , right , obtuse , obtuse

(2) 225° , 302° , 280° , 260° , 250° , $307\frac{1}{2}$° , 270°

2 (1) 15° , 105° (2) 23° , 157° (3) 26° , 64°

3 (1) 130° (2) 35° (3) 45°

4 (1) Yes , the reason : m ($\angle$ ACD) + m ($\angle$ DCB) = 180°

(2) No , the reason : m ($\angle$ ACE) + m ($\angle$ ECD) + m ($\angle$ DCF) + m ($\angle$ FCB) = 176° ≠ 180°

At the end of each lesson , you will find the final answers of try by yourself questions in the same form.

LESSON 2 The Relations between the Angles (Follow)

Vertically opposite angles (V.O.A.)

If two straight lines intersect , then the measures of each two vertically opposite angles are equal.

In the opposite figure :

If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at M

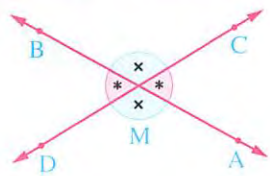
Then :

- ∠ AMC and ∠ BMD are vertically opposite angles

  , then m (∠ AMC) = m (∠ BMD)

- Also , ∠ CMB and ∠ AMD are vertically opposite angles

  , then m (∠ CMB) = m (∠ AMD)



For example:

In the opposite figure :

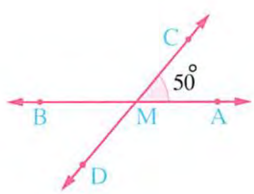
If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

, m (∠ AMC) = 50°

, then m (∠ DMB) = m (∠ AMC) = 50° (vertically opposite angles)

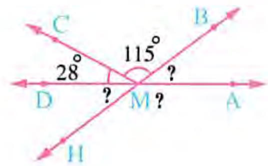
, m (∠ CMB) = 180° – m (∠ AMC) = 180° – 50° = 130°

, then m (∠ AMD) = m (∠ CMB) = 130° (vertically opposite angles)



Example 1

In the opposite figure :



$\overleftrightarrow{AD} \cap \overleftrightarrow{BH} = \{M\}$,

m (∠ CMD) = 28° and m (∠ BMC) = 115°

Find the measures of the angles marked by (?)

Solution

- m (∠ AMB) = 180° – (115° + 28°) = 180° – 143° = 37°
  Because : m (∠ AMB) + m (∠ BMD) = 180°
- m (∠ DMH) = 37°
  Because : m (∠ DMH) = m (∠ AMB) (vertically opposite angles)
- m (∠ AMH) = 143°
  Because : m (∠ AMH) = m (∠ BMD) (vertically opposite angles)

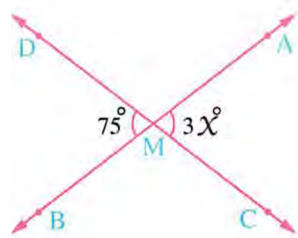
Example 2

Choose the correct answer from the given ones :

1 If the two vertically opposite angles are supplementary , then the measure of each one is ....

(a) 180° (b) 90° (c) 45° (d) 60°

2 **In the opposite figure :**



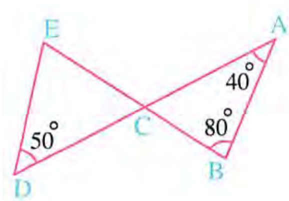
$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, m (∠ DMB) = 75°

, m (∠ AMC) = 3 $\mathcal{X}$° , then $\mathcal{X}$ = ...

(a) 20° (b) 25°

(c) 75° (d) 100°

3 **In the opposite figure :**



$\overline{AD} \cap \overline{BE} = \{C\}$

, then m (∠ E) = ...

(a) 40° (b) 50°

(c) 70° (d) 80°

Solution

1 (b) The reason : Since the two angles are supplementary

, then the sum of their measures is 180°

Since they are vertically opposite angles

, then they are equal in measure

i.e. The measure of each one = $\frac{180°}{2}$ = 90°

2 (b) The reason : Since m (∠ AMC) = m (∠ DMB)

(vertically opposite angles) , then $3\,x = 75°$

i.e. $x = \frac{75°}{3} = 25°$

3 (c) The reason : In Δ ABC : m (∠ ACB) = 180° – (40° + 80°) = 60°

, then m (∠ ECD) = m (∠ ACB) = 60°

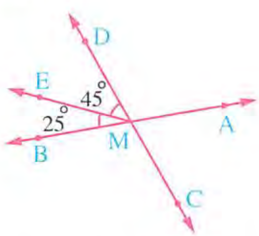
(vertically opposite angles)

, in Δ ECD : m (∠ E) = 180° – (50° + 60°) = 70°

TRY by yourself 1 ***In each of the following figures :***

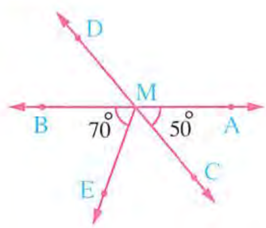
If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, find the measure of the required angle under each figure :

1



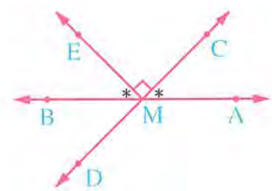
m (∠ AMC) = ...°

2



m (∠ DME) = ...°

3



m (∠ BMD) = ...°

Accumulative angles at a point

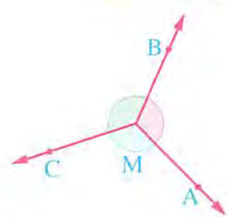
The sum of the measures of the accumulative angles at a point is 360°

In the opposite figure :

WATCH VIDEO

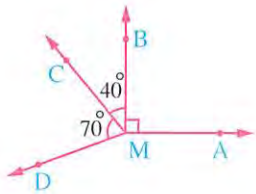
If $\overrightarrow{MA}$, $\overrightarrow{MB}$ and $\overrightarrow{MC}$ are rays having the same starting point M , then the angles ∠ AMB , ∠ BMC and ∠ CMA are called accumulative angles at the point M and

m (∠ AMB) + m (∠ BMC) + m (∠ CMA) = 360°



For example:

In the opposite figure :



If $\overrightarrow{MA}$, $\overrightarrow{MB}$, $\overrightarrow{MC}$ and $\overrightarrow{MD}$ are rays having the same starting point M

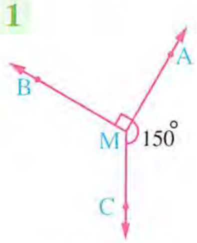
, then m (∠ AMB) + m (∠ BMC) + m (∠ CMD) + m (∠ DMA) = 360°

So , m (∠ DMA) = 360° – (90° + 40° + 70°) = 160°

Example 3

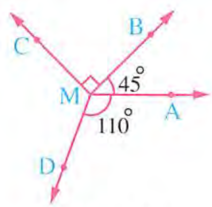
In each of the following figures , find the measure of the required angle under each figure :

1



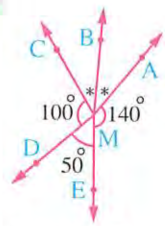
m (∠ BMC) = ... °

2



m (∠ CMD) = ... °

3



m (∠ AMB) = ... °

Solution

1 m (∠ BMC) = 360° – (150° + 90°) = 120°

2 m (∠ CMD) = 360° – (110° + 90° + 45°) = 115°

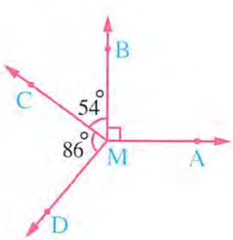
3 m (∠ AMC) = 360° – (140° + 50° + 100°) = 70°

, m (∠ AMB) = m (∠ BMC) = $\frac{70°}{2}$ = 35°

TRY by yourself 2

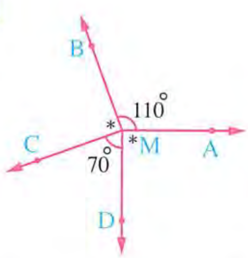
In each of the following figures , find the measure of the required angle under each figure :

1



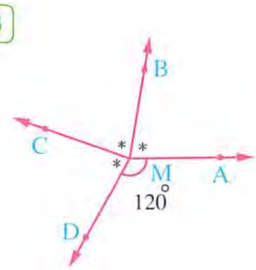
m (∠ AMD) = ... °

2



m (∠ AMD) = ... °

3

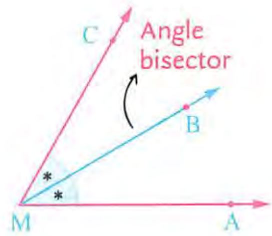


m (∠ BMD) = ... °

The angle bisector

It is the ray that divides the angle into two halves (two equal angles in measure)

In the opposite figure :



$\overrightarrow{MB}$ bisects $\angle$ AMC

i.e. m ($\angle$ AMB) = m ($\angle$ BMC) = $\frac{1}{2}$ m ($\angle$ AMC)

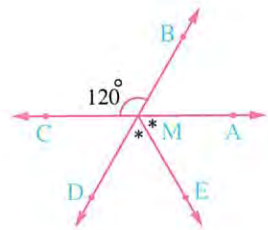
or m ($\angle$ AMC) = 2 m ($\angle$ AMB) = 2 m ($\angle$ BMC)

For example:

If m ($\angle$ AMB) = 30°

, then m ($\angle$ AMC) = 60°

Example 4 **In the opposite figure :**



$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$, m ($\angle$ BMC) = 120°

and $\overrightarrow{ME}$ bisects $\angle$ AMD

Find : m ($\angle$ EMC)

Solution

m ($\angle$ AMD) = 120°

Because : m ($\angle$ AMD) = m ($\angle$ BMC) (vertically opposite angles)

, m ($\angle$ EMD) = 60°

Because : $\overrightarrow{ME}$ bisects $\angle$ AMD

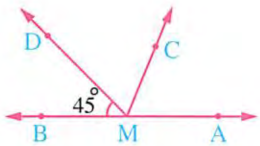
, m ($\angle$ CMD) = 180° – 120° = 60°

, then m ($\angle$ EMC) = 60° + 60° = 120°

TRY *by yourself* **3** ***In each of the following figures :***

If $\overrightarrow{MC}$ bisects $\angle$ AMD , find the measure of the required angle under each figure :

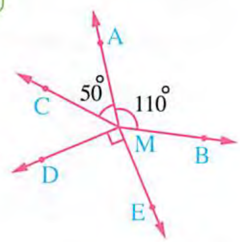
1



If M $\in \overleftrightarrow{AB}$, then

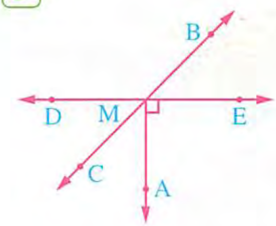
m ($\angle$ DMC) = ...°

2



m ($\angle$ EMB) = ...°

3



m ($\angle$ BME) = ...°

Wonders of numbers

Think of a number, add 2, multiply the sum by 3, subtract 6 from the product and divide the result by 3.

The answer is the same number you have chosen !

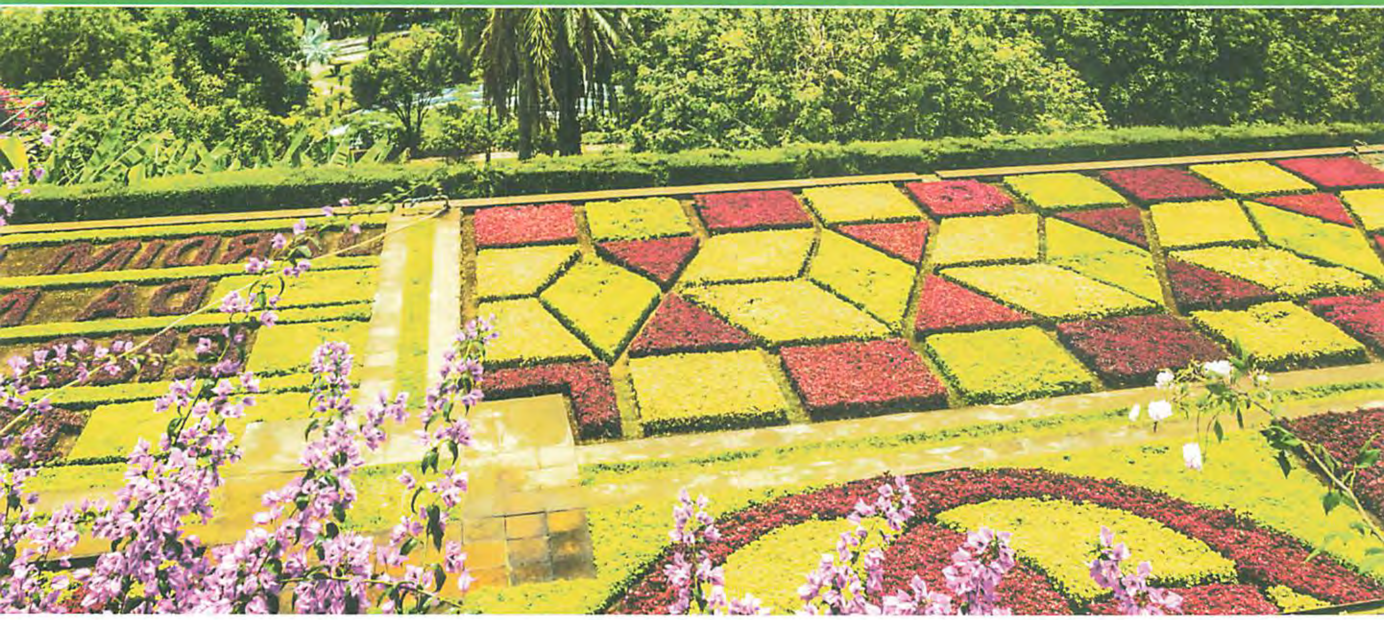
Try with your friend

Answers of try by yourself

1	(1) 70°	(2) 120°	(3) 45°
2	(1) 130°	(2) 90°	(3) 160°
3	(1) 67.5°	(2) 60°	(3) 45°

LESSON 3

Congruence

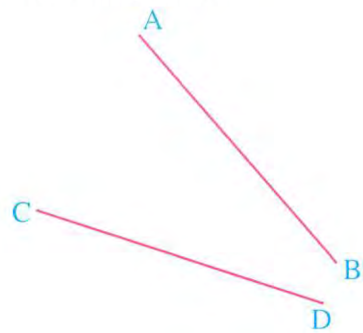


Two geometric figures are congruent if they are fit exactly on top of each other. We use the symbol ≡ to represent the congruence , and the following examples of congruence of some geometric figures :

First Congruence of two line segments

In the opposite figure :

The two line segments $\overline{AB}$ and $\overline{CD}$ are congruent and by measuring we find that they are equal in length and the length of each one is 4 cm.



Generally

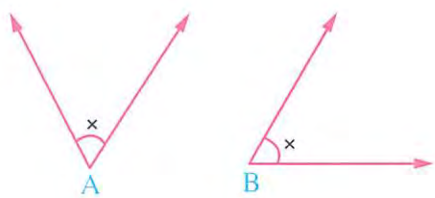
Two line segments are congruent if they are equal in length.

If the length of $\overline{XY}$ = the length of $\overline{ZL}$, then $\overline{XY} \equiv \overline{ZL}$

Second Congruence of two angles

In the opposite figure :

The two angles ∠ A and ∠ B are congruent and by measuring we find that they are equal in measure and the measure of each angle is 60°



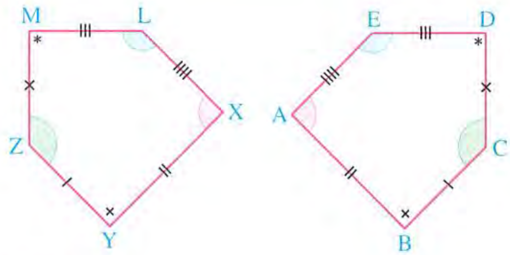
Generally

Two angles are congruent if they are equal in measure.

If m (∠ C) = m (∠ D) , then ∠ C ≡ ∠ D

Third Congruence of two polygons

Two polygons are congruent if there is correspondence between their vertices such that each side and each angle in the first polygon is congruent to its corresponding element in the other polygon.



For example:

The two opposite polygons are congruent because :

each two corresponding sides are equal in length.

i.e. AB = XY , BC = YZ , CD = ZM ,

DE = ML and EA = LX

and each two corresponding angles are equal in measure.

i.e. m (∠ A) = m (∠ X) , m (∠ B) = m (∠ Y) , m (∠ C) = m (∠ Z) ,

m (∠ D) = m (∠ M) and m (∠ E) = m (∠ L)

and we write the polygon ABCDE ≡ the polygon XYZML

! Remark

It is better to write the name of the two congruent polygons in the same order of their corresponding vertices.

For example:

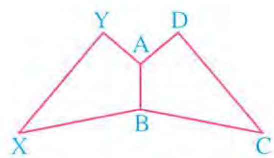
- The vertex A $\xleftrightarrow{\text{Corresponds to}}$ the vertex X
- The vertex B $\xleftrightarrow{\text{Corresponds to}}$ the vertex Y
- The vertex C $\xleftrightarrow{\text{Corresponds to}}$ the vertex Z
- The vertex D $\xleftrightarrow{\text{Corresponds to}}$ the vertex M
- The vertex E $\xleftrightarrow{\text{Corresponds to}}$ the vertex L

! Remark

If the two polygons are congruent , then each side and each angle in one of them is congruent to its corresponding element in the other polygon.

For example:

If the figure ABCD ≡ the figure ABXY , then :



1. BC = BX , AD = AY , CD = XY
2. m (∠ D) = m (∠ Y) , m (∠ C) = m (∠ X) ,

   m (∠ DAB) = m (∠ YAB)

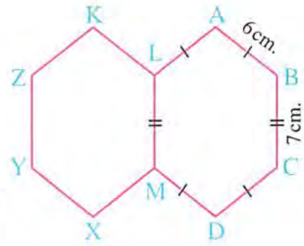
   and m (∠ ABC) = m (∠ ABX)

Notice that :

$\overleftrightarrow{AB}$ is the axis of symmetry of the polygon CDAYXB and divides it into two congruent polygons.

Example

In the opposite figure :



If the polygon ABCDML ≡ the polygon KZYXML

, AB = CD = AL = DM = 6 cm.

and BC = LM = 7 cm.

1. Write what you deduce from congruence of the two polygons.
2. Find the perimeter of the polygon MXYZKL

Solution

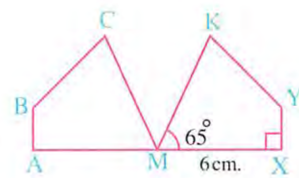
1. We deduce from the congruence of the two polygons ABCDML and KZYXML that :
   - The corresponding sides are equal in length.

     i.e. KL = AL = 6 cm. , KZ = AB = 6 cm. , ZY = BC = 7 cm.
     , YX = CD = 6 cm. and XM = DM = 6 cm.
   - The corresponding angles are equal in measure.

     i.e. m (∠ K) = m (∠ A) , m (∠ Z) = m (∠ B) , m (∠ Y) = m (∠ C) ,
     m (∠ X) = m (∠ D) , m (∠ XML) = m (∠ DML)
     and m (∠ MLK) = m (∠ MLA)
2. The perimeter of the polygon MXYZKL
   = MX + XY + YZ + ZK + KL + LM = 6 + 6 + 7 + 6 + 6 + 7 = 38 cm.

TRY by yourself

In the opposite figure :



If M ∈ $\overleftrightarrow{AX}$, m (∠ XMK) = 65° , $\overleftrightarrow{XY} \perp \overleftrightarrow{XM}$

the figure XYKM ≡ the figure ABCM and XM = 6 cm.

, ***complete the following :***

1 $\overline{XY}$ ≡	2 YK = ...	3 AM = ... cm.
4 AX = ... cm.	5 m (∠ Y) = m (∠ ....)	6 m (∠ X) = m (∠ ....)
7 m (∠ CMA) = ...°	8 m (∠ A) = ...°	9 m (∠ KMC) = ...°
10 m (∠ CMX) = ...°		

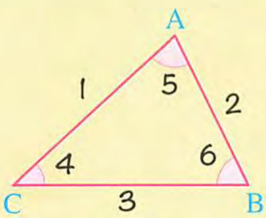
Answers of try by yourself

1 $\overline{AB}$ 2 BC 3 6 4 12 5 B

6 A 7 65° 8 90° 9 50° 10 115°

LESSON **4** | # Congruent Triangles

We know that any triangle has three sides and three angles , these three sides and three angles are known as the six elements of the triangle.



Six elements of the triangle ABC

For example:

The six elements of the triangle ABC are

three sides : $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

and three angles :

$\angle$ A , $\angle$ B and $\angle$ C

The two triangles are congruent if each element of the 6 elements of one of them is congruent to the corresponding element in the other triangle.

For example:

If ABC and XYZ are two triangles in which :

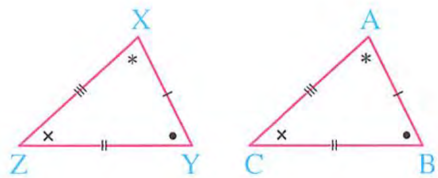
1 AB = XY , AC = XZ

and BC = YZ

2 m ($\angle$ A) = m ($\angle$ X) , m ($\angle$ B) = m ($\angle$ Y)

and m ($\angle$ C) = m ($\angle$ Z)

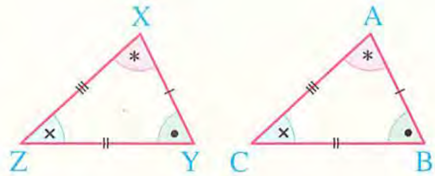
, then Δ ABC $\equiv$ Δ XYZ



! Remarks

❶ In the two previous triangles , we notice that :

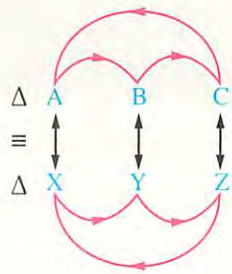
- The vertex X $\xleftrightarrow{\text{Corresponds to}}$ the vertex A
- The vertex Y $\xleftrightarrow{\text{Corresponds to}}$ the vertex B
- The vertex Z $\xleftrightarrow{\text{Corresponds to}}$ the vertex C



and when we write two congruent triangles , it is better to write them in the same order of their corresponding vertices.

$\Delta ABC \equiv \Delta XYZ$ or $\Delta ACB \equiv \Delta XZY$ or ...

❷ If two triangles are congruent , then each element of the six elements of one of the two triangles is congruent to the corresponding element of the other triangle.



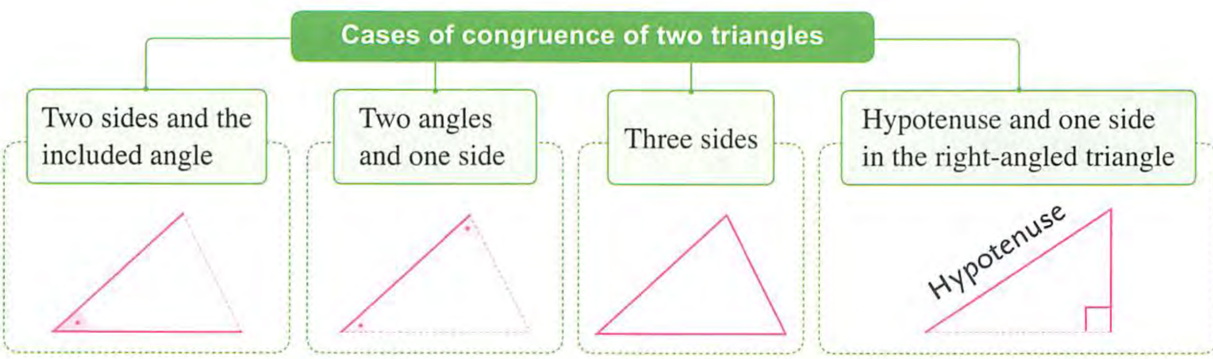
i.e. If $\Delta ABC \equiv \Delta XYZ$, then we deduce that :

- First : $\overline{AB} \equiv \overline{XY}$, $\overline{BC} \equiv \overline{YZ}$ and $\overline{CA} \equiv \overline{ZX}$
- Second : $\angle A \equiv \angle X$, $\angle B \equiv \angle Y$ and $\angle C \equiv \angle Z$

Cases of congruence of two triangles

From the previous , we knew that the two triangles be congruent when each element of the six elements of one is congruent to the corresponding element of the other triangle , and in the following we will study how to prove that two triangles are congruent by proving that three elements only of one of them are congruent to the corresponding elements of the other , in this case , the three other elements are congruent of the two triangles.

In the following , the different cases of congruence of two triangles :



The first case (Two sides and the included angle S.A.S.)

Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

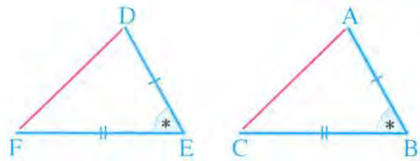
For example:

If ABC and DEF are two triangles in which :

$$\begin{cases} \overline{AB} \equiv \overline{DE} \\ \overline{BC} \equiv \overline{EF} \\ \angle B \equiv \angle E \end{cases}$$

, then $\Delta\, ABC \equiv \Delta\, DEF$ and we deduce that :

$$\begin{cases} \overline{AC} \equiv \overline{DF} \\ \angle A \equiv \angle D \\ \angle C \equiv \angle F \end{cases}$$



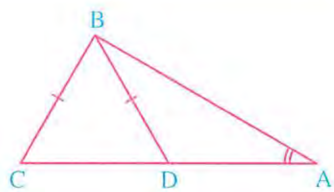
Remark

In the case of congruence of two triangles by two sides and an angle , the angle should be included between the two sides.

For example:

Although ABC and ABD are two triangles in which :

$$\begin{cases} BC = BD \\ \overline{AB} \text{ is a common side} \\ \angle A \text{ is a common angle} \end{cases}$$



but it is clear that $\Delta\, ABC$ is not congruent to $\Delta\, ABD$ because $\angle A$ is not included between the two sides in each of the two triangles.

The second case (Two angles and one side A.S.A.)

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.

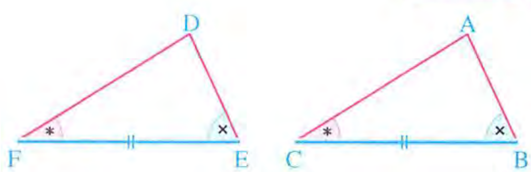
For example:

If ABC and DEF are two triangles in which :

$$\begin{cases} \overline{BC} \equiv \overline{EF} \\ \angle B \equiv \angle E \\ \angle C \equiv \angle F \end{cases}$$

, then $\Delta\, ABC \equiv \Delta\, DEF$ and we deduce that :

$$\begin{cases} \overline{AB} \equiv \overline{DE} \\ \overline{AC} \equiv \overline{DF} \\ \angle A \equiv \angle D \end{cases}$$



The third case (Three sides S.S.S.)

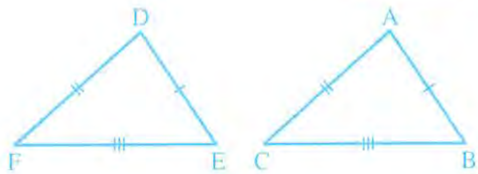
Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.

WATCH VIDEO

For example:

If ABC and DEF are two triangles in which :

$$\begin{cases} \overline{AB} \equiv \overline{DE} \\ \overline{BC} \equiv \overline{EF} \\ \overline{AC} \equiv \overline{DF} \end{cases}$$



, then Δ ABC $\equiv \Delta$ DEF and we deduce that :

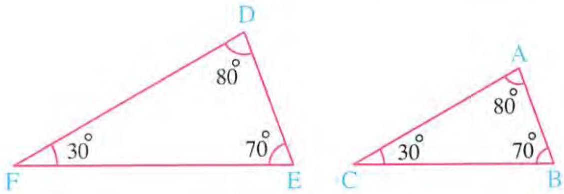
$$\begin{cases} \angle A \equiv \angle D \\ \angle B \equiv \angle E \\ \angle C \equiv \angle F \end{cases}$$

! Remark

If each angle of one triangle is congruent to the corresponding angle of the other triangle , it is not necessary for the two triangles to be congruent.

For example:

Although the measures of the corresponding angles of the two triangles ABC and DEF are equal , but it is clear that the two triangles are not congruent.



The fourth case (Hypotenuse and one side in the right-angled triangle R.H.S.)

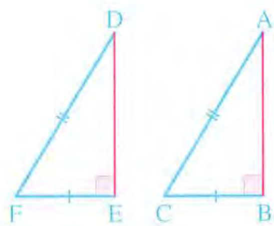
Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

WATCH VIDEO

For example:

If ABC and DEF are two triangles in which :

$$\begin{cases} \overline{AC} \equiv \overline{DF} \\ \overline{BC} \equiv \overline{EF} \\ m(\angle B) = m(\angle E) = 90^\circ \end{cases}$$



, then Δ ABC $\equiv \Delta$ DEF and we deduce that :

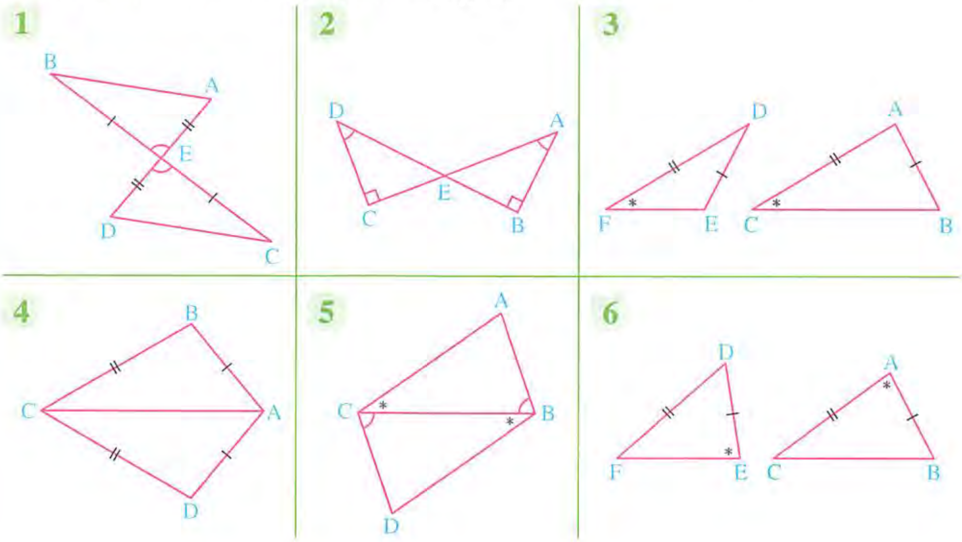
$$\begin{cases} \overline{AB} \equiv \overline{DE} \\ \angle A \equiv \angle D \\ \angle C \equiv \angle F \end{cases}$$

! Remark

The two right-angled triangles are congruent if the two sides of the right angle in one of them are congruent to the corresponding elements in the other triangle. (This case is an application of the first case of congruence of two triangles)

Example 1

In each of the following figures , show if the two triangles are congruent or not , give reason (Given that the similar signs denote the congruency of the elements marked by these signs)



Solution

1. The two triangles are congruent (two sides and the included angle "S.A.S.")
2. The given data is not enough to prove the congruence of the two triangles.
3. The two triangles are not congruent because the given angle is not included between the two sides.
4. The two triangles are congruent (three sides "S.S.S.")
5. The two triangles are congruent (two angles and a side "A.S.A.")
6. The two triangles are not congruent because the two congruent angles are not corresponding.

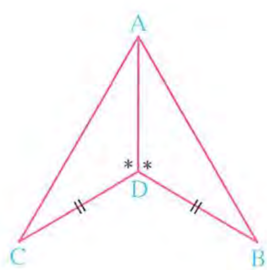
Example 2

In the opposite figure :

BD = CD and m (∠ ADB) = m (∠ ADC)

Is Δ ABD ≡ Δ ACD ?

, then explain why $\overrightarrow{AD}$ bisects ∠ BAC



Solution

Yes , Δ ABD $\equiv$ Δ ACD "two sides and included angle" we deduce from the congruence that : m ($\angle$ BAD) = m ($\angle$ CAD)

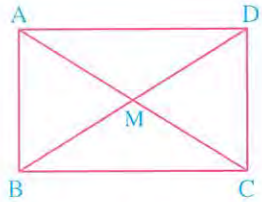
i.e. $\overrightarrow{AD}$ bisects $\angle$ BAC

Example 3

In the opposite figure :

ABCD is a rectangle whose diagonals intersect at M

Is Δ ABC $\equiv$ Δ DCB ? Why ?



Solution

Yes , Δ ABC $\equiv$ Δ DCB , because : m ($\angle$ ABC) = m ($\angle$ DCB) = 90°

, AC = DB (two diagonals of the rectangle)

and $\overline{BC}$ is a common side.

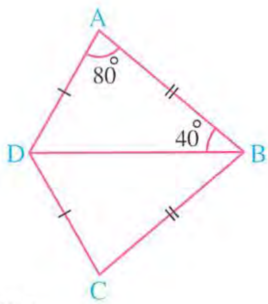
Example 4

In the opposite figure :

BA = BC , DA = DC ,

m ($\angle$ ABD) = 40° and m ($\angle$ BAD) = 80°

Find : m ($\angle$ ADC) showing the steps of the solution.



Solution

In Δ ABD , since m ($\angle$ ABD) = 40° , m ($\angle$ BAD) = 80°

Then m ($\angle$ ADB) = 180° – (40° + 80°) = 60°

Since Δ ABD $\equiv$ Δ CBD (S.S.S.)

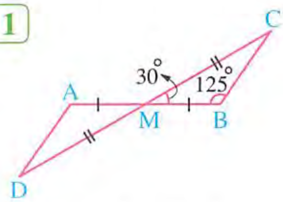
Then m ($\angle$ ADB) = m ($\angle$ CDB) = 60°

Then m ($\angle$ ADC) = 60° + 60° = 120°

TRY by yourself

In each of the following figures, find the required under each figure :

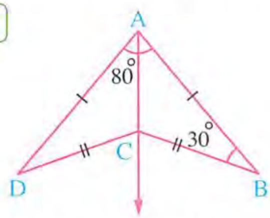
1



$\overline{AB} \cap \overline{CD} = \{M\}$

m ($\angle$ D) = ... °

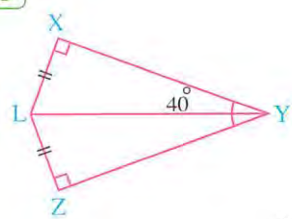
2



m ($\angle$ D) = ... °

, m ($\angle$ BAC) = ... °

3



m ($\angle$ XLY) = ... °

Answers of try by yourself

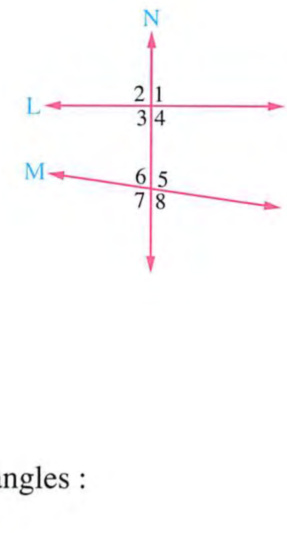
1 25° **2** 30° , 40° **3** 70°

LESSON 5

Parallelism

Angles formed from two straight lines and a transversal

In the opposite figure :



The straight line N intersects the two straight lines L and M

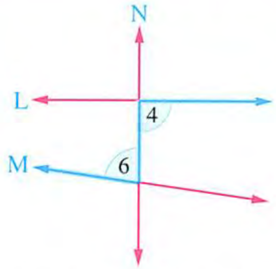
The straight line N is called "a transversal".

In this case , we get eight angles (at each point of intersection four angles are formed) and these eight angles could be classified according to their position relative to the transversal as follows :

- Alternate angles.
- Corresponding angles.
- Interior angles on the same side of the transversal.

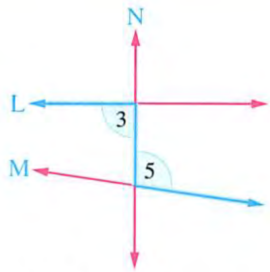
In the following , we will represent each pair of the previous pairs of angles :

1 Pairs of alternate angles



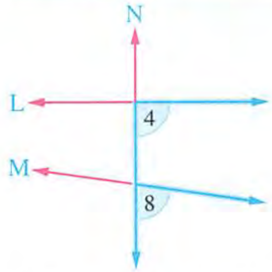
$\angle$ 4 and $\angle$ 6 are alternate angles.

Alternate angles are identified using a"Z"

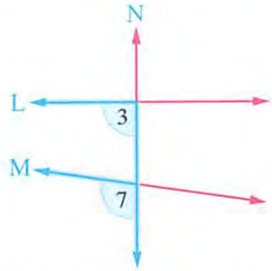


$\angle$ 3 and $\angle$ 5 are alternate angles.

2 Pairs of corresponding angles

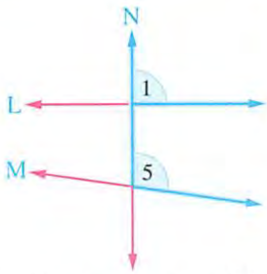


∠ 4 and ∠ 8 are corresponding angles.

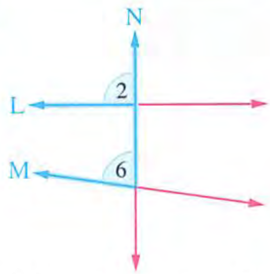


∠ 3 and ∠ 7 are corresponding angles.

Corresponding angles are identified using a "F"

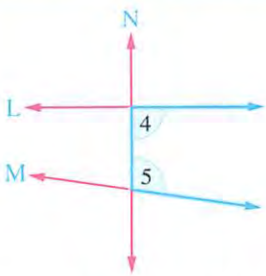


∠ 1 and ∠ 5 are corresponding angles.



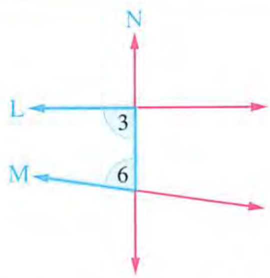
∠ 2 and ∠ 6 are corresponding angles.

3 Pairs of interior angles on the same side of the transversal



∠ 4 and ∠ 5 are interior angles on the same side of the transversal.

interior angles are identified using a ⊏ or a ⊐

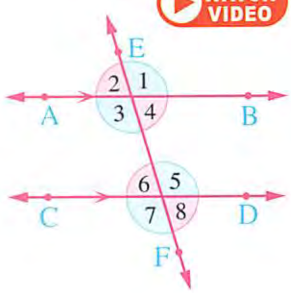


∠ 3 and ∠ 6 are interior angles on the same side of the transversal.

Relation between pairs of angles formed from two parallel straight lines and a transversal to them

WATCH VIDEO

If two parallel straight lines are intersected by a transversal , then any two result angles from the intersection either congruent or supplementary.



For example:

If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ is a transversal to them

by measuring , you find that :

1

- $m(\angle 3) = m(\angle 5)$
- $m(\angle 4) = m(\angle 6)$

Generally

If a straight line intersects two parallel straight lines , then each two alternate angles are equal in measure.

2

- $m(\angle 1) = m(\angle 5)$
- $m(\angle 2) = m(\angle 6)$
- $m(\angle 3) = m(\angle 7)$
- $m(\angle 4) = m(\angle 8)$

Generally

If a straight line intersects two parallel straight lines , then each two corresponding angles are equal in measure.

3

- $m(\angle 3) + m(\angle 6) = 180°$
- $m(\angle 4) + m(\angle 5) = 180°$

Generally

If a straight line intersects two parallel straight lines , then each two interior angles in the same side of the transversal are supplementary.

Example 1

In each of the following figures , find the measure of the angle which is marked by " ? " giving reason.

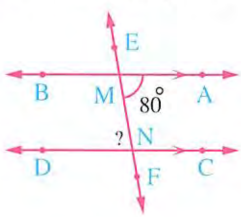


Fig. (1)

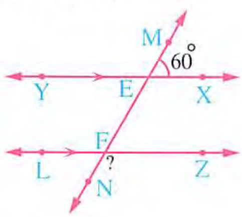


Fig. (2)

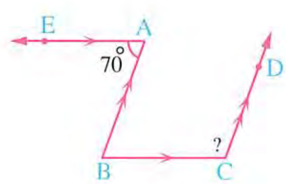


Fig. (3)

Solution

Fig. (1) : $m(\angle MND) = 80°$

because : $m(\angle MND) = m(\angle AMN)$ (alternate angles)

Fig. (2) : $m(\angle ZFN) = 120°$

because : $m(\angle ZFE) = m(\angle XEM) = 60°$ (corresponding angles)

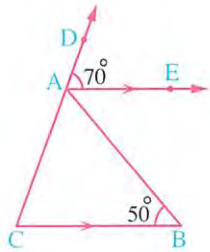
Then : $m(\angle ZFN) = 180° - 60° = 120°$

Fig. (3) : $m(\angle BCD) = 110°$

because : $m(\angle B) = m(\angle A) = 70°$ (alternate angles)

, since $\angle B$, $\angle BCD$ are two interior angles in the same side of the transversal , then $m(\angle BCD) = 180° - 70° = 110°$

Example 2 **In the opposite figure : $\overrightarrow{AE} \parallel \overline{BC}$, D $\in \overrightarrow{CA}$, m ($\angle$ DAE) = 70° and m ($\angle$ B) = 50° Find giving reason :**



1. m ($\angle$ EAB)
2. m ($\angle$ C)
3. m ($\angle$ EAC)

Solution

1. m ($\angle$ EAB) = 50° because : m ($\angle$ EAB) = m ($\angle$ B) (alternate angles)
2. m ($\angle$ C) = 70° because : m ($\angle$ C) = m ($\angle$ EAD) (corresponding angles)
3. m ($\angle$ EAC) = 110° because : $\angle$ EAC , $\angle$ C are two interior angles in the same side of the transversal , then m ($\angle$ EAC) = 180° – 70° = 110°

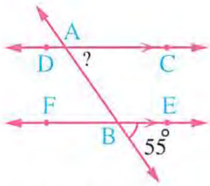
   or : because : m ($\angle$ DAE) + m ($\angle$ EAC) = 180°

   , then m ($\angle$ EAC) = 180° – 70° = 110°

TRY by yourself 1

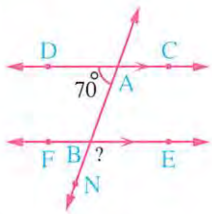
In each of the following figures , find the measure of the angle which is written under each figure :

1.



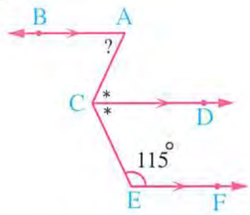
m ($\angle$ CAB) = ...°

2.



m ($\angle$ EBN) = ...°

3.



m ($\angle$ A) = ...°

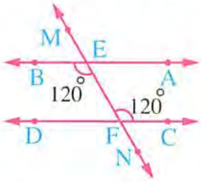
How to prove that two straight lines are parallel ?

The two straight lines are parallel if a third straight line intersects them (as a transversal) and one of the following cases is satisfied :

1. Two alternate angles have the same measure.
2. Two corresponding angles have the same measure.
3. Two interior angles in the same side of the transversal are supplementary.

Notice that :

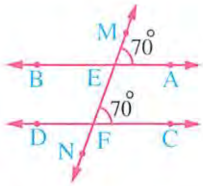
In each of the following figures where : $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two straight lines and $\overleftrightarrow{MN}$ is a transversal to them.



$\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ because :

m (∠ BEF) = m (∠ EFC) = 120°

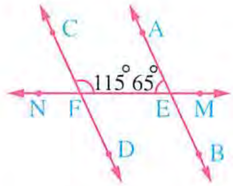
and they are two alternate angles.



$\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ because :

m (∠ AEM) = m (∠ CFE) = 70°

and they are two corresponding angles.



$\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ because :

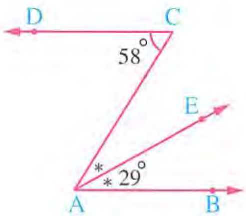
m (∠ AEF) + m (∠ CFE) = 65° + 115° = 180°

and they are interior angles in the same side of the transversal.

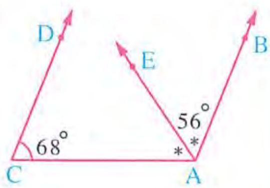
Example 3

In each of the following figures, show why $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$:

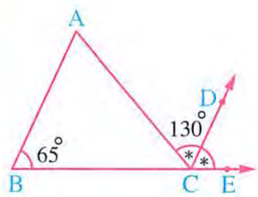
1



2



3



Solution

1 m (∠ BAC) = 29° × 2 = 58°

i.e. m (∠ BAC) = m (∠ C) and they are two alternate angles ,

therefore : $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

2 m (∠ CAB) = 56° × 2 = 112°

i.e. m (∠ CAB) + m (∠ C) = 112° + 68° = 180°

and they are interior angles in one side of the transversal ,

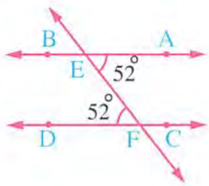
therefore : $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

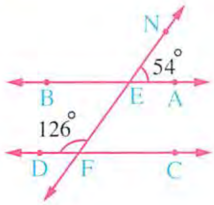
3 m (∠ ECD) = $\frac{130°}{2}$ = 65°

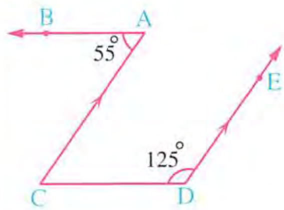
i.e. m (∠ ECD) = m (∠ B) and they are corresponding angles ,

therefore : $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

TRY by yourself 2 *In each of the following figures , why is $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$?*

1


2


3


Geometric facts

1 The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other.
And vice versa , if two coplaner straight lines are perpendicular to a third one , then the two straight lines are parallel.

For example:

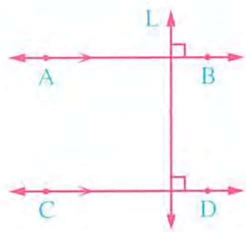
In the opposite figure :

If $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$, the straight line L is drawn perpendicular to $\overleftrightarrow{AB}$, then the straight line L $\perp \overleftrightarrow{CD}$

and if $\overleftrightarrow{AB} \perp$ the straight line L

, $\overleftrightarrow{CD} \perp$ the straight line L

, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



2 If two straight lines are parallel to a third straight line , then these two straight lines are parallel.

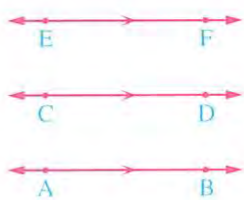
For example:

In the opposite figure :

If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

, $\overleftrightarrow{EF}$ is drawn parallel to $\overleftrightarrow{CD}$

, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$

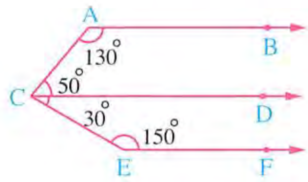


Example 4 **In the opposite figure :**

m (∠ A) = 130° , m (∠ ACD) = 50°

, m (∠ DCE) = 30° and m (∠ E) = 150°

Is $\overrightarrow{AB}$ // $\overrightarrow{EF}$? Why ?



Solution

$\overrightarrow{AB}$ // $\overrightarrow{CD}$ because : m (∠ A) + m (∠ ACD) = 130° + 50° = 180°

"interior angles on the same side of the transversal"

, $\overrightarrow{EF}$ // $\overrightarrow{CD}$ because : m (∠ E) + m (∠ DCE) = 150° + 30° = 180°

"interior angles on the same side of the transversal"

, then $\overrightarrow{AB}$ // $\overrightarrow{EF}$

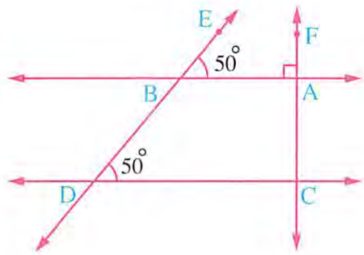
TRY by yourself 3

In the opposite figure :

m (∠ ABE) = m (∠ CDB) = 50°

and $\overleftrightarrow{FC} \perp \overleftrightarrow{AB}$

Is $\overleftrightarrow{FC} \perp \overleftrightarrow{CD}$? Why ?



3 If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of equal lengths.

For example:

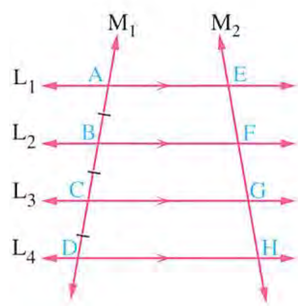
In the opposite figure :

If L_1 // L_2 // L_3 // L_4

, M_1 and M_2 are two transversals

where AB = BC = CD

, then EF = FG = GH

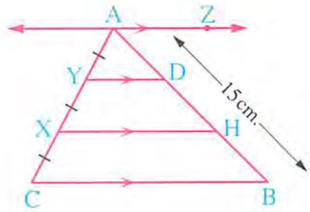


Example 5 **In the opposite figure :**

$\overleftrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XH} \parallel \overline{CB}$

, AY = YX = XC and AB = 15 cm.

Find the length of $\overline{BD}$ showing the reason.



Solution

Since , $\overleftrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XH} \parallel \overline{CB}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are their transversals

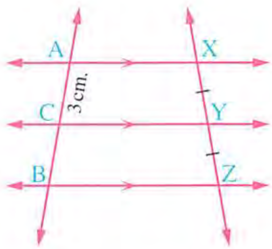
, AY = YX = XC

, then AD = DH = HB = $\frac{15}{3}$ = 5 cm.

, then BD = 5 + 5 = 10 cm.

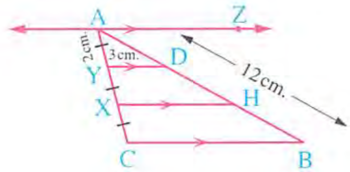
TRY by yourself 4 ***Complete under each figure of the following figures :***

1



AB = ... cm.

2



BH = ... cm.

The perimeter of Δ ADY = ... cm.

Answers *of try by yourself*

1 (1) 55° (2) 110° (3) 65°

2 (1) The reason : m($\angle$ AEF) = m($\angle$ EFD) = 52° and they are alternate angles.

(2) The reason : m($\angle$ BEF) = m($\angle$ AEN) = 54° (vertically opposite angles) , then m($\angle$ BEF) + m($\angle$ EFD) = 54° + 126° = 180° and they are interior angles in the same side of the transversal.

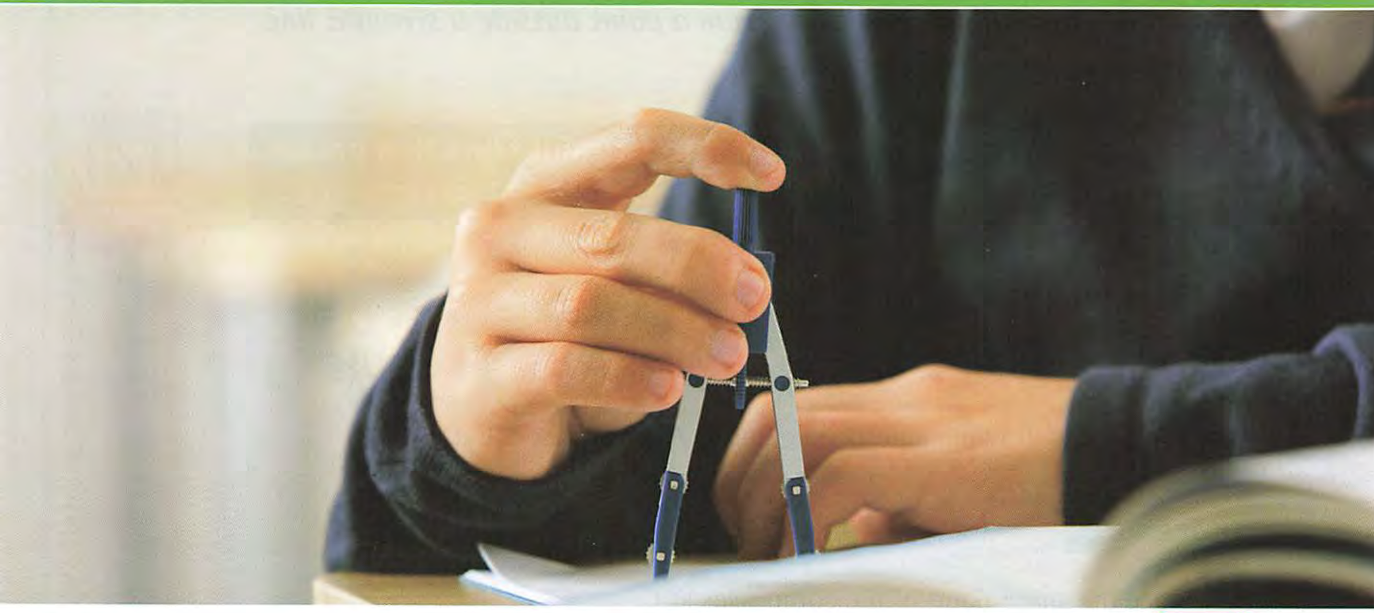
(3) The reason : m($\angle$ C) = 180° − 125° = 55° , then m($\angle$ A) = m($\angle$ C) = 55° and they are alternate angles.

3 Yes , the reason : m($\angle$ ABE) = m($\angle$ CDB) = 50° and they are corresponding angles , therefore $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and since $\overleftrightarrow{FC} \perp \overleftrightarrow{AB}$, therefore $\overleftrightarrow{FC} \perp \overleftrightarrow{CD}$

4 (1) 6 (2) 4 , 9

LESSON 6

Geometric Constructions



First Constructing a perpendicular from a point outside a straight line :

WATCH VIDEO

If $\overleftrightarrow{AB}$ is a given straight line and $C \notin \overleftrightarrow{AB}$ as shown in fig. (1)

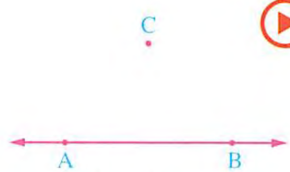


Fig. (1)

The required is constructing the perpendicular to $\overleftrightarrow{AB}$ from C

Procedure :

1. Using the compasses at C as a centre and with a suitable radius , draw an arc to intersect $\overleftrightarrow{AB}$ at the two points D and E as shown in fig. (2)

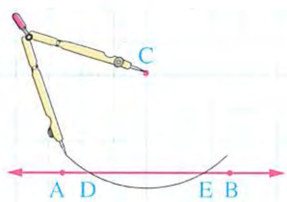


Fig. (2)

2. At D and E as centres and with a suitable radius (greater than $\frac{1}{2}$ DE) draw two arcs to intersect each other at L as shown in fig. (3)

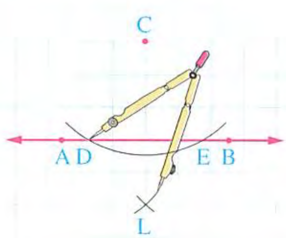


Fig. (3)

3. Draw $\overleftrightarrow{CL}$ to be the straight line passing through C perpendicular to $\overleftrightarrow{AB}$ as shown in fig. (4)

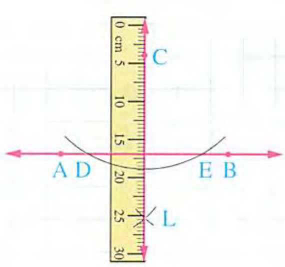


Fig. (4)

TRY by yourself 1 *Draw a perpendicular from a point outside a straight line.*

Second Drawing a perpendicular to a straight line that passes through a point which belongs to that straight line :

If $\overleftrightarrow{AB}$ is a given straight line.

$C \in \overleftrightarrow{AB}$ as shown in fig. (1)

Fig. (1)

The required is drawing a perpendicular to $\overleftrightarrow{AB}$ from the point C

Procedure :

1. Place the sharp point of the compasses at C and adjust it with suitable length , then draw two arcs in two different sides of C to intersect $\overleftrightarrow{AB}$ at D and E as shown in fig. (2)

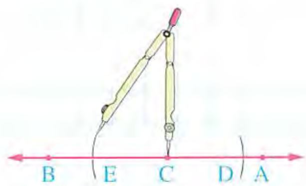


Fig. (2)

2. Place the sharp point of the compasses at each of the points D and E and adjust it with length greater than half the length of $\overline{DE}$, then draw two arcs to intersect at point X as shown in fig. (3)

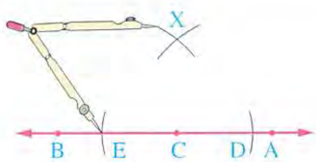


Fig. (3)

3. Draw $\overline{XC}$, then $\overline{XC}$ is perpendicular to $\overleftrightarrow{AB}$ as shown in fig. (4)

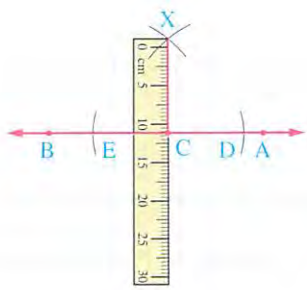


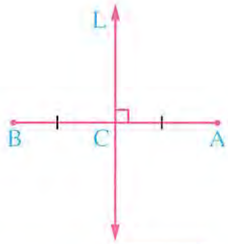
Fig. (4)

TRY by yourself 2 *Draw a perpendicular to a straight line from a point which belongs to that straight line.*

The axis of symmetry of a line segment :

It is the straight line perpendicular to it from its midpoint.

In the opposite figure :



If C is the midpoint of $\overline{AB}$ and the straight line

L $\perp$ $\overline{AB}$ from the point C

Then the straight line L is the axis of symmetry of the line segment $\overline{AB}$

Third Bisecting a given line segment "Constructing the symmetry axis of a given line segment" :

WATCH VIDEO

If $\overline{AB}$ is a given line segment as shown in fig. (1)

Fig. (1)

The required is constructing the symmetry axis of the line segment $\overline{AB}$ (The perpendicular to $\overline{AB}$ from its midpoint).

Procedure :

1. Using the compasses at A as a centre and with a radius greater than $\frac{1}{2}$ AB , draw two arcs in the opposite sides of $\overline{AB}$ as shown in fig. (2)

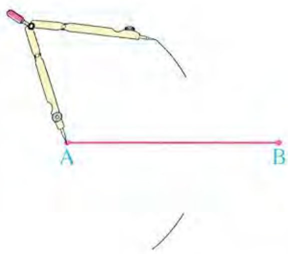


Fig. (2)

2. Using the compasses at B as a centre and with the same radius , draw two other arcs to intersect the previous two arcs at D and E as shown in fig. (3)

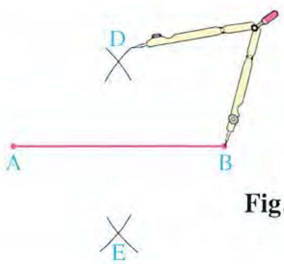


Fig. (3)

3. Draw $\overleftrightarrow{DE}$ to cut $\overline{AB}$ at a point as C which is the midpoint of $\overline{AB}$,

   $\overleftrightarrow{DE} \perp \overline{AB}$

   Then $\overleftrightarrow{DE}$ is the perpendicular to $\overline{AB}$ from its midpoint

   i.e. $\overleftrightarrow{DE}$ is the axis of symmetry of $\overline{AB}$

   as shown in fig. (4)

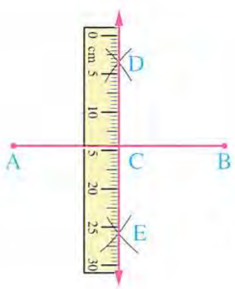


Fig. (4)

TRY by yourself 3 *Draw a line segment of length 5 cm. , then draw its symmetry axis.*

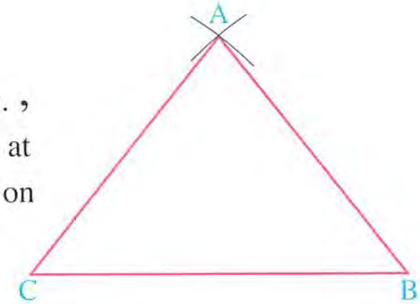
Example 1

Using the geometric instruments , draw Δ ABC in which AB = AC = 4 cm. , BC = 5 cm. , then draw the axes of symmetry of its three sides.

Are the axes of symmetry of the three sides concurrent (i.e. intersecting at one point) ? (Don't remove the arcs)

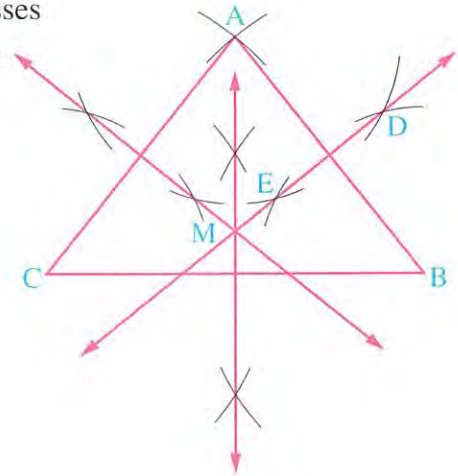
Solution

First : Drawing Δ ABC



1. Draw $\overline{BC}$ such that BC = 5 cm.
2. Using the compasses with length 4 cm. , place the sharp point of the compasses at each point of B and C , draw two arcs on one side of $\overline{BC}$ to intersect at A
3. Draw $\overline{BA}$ and $\overline{CA}$ to get Δ ABC

Second : Drawing the axes of symmetry of the sides of Δ ABC

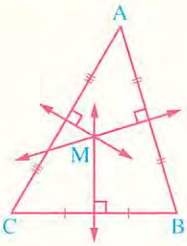
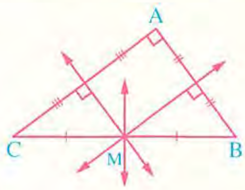
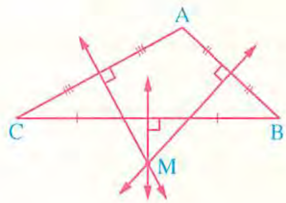


1. Place the sharp point of the compasses at A with length greater than half the length of $\overline{AB}$
   i.e. more than 2 cm. , draw two arcs on two different sides of $\overline{AB}$
2. Place the sharp point of the compasses at B and with the same previous length, draw two other arcs to intersect the previous arcs at the two points D and E
3. Draw $\overleftrightarrow{DE}$, then $\overleftrightarrow{DE}$ is the axis of symmetry of the side $\overline{AB}$
4. Do the same previous steps to draw the axes of symmetry of $\overline{AC}$ and $\overline{BC}$
5. We notice that the three axes of symmetry are concurrent
   (i.e. they are intersecting at one point M)

Notice that :
You can draw without writing the steps of the construction but don't remove the arcs.

! Remarks

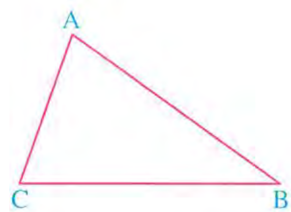
- The axes of symmetry of the sides of any triangle are intersecting at one point (say M).
  The position of M differs according to the type of the triangle as follows :

Acute-angled triangle	Right-angled triangle	Obtuse-angled triangle
		
M is inside the triangle.	**M is the midpoint of the hypotenuse.**	**M is outside the triangle.**

- The lengths of the line segments joining the point of intersection of the axes of symmetry and the vertices of the triangle are equal in all previous cases.
  i.e. AM = BM = CM

TRY by yourself 4

Draw the axis of symmetry of each side of Δ ABC and check that the three axes intersect at one point.



Fourth Constructing the bisector of a given angle :

If $\angle$ ABC is a given angle as shown in fig. (1)

The required is constructing the bisector of $\angle$ ABC "Using the compasses and the ruler"

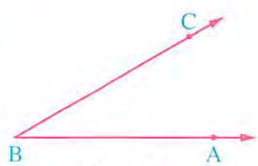


Fig. (1)

Procedure :

1. Using the compasses and with a suitable radius at the vertex of the angle B as a centre , draw an arc to intersect $\overrightarrow{BA}$ and $\overrightarrow{BC}$ (the two sides of $\angle$ ABC) at D and E respectively as shown in fig. (2)

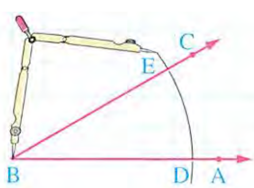


Fig. (2)

2 Taking D and E as centres and using the compasses with a suitable radius , draw two arcs to intersect at the point X as shown in fig. (3)

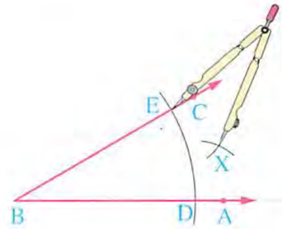


Fig. (3)

3 Draw $\overrightarrow{BX}$ to be the bisector of $\angle$ ABC as shown in fig. (4)

Notice that :

$\overleftrightarrow{BX}$ is the axis of symmetry of $\angle$ ABC

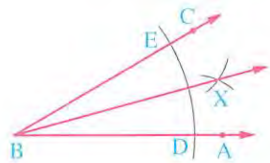


Fig. (4)

TRY by yourself 5 ***Draw an angle of measure 80° , then construct the bisector of this angle.***

Fifth Constructing an angle to be congruent to a given angle (without using protractor) :

WATCH VIDEO

$\angle$ ABC is a given angle as shown in fig. (1)

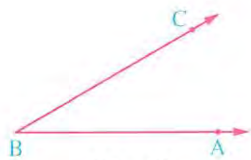


Fig. (1)

The required is drawing $\angle$ XYZ such that $\angle$ XYZ is congruent to $\angle$ ABC
i.e. m ($\angle$ XYZ) = m ($\angle$ ABC)

Procedure :

1 Draw $\overrightarrow{YL}$ to represent one of the sides of the required angle as shown in fig. (2)

Fig. (2)

2 Using the compasses with B as a centre and with a suitable radius , draw an arc to cut $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at D and E respectively as shown in fig. (3)

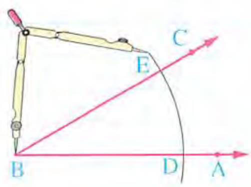


Fig. (3)

3 With Y as a centre and with the same radius , draw an arc to cut the ray $\overrightarrow{YL}$ at X as shown in fig. (4)

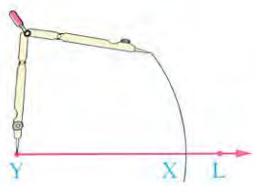


Fig. (4)

4 With X as a centre and with radius equal to the length of $\overline{DE}$, draw another arc to cut the previous arc at Z as shown in fig. (5)

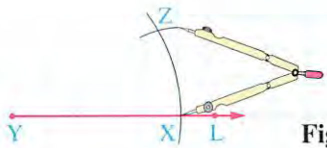


Fig. (5)

5 Draw $\overrightarrow{YZ}$, then $\angle$ XYZ is the required angle as shown in fig. (6)

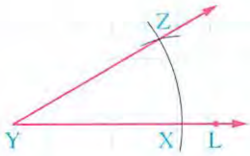


Fig. (6)

TRY by yourself 6 ***Draw $\angle$ B of measure 50° , then without using the protractor draw $\angle$ C congruent to $\angle$ B***

Sixth | Drawing a straight line from a given point parallel to a given straight line :

$\overleftrightarrow{AB}$ is a given straight line and C $\notin$ $\overleftrightarrow{AB}$ as shown in fig. (1)

Fig. (1)

WATCH VIDEO

The required is drawing a straight line passing through the point C parallel to $\overleftrightarrow{AB}$

Procedure :

1 Draw the straight line $\overleftrightarrow{XY}$ passing through the point C and cutting $\overleftrightarrow{AB}$ at Y as shown in fig. (2)

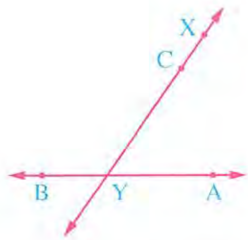


Fig. (2)

2 Draw at C the angle XCD corresponding to $\angle$ AYX such that $\angle$ XCD $\equiv$ $\angle$ XYA using the previous construction , then $\overleftrightarrow{CD}$ is the straight line which passes through the point C and parallel to $\overleftrightarrow{AB}$ as shown in fig. (3)

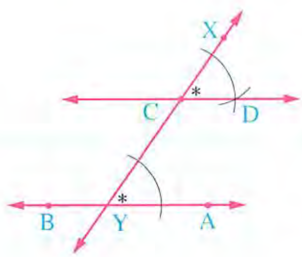
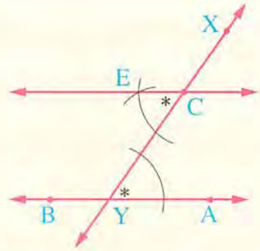


Fig. (3)

! Remark

In the previous activity , we can replace the second step by drawing ∠ YCE at the point C in the alternate position with ∠ AYC such that ∠ YCE ≡ ∠ AYC , then $\overleftrightarrow{CE}$ will be the straight line which passes through the point C and parallel to $\overleftrightarrow{AB}$ as shown in the opposite figure.



Example 2

Draw Δ ABC in which AB = 7 cm. , m (∠ A) = 50° , m (∠ B) = 70° , then bisect $\overline{AC}$ at D , then draw $\overleftrightarrow{DE}$ // $\overline{AB}$ to cut $\overline{BC}$ at E , then by measuring , find :

1. The length of each of $\overline{BE}$ and $\overline{CE}$ What do you notice ?
2. The length of $\overline{DE}$ What do you notice ?

Solution

- Using the ruler and the protractor , draw Δ ABC
- Using the campasses , bisect $\overline{AC}$ at D
- Using the ruler and the compasses , draw ∠ CDE such that : ∠ CDE ≡ ∠ A

So $\overrightarrow{DE}$ // $\overline{AB}$, then by measuring , we find that :

1. BE ≃ 3.1 cm. , CE ≃ 3.1 cm.

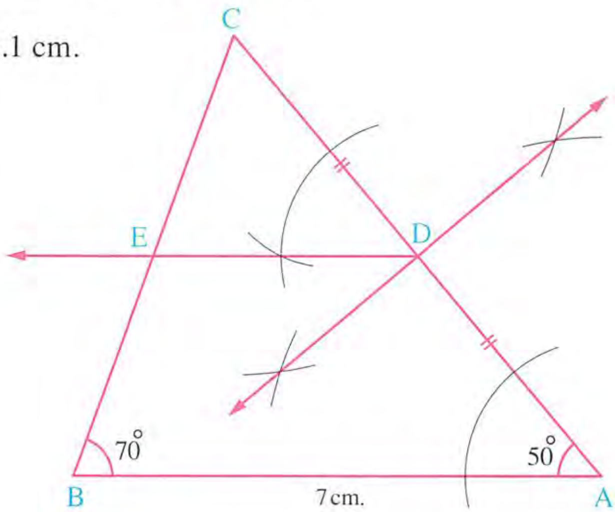
   We notice that E is the midpoint of $\overline{BC}$

   i.e. BE = CE

2. DE = 3.5 cm.

   We notice that :

   $DE = \frac{1}{2} AB$



TRY by yourself 7

Using the geometric tools , draw the equilateral triangle ABC whose side length is 6 cm. , then bisect ∠ A by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D , then draw $\overrightarrow{DE}$ // $\overline{AB}$ to cut $\overline{AC}$ at E , then find by measuring the length of $\overline{DE}$, and the length of $\overline{AE}$ what do you notice ? (Don't remove the arcs)

EL-MOASSER

By a group of supervisors

EXERCISES

1st PREP. 2023

FIRST TERM

Maths

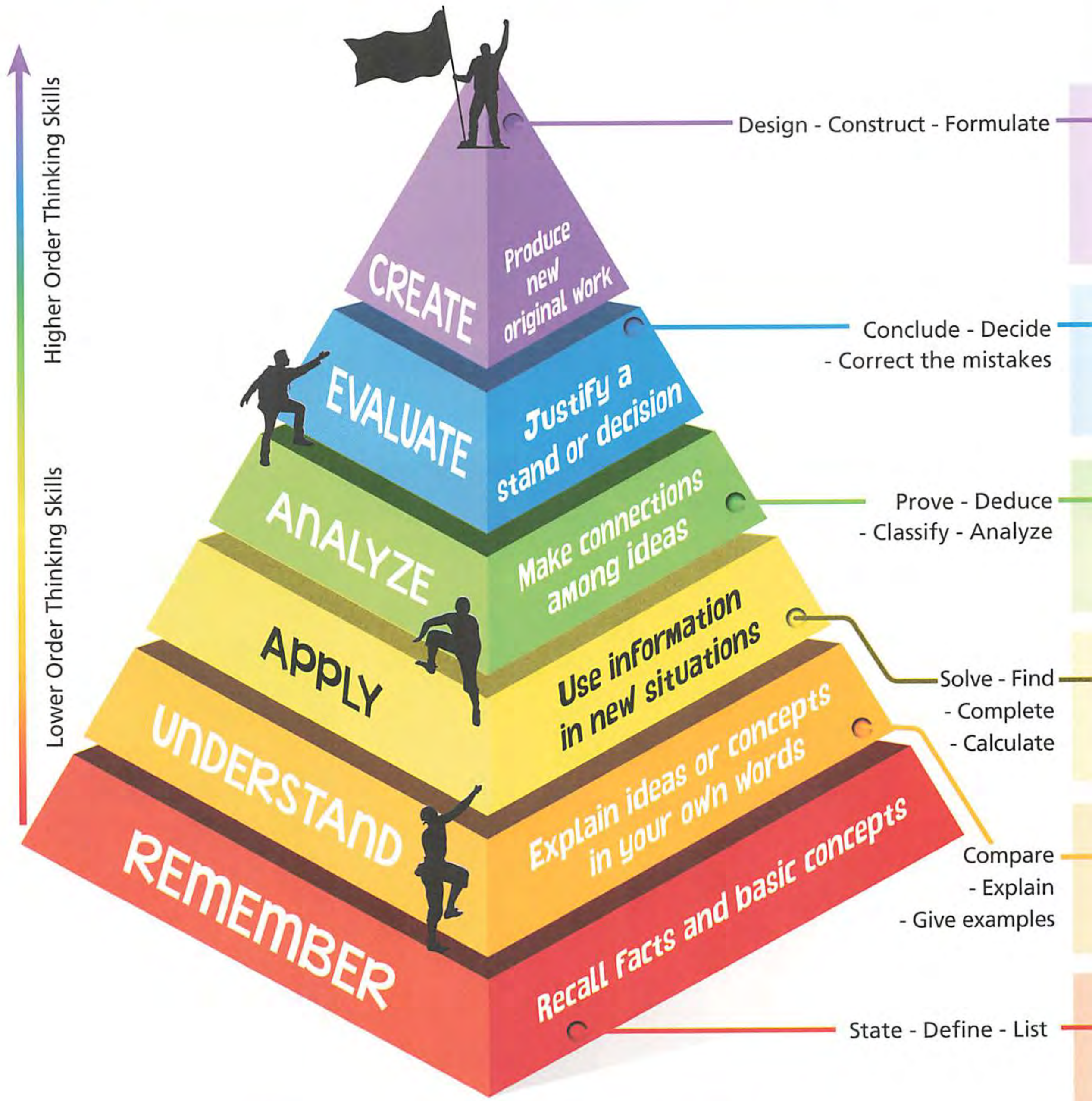
Contents

A research project on each unit

Bloom's Taxonomy Of Cognitive Levels

Bloom's Taxonomy is an educational classification created by Benjamin Bloom, it is often represented as a pyramid. This taxonomy was revised to include six cognitive levels graded from the lower level to the higher level as follows:

REMEMBER → UNDERSTAND → APPLY → ANALYZE → EVALUATE → CREATE



Bloom's Revised Pyramid

Learning Strategies That Enhance Higher Order Thinking Skills Of Bloom's Pyramid

The Bloom's Pyramid shows that each cognitive level depends on the levels that precede it, and it is necessary to achieve deep learning to reach the higher levels of thinking and this is done by first making sure that the lower levels of thinking are achieved.

Here are some learning strategies that will enable you to achieve the levels' objectives:

Deep Learning

REMEMBER / UNDERSTAND + APPLY / ANALYZE / EVALUATE +

- **Imagining** – Form a mental image
- **Researching** – Investigate what others have done
- **Define & Developing** – Clarify your scope and put idea in motion
- **Test & Adjusting** – Get feedback and modify as needed

REMEMBER + UNDERSTAND + APPLY + ANALYZE +

- **Interpreting** – Giving your own meaning to information

REMEMBER + UNDERSTAND + APPLY +

- **Idea Mapping** – Visual representation of ideas
- **Deducing** – Reaching logical conclusions from information

REMEMBER + UNDERSTAND +

- **Concrete Examples** – Collecting examples from class or textbook and create your own
- **Dual Coding** – Combining words and visuals

REMEMBER +

- **Elaborative Interrogation** – Asking yourself "why" as you read
- **Self-Explanation** – Making sense of information

- **Re-reading** – Reviewing information multiple times
- **Highlighting & Underlining** – Emphasizing key points in text
- **Summarizing / Key Words** – Most effective after pausing

Shallow Learning

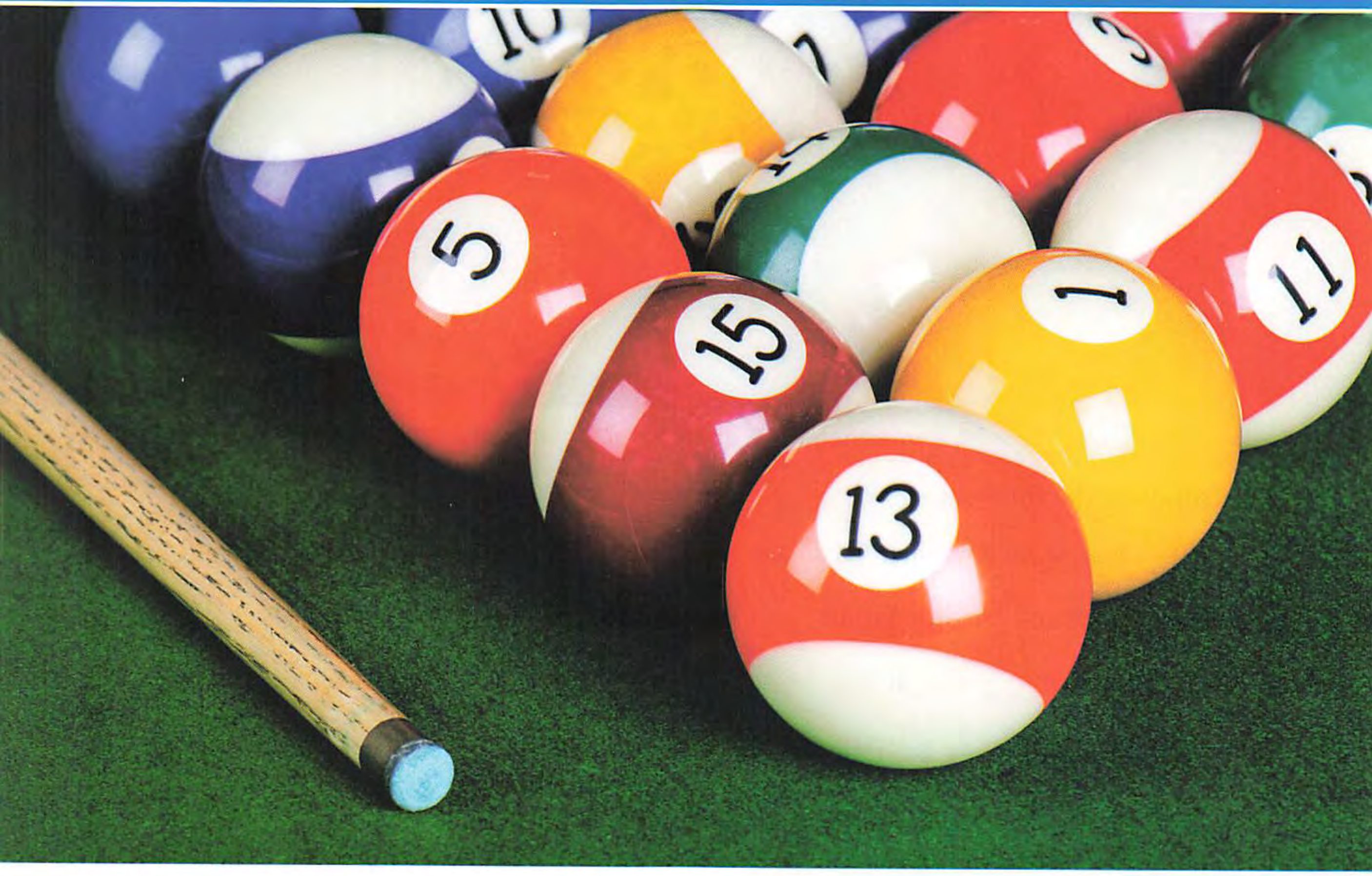
Note: The questions within each exercise are classified according to the levels of Bloom's Pyramid and are referred to as follows:

● REMEMBER ● UNDERSTAND ● APPLY ● PROBLEM SOLVING (ANALYZE - EVALUATE - CREATE)

First Algebra and Statistics

UNIT 1

Rational Numbers



Exercises of the unit :

1. Set of rational numbers.
2. Comparing and ordering rational numbers.
3. Adding and subtracting rational numbers.
4. Multiplying and dividing rational numbers.
5. Applications on rational numbers.

A research project on unit one

Scan the QR code to solve an interactive test on each lesson

EXERCISE 1

Set of Rational Numbers

Interactive test

From the school book

Remember Understand Apply Problem Solving

1 **Complete the following :**

1 If $\frac{5}{a}$ is a rational number , then $a \neq$ ....

2 The necessary condition to be $\frac{3}{X-2}$ is a rational number is $X \neq$ ....

3 The number $\frac{2}{3X} \in \mathbb{Q}$ if $X \neq$ ....

4 The number $\frac{X-3}{3X+6}$ is a rational number if $X \neq$ ....

5 The number $\frac{a-6}{a-4}$ is not rational if $a =$ ....

6 The rational number $\frac{X-5}{X} = 0$ if $X =$ ....

7 The rational number $\frac{4-X}{X-3} = 0$ if $X =$ ....

8 The rational number $\frac{5X+15}{X-5} = 0$ if $X =$ ....

9 If $\frac{X+4}{X-3}$ is not rational , then $X-2 =$ ....

10 $\frac{3}{4} = \frac{9}{\cdots\cdots}$

11 $-\frac{16}{20} = \frac{\cdots\cdots}{10}$

12 $\frac{7}{20} =$ .... %

13 $\frac{21}{1000} =$ .... %

14 $|-0.4| =$ .... %

2 **Choose the correct answer from the given ones :**

1 All the following numbers are rational except ....

(a) 0 (b) $\frac{2}{5}$ (c) $\frac{3-3}{7}$ (d) $\frac{4}{5-5}$

2 Which of the following numbers is an integer ?

(a) $-\frac{24}{5}$ (b) $\frac{4}{8}$ (c) $\frac{15}{5}$ (d) $3\frac{1}{4}$

3 Which of the following rational numbers is negative ?

(a) $\frac{0}{-3}$ (b) $-\left|-\frac{1}{2}\right|$ (c) $\frac{-3}{-4}$ (d) $(-7)^2$

4 Which of the following rational numbers is positive ?

(a) $-\frac{3}{4}$ (b) $\frac{0}{5}$ (c) $(-5)^3$ (d) $\frac{-2}{-9}$

5 Which of the following equals $\frac{4}{5}$?

(a) 4 % (b) 54 % (c) 120 % (d) 80 %

6 If $-\frac{4}{5} = \frac{20}{x}$, then $x =$ ....

(a) 25 (b) – 25 (c) 5 (d) 100

7 The rational number $\frac{a}{b}$ is positive if ....

(a) $a\,b > 0$ (b) $a\,b < 0$ (c) $a + b = 0$ (d) $a > b$

8 The rational number $\frac{-7}{a}$ is positive if a .... zero

(a) $>$ (b) $\geq$ (c) $<$ (d) $=$

9 The rational number $\frac{x}{-5}$ is negative if x .... zero

(a) $>$ (b) $<$ (c) $\leq$ (d) $=$

10 If $a = 2$, $b = 6$, then which of the following is not a rational number ?

(a) $\frac{b}{a}$ (b) $-\frac{2}{a}$ (c) $\frac{0}{a+b}$ (d) $\frac{2\,b}{a-2}$

11 $0.\dot{5}\dot{7} =$ ....

(a) $\frac{57}{100}$ (b) $\frac{75}{99}$ (c) $\frac{575}{1000}$ (d) $\frac{19}{33}$

12 $\left|-\frac{8}{25}\right| =$ ....

(a) $-\frac{8}{25}$ (b) $-0.3\dot{2}$ (c) $0.\dot{3}\dot{2}$ (d) 32%

13 12% = ...

(a) $0.\dot{3}$ (b) 1.2 (c) $\frac{3}{25}$ (d) 0.012

3 **Put each of the following numbers in the simplest form :**

1 $\frac{15}{25}$ 2 $-\frac{24}{56}$ 3 $\frac{45}{20}$ 4 $-\frac{132}{88}$

4 **Which of the following rational numbers can be written as a terminating decimal ?**

1 $\frac{7}{15}$ 2 $\frac{7}{20}$ 3 $\frac{5}{8}$ 4 $-\frac{8}{9}$ 5 $\frac{5}{11}$

6 $-\frac{13}{22}$ 7 $\frac{17}{6}$ 8 $2\frac{2}{5}$ 9 $-1\frac{2}{3}$ 10 $\left|-1\frac{2}{9}\right|$

5 **Write each of the following two numbers in the form of a recurring decimal :**

1 $\frac{6}{11}$ 2 $-3\frac{1}{15}$

6 **Write each rational number in the form $\frac{a}{b}$:**

1 -5 2 zero 3 0.75 4 -0.01

5 5.4 6 30% 7 4.5% 8 $8\frac{2}{3}$

7 **Write each of the following rational numbers as a decimal and a percentage :**

1 $2\frac{1}{2}$ 2 $-\frac{3}{20}$ 3 $7\frac{3}{16}$ 4 $\frac{1}{6}$

8 Why does the definition of a rational number $\frac{a}{b}$ state that $b \neq 0$?

For excellent pupils

9 **Choose the correct answer from the given ones :**

1 If $\frac{a}{b}$ is a rational number and ab = zero, then ....

(a) $a = 0$, $b \neq 0$ (b) $a \neq 0$, $b \neq 0$ (c) $a = 0$, $b = 0$ (d) $a \neq 0$, $b = 0$

2 The number $\frac{5x}{|x|-2} \notin \mathbb{Q}$ if $x =$ ....

(a) zero (b) -1 (c) ± 2 (d) 5

10 Write the rational number $\frac{a}{b}$ that equals $\frac{3}{5}$ and the sum of its two terms is 24

11 **If $x \in \mathbb{N}$, find the values of x which make each of the following an integer :**

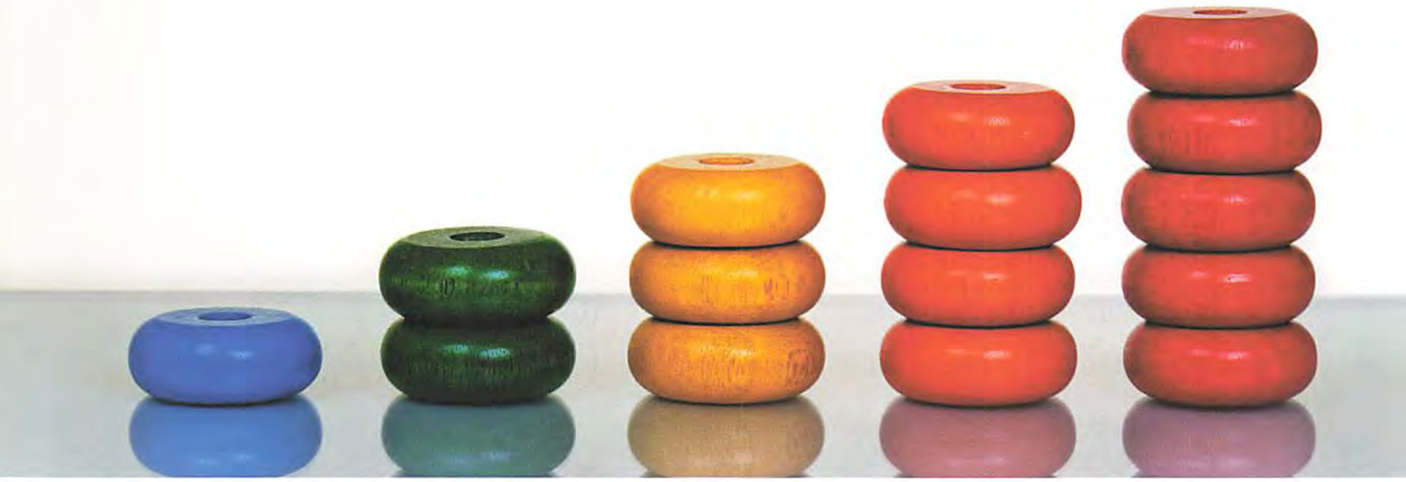
1 $\frac{75}{x}$ 2 $\frac{15}{x+1}$

EXERCISE **2**

Comparing and Ordering Rational Numbers

Interactive test

From the school book



Remember Understand Apply Problem Solving

1 **Represent each of the following rational numbers on the number line :**

1 $\frac{1}{3}$	2 $-\frac{1}{2}$	3 $\frac{5}{3}$	4 $-\frac{7}{4}$
5 $1\frac{1}{5}$	6 $-3\frac{1}{2}$	7 0.4	8 $\left\|-\frac{3}{5}\right\|$

2 **Write the correct sign "< , = or >" :**

1 $-\frac{1}{2}$ $\square$ zero	2 $-\frac{3}{4}$ $\square$ $\frac{1}{4}$	3 $-4\frac{1}{2}$ $\square$ -5
4 $4\frac{1}{2}$ $\square$ 5	5 $\left\|-\frac{3}{2}\right\|$ $\square$ $\frac{1}{2}$	6 $\left\|\frac{15}{2}\right\|$ $\square$ $7\frac{1}{2}$

3 **Put the suitable sign " > , < or = " :**

1 $\frac{1}{4}$ $\square$ $\frac{1}{6}$	2 $-\frac{5}{7}$ $\square$ $-\frac{3}{2}$	3 $\frac{9}{5}$ $\square$ $1\frac{2}{3}$
4 $-3\frac{1}{2}$ $\square$ $-\frac{20}{6}$	5 0.5 $\square$ $\frac{2}{8}$	6 1.6 $\square$ $\left\|-\frac{8}{5}\right\|$

4 **Arrange the following rational numbers descendingly :**

$\frac{3}{10}$, $\frac{7}{30}$, $-\frac{1}{3}$, $-\frac{1}{5}$ and $\frac{4}{15}$

5 **Arrange the following rational numbers in an ascending order :**

$\frac{3}{4}$, $-\frac{5}{8}$, $-\frac{7}{12}$ and $\frac{2}{3}$

6 **Choose the correct answer from the given ones :**

1 The smallest non negative rational number is ....

(a) 0.1 (b) $\frac{1}{2}$ (c) 1 (d) 0

2 The rational number opposite to $\frac{1}{5}$ on the number line is ....

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) 0.4 (d) 5

3 The rational number opposite to $-\frac{2}{3}$ on the number line is ....

(a) $\frac{3}{2}$ (b) 0 (c) 0.6 (d) $0.\dot{6}$

4 Between each two successive integers there exists ....

(a) a unique rational number. (b) a unique integer.

(c) an infinite number of rational numbers. (d) an infinite numbers of integers.

5 The number of integers lying between $\frac{1}{5}$, $\frac{3}{5}$ is ....

(a) 0 (b) 1

(c) 2 (d) an infinite number.

6 The number of rational numbers lying between $\frac{1}{5}$, $\frac{3}{5}$ is ....

(a) 0 (b) 1

(c) 2 (d) an infinite number.

7 The number of integers lying between $\frac{3}{5}$, $\frac{8}{7}$ is ....

(a) 0 (b) 1

(c) 2 (d) an infinite number.

8 The number of integers lying between $\frac{9}{5}$, $\frac{11}{7}$ is ....

(a) 0 (b) 1

(c) 2 (d) an infinite number.

9 The integer lying between $\frac{5}{7}$, $\frac{5}{3}$ is ....

(a) 1 (b) 3 (c) 5 (d) 7

10 $\frac{7}{5}$ > ....

(a) $\frac{14}{5}$ (b) $\frac{14}{10}$ (c) $\frac{5}{7}$ (d) $\frac{21}{15}$

11 Which of the following rational numbers is the smallest ?

(a) $\frac{2}{5}$ (b) $-\frac{2}{5}$ (c) $\frac{5}{2}$ (d) $-\frac{5}{2}$

12 If $\frac{a}{5} > \frac{b}{5}$, then a .... b

(a) < (b) > (c) ≤ (d) =

13 If $\frac{3}{a} < \frac{3}{b}$ where a b > 0 , then a .... b

(a) > (b) < (c) ≤ (d) =

14 If $\frac{a}{7} > \frac{b}{9}$, then 9 a .... 7 b

(a) < (b) > (c) ≤ (d) =

7 **Write a rational number in each of the following :**

1 $\frac{2}{5} < \square < \frac{3}{5}$ 2 $-\frac{2}{3} < \square < -\frac{1}{3}$ 3 $\frac{1}{8} < \square < \frac{1}{4}$ 4 $-\frac{2}{7} < \square < -\frac{3}{14}$

8 **Write two rational numbers lying between :**

1 $\frac{1}{2}$ and $\frac{4}{5}$ 2 $-\frac{3}{4}$ and $-\frac{2}{3}$ 3 0.3 and $\frac{3}{5}$ 4 75 % and $0.\dot{6}$

9 **Write four rational numbers between each of the following pairs of numbers :**

1 $\frac{1}{2}$ and $\frac{11}{12}$ 2 $-\frac{4}{9}$ and $-\frac{5}{6}$ 3 zero and 3

10 **Complete by rational numbers on the number line :**

11 Identify and write four rational numbers between $\frac{3}{2}$ and $\frac{3}{4}$, such that one of them is an integer.

12 If $\frac{x-3}{x+2} = 0$, find three rational numbers lying between $\frac{1}{x}$, $\frac{x-1}{x+2}$

For excellent pupils

13 **Choose the correct answer from the given ones :**

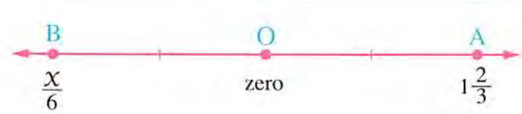
If $x < 0 < y$, $|x| > y$, then $x + y$ .... zero

(a) > (b) ≥ (c) < (d) =

14 Find the integer lying between $\frac{11}{3}$, $\frac{11}{2}$, and between $\frac{9}{4}$, $\frac{25}{6}$ at the same time. « 4 »

15 **In the opposite number line :**

If OA = OB , find the value of x



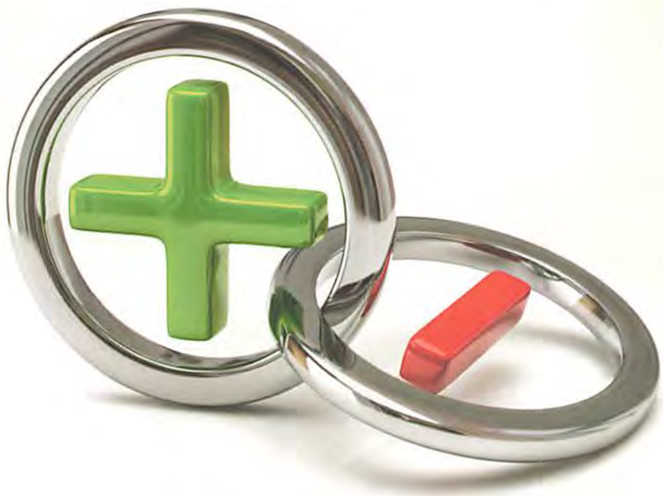
« – 10 »

EXERCISE 3

Adding and Subtracting Rational Numbers

Interactive test

From the school book



Remember   Understand   Apply   Problem Solving

1 Complete the following :

1. The additive identity element in $\mathbb{Q}$ is ....
2. The additive inverse of the number $\frac{3}{7}$ is ....
3. The additive inverse of the number $-\frac{4}{9}$ is ....
4. The additive inverse of the number -2.3 is ....
5. $\frac{-6}{-11}$ is the additive inverse of the number ....
6. The additive inverse of the number $\left(\frac{2}{3}\right)^{\text{zero}}$ is ....
7. The additive inverse of the number $\left(-\frac{2}{7}\right)^{\text{zero}}$ is ....
8. The additive inverse of the number $(-2)^3$ is ....
9. The additive inverse of the number $\left|-\frac{4}{5}\right|$ is ....
10. The additive inverse of the number zero is ....

2 Find the result of each of the following in the simplest form :

1. $\frac{3}{7} + \frac{2}{7}$
2. $-\frac{2}{9} + \frac{2}{9}$
3. $\frac{7}{8} - \frac{3}{8}$
4. $-\frac{3}{5} - \frac{9}{5}$
5. $\frac{5}{6} + \left(-\frac{4}{6}\right)$
6. $\frac{5}{9} + \left|-\frac{4}{9}\right|$

3 **Calculate the value of each of the following in its simplest form :**

1. $\frac{1}{4} + \frac{25}{8}$
2. $\frac{1}{5} - \frac{2}{3}$
3. $-\frac{9}{12} + \frac{3}{16}$
4. $-\frac{3}{10} + \left(-\frac{2}{5}\right)$
5. $-\frac{15}{18} + \frac{12}{16}$
6. $-\frac{2}{5} - \frac{3}{15}$
7. $\frac{3}{7} - \left(-\frac{2}{5}\right)$
8. $-\frac{5}{6} - \left(-\frac{3}{4}\right)$
9. $\frac{19}{10} + \left(-\frac{39}{100}\right)$

4 **Find the value of each of the following in its simplest form :**

1. $3\frac{2}{7} + 2\frac{3}{7}$
2. $9\frac{1}{5} - 7\frac{3}{5}$
3. $-10\frac{7}{8} - \left(-4\frac{5}{8}\right)$
4. $\frac{1}{4} + 2\frac{3}{8}$
5. $6\frac{2}{3} - 3\frac{1}{6}$
6. $-15\frac{1}{2} + 2\frac{3}{8}$
7. $-2\frac{1}{2} - 12\frac{1}{16}$
8. $2\frac{3}{8} - \frac{1}{4}$
9. $-2 + 13\frac{3}{7}$

5 **Calculate each of the following in its simplest form :**

1. $\frac{2}{5} + 0.2$
2. $\left|-5\frac{1}{2}\right| - \frac{1}{4}$
3. $25\% + \left(-\frac{1}{4}\right)$
4. $\frac{2}{3} - 0.\dot{3}$

6 **Choose the correct answer from the given ones :**

1. $\frac{3}{4} + 50\ \% =$ ....
   (a) 75 % (b) 150 % (c) $\frac{5}{4}$ (d) $\frac{3}{2}$
2. $1 - 40\ \% =$ ....
   (a) – 39 (b) 39 (c) 60 % (d) 60
3. $0.25 + \frac{2}{5} =$ ....
   (a) $\frac{11}{20}$ (b) $\frac{3}{5}$ (c) 0.65 (d) 0.9
4. $\frac{4}{9} + \frac{5}{9} =$ .... %
   (a) 1 (b) 9 (c) 10 (d) 100
5. $\frac{1}{5} + \left(-\frac{6}{5}\right) =$ ....
   (a) 1 (b) – 1 (c) $\frac{7}{5}$ (d) $-\frac{7}{5}$
6. The result of adding $\frac{2}{7}$ and $-\frac{3}{7}$ equals the additive inverse of ....
   (a) $-\frac{5}{7}$ (b) $-\frac{1}{7}$ (c) $\frac{1}{7}$ (d) $\frac{5}{7}$
7. Which of the following results is a negative number ?
   (a) $\frac{6}{7} + \left(-\frac{3}{7}\right)$ (b) $-\frac{1}{5} + \frac{3}{5}$ (c) $-\frac{10}{100} + \left(-\frac{1}{10}\right)$ (d) $\frac{4}{3} + \left(-\frac{4}{3}\right)$

8 Subtracting $\frac{1}{7}$ from $\frac{8}{7}$ gives ....

(a) 1 (b) – 1 (c) $-\frac{9}{7}$ (d) $\frac{9}{7}$

9 Subtracting $\frac{1}{3}$ from $-\frac{4}{3}$ gives ....

(a) – 1 (b) 1 (c) $-\frac{5}{3}$ (d) $\frac{5}{3}$

10 Subtracting $-\frac{3}{2}$ from zero gives ....

(a) zero (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) 1

11 The additive inverse of the result of subtracting $-\frac{2}{9}$ from $\frac{5}{9}$ is ....

(a) $-\frac{7}{9}$ (b) $-\frac{3}{9}$ (c) $\frac{3}{9}$ (d) $\frac{7}{9}$

12 $\frac{3}{4}$ exceeds $\frac{3}{8}$ by ....

(a) $-\frac{3}{8}$ (b) $\frac{3}{8}$ (c) $-\frac{9}{8}$ (d) $\frac{9}{8}$

13 $\frac{3}{4} + \frac{3}{4} = \frac{\cdots\cdots}{8}$

(a) 3 (b) 6 (c) 12 (d) 24

14 .... $- \frac{1}{2} = -1$

(a) $1\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-1\frac{1}{2}$

15 If $A + \frac{6}{7}$ = zero , then A = ...

(a) zero (b) 1 (c) $\frac{6}{7}$ (d) $-\frac{6}{7}$

16 If $x + \left(-\frac{1}{3}\right)$ = zero , then x = ...

(a) zero (b) 1 (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$

17 If $\frac{3}{5} + x = \frac{3}{5}$, then x = ...

(a) 0 (b) 1 (c) $\frac{3}{5}$ (d) $-\frac{3}{5}$

18 If $\left(A + \frac{1}{4}\right)$ is the additive inverse of the number $\frac{3}{4}$, then A = ...

(a) $-\frac{3}{4}$ (b) $-\frac{1}{4}$ (c) – 1 (d) 1

19 $-\left(\frac{2}{7} - \frac{4}{7}\right)$ = ...

(a) $-\frac{6}{7}$ (b) $-\frac{2}{7}$ (c) $\frac{2}{7}$ (d) $\frac{6}{7}$

20 If $\frac{5}{7} + \frac{x}{2} = \frac{25}{35}$, then $2x$ = ...

(a) 2 (b) $\frac{5}{7}$ (c) zero (d) $\frac{11}{2}$

7 **Use the number line to find the result of each of the following :**

1 $\frac{1}{5}+\frac{2}{5}$ 2 $\frac{5}{8}-\frac{3}{8}$ 3 $-\frac{1}{3}+\frac{5}{3}$ 4 $-\frac{3}{4}+\left(-\frac{1}{4}\right)$

8 **Write the property of addition used in each of the following :**

1 $\frac{7}{2}+\frac{9}{16}=\frac{9}{16}+\frac{7}{2}$

2 $\left[\frac{2}{3}+\left(-\frac{1}{3}\right)\right]+\left(-\frac{1}{6}\right)=\frac{2}{3}+\left[\left(-\frac{1}{3}\right)+\left(-\frac{1}{6}\right)\right]$

3 $\frac{3}{4}+\left(-\frac{3}{4}\right)=\text{zero}$

4 $\text{zero}+\left(-\frac{3}{4}\right)=-\frac{3}{4}$

9 **Find the sum of each of the following :**

1 $\frac{4}{7}+\text{zero}$

2 $\text{zero}+\left(-\frac{7}{10}\right)$

3 $\text{zero}-\left(-\frac{17}{4}\right)$

4 $\left[\frac{1}{4}+\left(-\frac{1}{4}\right)\right]+\frac{3}{4}$

5 $\frac{5}{6}+\left(-\frac{3}{6}+\frac{3}{6}\right)$

6 $\left[\frac{2}{9}+\left(-\frac{4}{9}\right)\right]+\left(-\frac{3}{9}\right)$

10 **Using the addition properties in $\mathbb{Q}$, find the result of each of the following in the simplest form :**

1 $\frac{1}{4}+\frac{1}{2}+\frac{3}{4}$

2 $\frac{2}{7}+\frac{3}{4}+\frac{5}{7}+\frac{1}{4}$

3 $\frac{5}{4}+\left(-\frac{13}{5}\right)+\left(-\frac{25}{4}\right)+\frac{28}{5}$

4 $\frac{5}{8}+\left(-\frac{3}{4}\right)+\frac{3}{8}+\frac{3}{4}$

5 $\frac{2}{13}+\frac{1}{5}+\frac{11}{13}+\left(-\frac{6}{5}\right)$

6 $-\frac{3}{7}+\frac{1}{2}+\left(-\frac{1}{14}\right)$

7 $\frac{12}{18}+\frac{5}{9}+\frac{1}{3}+\left(-\frac{15}{27}\right)$

8 $\frac{2}{3}+\frac{4}{5}+\frac{3}{4}$

9 $7\frac{1}{4}+\left(-11\frac{1}{4}\right)$

10 $-13\frac{1}{8}+7\frac{3}{8}$

11 **If $x=\frac{5}{6}$, $y=-\frac{1}{3}$ and $z=\frac{1}{2}$, find the value of each of the following :**

1 $x+z$ « $\frac{4}{3}$ »

2 $x+y$ « $\frac{1}{2}$ »

3 $x-y$ « $\frac{7}{6}$ »

4 $(y+z)-x$ « $-\frac{2}{3}$ »

12 If $a=\frac{1}{2}$, $b=-\frac{3}{2}$, find the value of $(a-b)^3$ « 8 »

13 Complete the following :

1. $14\frac{1}{2} + \left(-11\frac{1}{2}\right) = \dots\dots\dots + \left[11\frac{1}{2} + \left(-11\frac{1}{2}\right)\right]$

2. $\frac{3}{32} + \left(-\frac{17}{32}\right) = \left[\frac{3}{32} + \left(-\frac{3}{32}\right)\right] + \dots\dots\dots$

14 Complete in the same pattern :

1. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, ... , ...

2. 6 , $5\frac{1}{4}$, $4\frac{1}{2}$, ... , ... , ... , ... , $\frac{3}{4}$

For excellent pupils

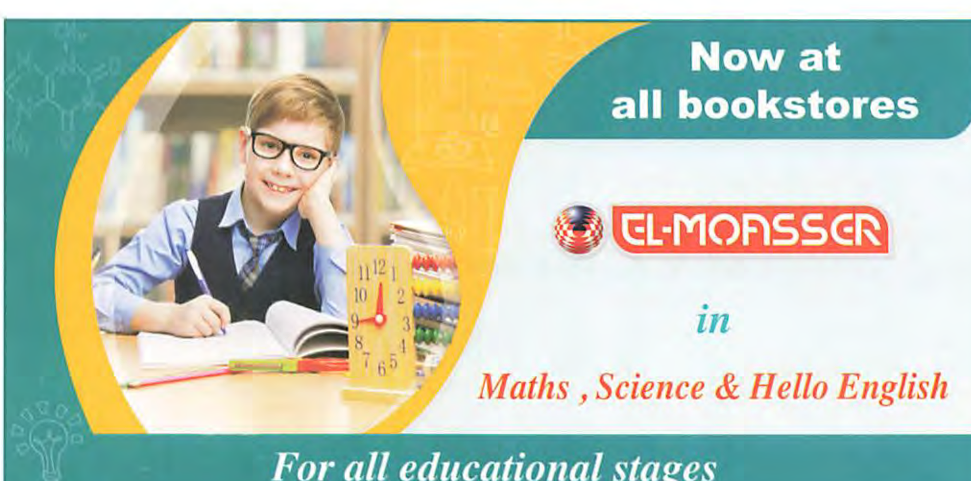
15 In each of the following , find the value of x :

1. $\left|x + \frac{1}{5}\right| = \frac{2}{5}$ « $\frac{1}{5}$ or $-\frac{3}{5}$ »

2. $\left|\frac{3}{4} - x\right| = \frac{1}{4}$ « $\frac{1}{2}$ or 1 »

16 Find the result of the following :

$\left(51\frac{1}{2} - 1\frac{1}{2}\right) + \left(52\frac{1}{2} - 2\frac{1}{2}\right) + \cdots + \left(99\frac{1}{2} - 49\frac{1}{2}\right) + \left(100\frac{1}{2} - 50\frac{1}{2}\right)$ « 2500 »



EXERCISE 4

Multiplying and Dividing Rational Numbers

Interactive test

From the school book

Remember Understand Apply Problem Solving

1 Complete the following :

1. The multiplicative identity of the rational numbers is ....
2. The multiplicative inverse of the number $\frac{3}{7}$ is ....
3. The multiplicative inverse of the number $-\frac{4}{9}$ is ....
4. The multiplicative inverse of the number -6 is ....
5. The multiplicative inverse of the number $3\frac{1}{2}$ is ....
6. The multiplicative inverse of the number 0.5 is ....
7. The multiplicative inverse of the number 1 is ....
8. The multiplicative inverse of the number -1 is ....
9. The multiplicative inverse of the number $\left(-\frac{3}{5}\right)^{\text{zero}}$ is ....
10. The multiplicative inverse of the number $\left|-\frac{3}{5}\right|$ is ....
11. The rational number $\frac{a-1}{5}$ has a multiplicative inverse if $a \neq$ ....
12. The rational number which has no multiplicative inverse is ....

2 Complete the following :

1. $\frac{2}{3} \times \left(-\frac{4}{5}\right) = -\frac{4}{5} \times$ ....
2. $\frac{2}{3} \times \left(-\frac{5}{7}\right) = \frac{5}{7} \times$ ....
3. $\frac{2}{3} \times \frac{3}{2} =$ ....
4. $\frac{4}{5} \div 1 =$ ....
5. $1 \div \frac{2}{7} =$ ....
6. $\frac{1}{4} \div 25\ \% =$ ....

7 $-\frac{4}{5} \times \cdots\cdots = -\frac{4}{5}$

8 $-\frac{4}{11} \times \cdots\cdots = 1$

9 $2\frac{3}{5} \times \cdots\cdots = 1$

10 $\cdots\cdots \times 0.8 = 1$

11 $4 \times \cdots\cdots = -5$

12 $\frac{2}{3}\left(2 + \frac{1}{2}\right) = \frac{2}{3} \times 2 + \cdots\cdots$

3 Choose the correct answer from the given ones :

1 If $\frac{2}{3} \times X = \frac{5}{7} \times \frac{2}{3}$, then $X = \cdots\cdots$

(a) $\frac{2}{3}$ (b) $\frac{5}{7}$ (c) $\frac{3}{2}$ (d) $\frac{7}{5}$

2 If $\frac{2}{5} \div X = \frac{2}{5} \times \frac{-7}{9}$, then $X = \cdots\cdots$

(a) $-\frac{9}{7}$ (b) $-\frac{7}{9}$ (c) $\frac{7}{9}$ (d) $\frac{9}{7}$

3 If $(X - 1)$ is the multiplicative inverse of $\frac{1}{5}$, then $X = \cdots\cdots$

(a) 4 (b) 5 (c) 6 (d) $1\frac{1}{5}$

4 $\left(\frac{2}{7} + \frac{3}{5}\right)$ is the multiplicative inverse of $\cdots\cdots$

(a) $-\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{31}{35}$ (d) $\frac{35}{31}$

5 If three times a number is 27 , then $\frac{1}{3}$ of that number equals $\cdots\cdots$

(a) -3 (b) 3 (c) -9 (d) 9

6 If $\frac{X}{y} = \frac{2}{3}$, then $\frac{3X}{2y} = \cdots\cdots$

(a) $\frac{1}{3}$ (b) 1 (c) $\frac{3}{2}$ (d) $\frac{9}{4}$

7 If $\frac{a}{b} = 70$, then $\frac{a}{2b} = \cdots\cdots$

(a) 35 (b) 68 (c) 72 (d) 140

8 If $\frac{|X|}{5} = 3$, then $X = \cdots\cdots$

(a) 5 (b) 10 (c) 15 (d) ± 15

4 State the property of the multiplication of rational numbers used in each of the following statements :

1 $-\frac{1}{2} \times \frac{2}{3} = \frac{2}{3} \times \left(-\frac{1}{2}\right)$

2 $-\frac{3}{7} \times \left(-\frac{7}{3}\right) = 1$

3 $-\frac{7}{20} \times \left(\frac{5}{2} \times 4\right) = \left(\frac{5}{2} \times 4\right) \times -\frac{7}{20}$

4 $\frac{5}{4} \times 1 = \frac{5}{4}$

5 $0.8 \times 0 = 0$

5 **Find the result of each of the following in the simplest form :**

1. $\frac{3}{5} \times \frac{2}{7}$
2. $-\frac{1}{2} \times \frac{2}{3}$
3. $-\frac{3}{8} \times \left(-\frac{5}{3}\right)$
4. $\frac{2}{6} \times -\frac{3}{4}$
5. $-\frac{2}{3} \times \frac{5}{8}$
6. $\frac{4}{5} \times \left(-\frac{3}{7}\right)$
7. $\left|-\frac{3}{7}\right| \times \left(-\frac{4}{3}\right)$
8. $\frac{1}{2} \times \left|-12\right|$
9. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$

6 **Find the result of each of the following in the simplest form :**

1. $\frac{4}{5} \div \frac{3}{7}$
2. $-\frac{1}{6} \div \frac{5}{2}$
3. $-\frac{4}{11} \div \left(-\frac{4}{11}\right)$
4. $\frac{5}{27} \div \frac{1}{9}$
5. $\frac{5}{6} \div \left(-\frac{15}{2}\right)$
6. $-\frac{5}{16} \div \left(-\frac{11}{8}\right)$
7. $-\frac{5}{8} \div \frac{5}{8}$
8. zero $\div \frac{3}{5}$
9. $\frac{3}{4} \div (-9)$

7 **Find the result of each of the following in the simplest form :**

1. $3\frac{1}{2} \times (-4)$
2. $1\frac{1}{2} \times \left(-\frac{3}{2}\right)$
3. $-4\frac{2}{7} \times \left(-5\frac{1}{6}\right)$
4. $3\frac{1}{8} \times \left(-4\frac{1}{5}\right)$
5. $-0.5 \times \frac{2}{5}$
6. $2\frac{1}{2} \times 0.8$
7. $\left|-1\frac{1}{2}\right| \times \left|-\frac{5}{3}\right|$
8. $\left|-0.\dot{6}\right| \times 1\frac{1}{3}$

8 **Find the result of each of the following in the simplest form :**

1. $2\frac{1}{5} \div \frac{11}{5}$
2. $5\frac{1}{2} \div 2\frac{1}{5}$
3. $-4\frac{2}{7} \div 1\frac{1}{14}$
4. $-1 \div 2\frac{1}{4}$
5. $-4\frac{1}{3} \div \left(-3\frac{1}{4}\right)$
6. $0.5 \div 5\frac{1}{2}$
7. $-2\frac{3}{4} \div \left(-3\frac{1}{8}\right)$
8. $6\frac{1}{4} \div (-15)$
9. $2\frac{3}{5} \div \left(-1\frac{11}{15}\right)$

9 **Using the distribution property, find the value of each of the following in the simplest form :**

1. $\frac{5}{12} \times 3 + \frac{5}{12} \times 9$
2. $\frac{4}{9} \times 11 + \frac{4}{9} \times 16$
3. $4 \times \frac{8}{17} + 9 \times \frac{8}{17} + 4 \times \frac{8}{17}$
4. $\frac{6}{37} \times 7 + \frac{6}{37} \times 5 + \frac{6}{37} \times (-11)$
5. $\frac{4}{5} \times 13 - \frac{4}{5} \times 22 + \frac{4}{5} \times 9$
6. $\frac{7}{12} \times 5 + 9 \times \frac{7}{12} - 2 \times \frac{7}{12}$
7. $\frac{27}{11} \times \frac{9}{4} - \frac{27}{11} \times \frac{1}{4} + \frac{27}{11} \times 9$
8. $\frac{7}{13} \times 6 + \frac{7}{13} \times 8 - \frac{7}{13}$
9. $-\frac{3}{7} \times 8 + 5 \times \left(-\frac{3}{7}\right) + \left(-\frac{3}{7}\right)$
10. $\frac{22}{25} \times \frac{7}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25}$
11. $35 \times \frac{3}{4} + 35 \times \frac{1}{2} - 35 \times \frac{1}{4}$

10 **Find the result of each of the following in the simplest form :**

1. $\left(\frac{5}{6}+\frac{2}{3}\right)\div\frac{3}{5}$
2. $\frac{3}{4}\times\left(\frac{1}{2}-\frac{1}{3}\right)$
3. $\left(-\frac{18}{5}\div\frac{9}{35}\right)\times\left(-\frac{3}{7}\right)$
4. $\left[-\frac{12}{25}\times\left(-\frac{5}{7}\right)\right]\div\left(-\frac{9}{14}\right)$
5. $\left(-1\frac{2}{3}\times4\frac{2}{3}\right)\div6\frac{1}{9}$
6. $\left(5\frac{1}{16}\div6\frac{3}{4}\right)\times\left(-7\frac{5}{9}\right)$

11 **Find the value of n in each of the following :**

1. $-\frac{7}{3}\times\left(-\frac{3}{7}\right)=n$
2. $n\times\frac{17}{3}=1$
3. $-\frac{7}{3}\times n=0$
4. $\frac{5}{7}\times n=\frac{5}{7}$
5. $n\times\left[\frac{1}{2}+\left(-\frac{3}{5}\right)\right]=n\times\frac{1}{2}+5\times\left(-\frac{3}{5}\right)$

12 **If $x=-\frac{1}{3}$, $y=\frac{3}{4}$ and $z=-3$, find the numerical value of each of the following :**

1. $x\,y\,z$
2. $x\,y+y\,z$

« $\frac{3}{4}$, $-\frac{5}{2}$ »

13 **If $a=1\frac{3}{4}$, $b=\frac{12}{7}$ and $c=\frac{2}{3}$, then find the numerical value of each of the following :**

1. $a\,b\,c+3$
2. $a\,b-c$

« 5 , $\frac{7}{3}$ »

14 If $x=\frac{5}{8}$ and $y=\frac{1}{2}$, find in the simplest form the numerical value of : $\frac{x+y}{x-y}$ « 9 »

15 **If $x=\frac{3}{2}$, $y=-\frac{1}{4}$ and $z=-2$, find in the simplest form the numerical value of each of the following :**

1. $\frac{1}{x\,y\,z}$ « $\frac{4}{3}$ »
2. $x-(z\div y)$ « $-\frac{13}{2}$ »
3. $\frac{x}{y}-\frac{z}{y}$ « -14 »
4. $(x+z)\div(y-z)$ « $-\frac{2}{7}$ »
5. $\frac{x+y}{z}$ « $-\frac{5}{8}$ »

Life Applications

16 The weights of things on the surface of the moon = $\frac{1}{6}$ their weights on the surface of the Earth.
If the weight of a man on the Earth = $76\frac{4}{5}$ kg.
, find his weight on the moon.

« $12\frac{4}{5}$ kg. »

17 If water flows through a pipe at a rate of $2\frac{1}{2}$ litres per minute, how long will it take to fill three containers 20 litres each ?

« 24 minutes »

18 How many pieces of wire the length of each is $3\frac{3}{4}$ metres can be cut from a wire of length 60 metres ?

Will any piece of wire be left over ?

If so , how long will it be ?

« 16 pieces »

For excellent pupils

19 **Use the distribution property to find the value of each of the following in its simplest form :**

1. $\frac{7}{15} \times \frac{4}{25} + \frac{16}{25} \times \frac{2}{3} + \frac{7}{15} \times \frac{1}{5} + \frac{16}{25} \times \left(-\frac{1}{5}\right)$

2. $\frac{2}{13} \times 3 + \frac{2}{13} \times 8 + \frac{4}{13}$

20 **Find the product of :**

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \cdots \times \frac{99}{100}$

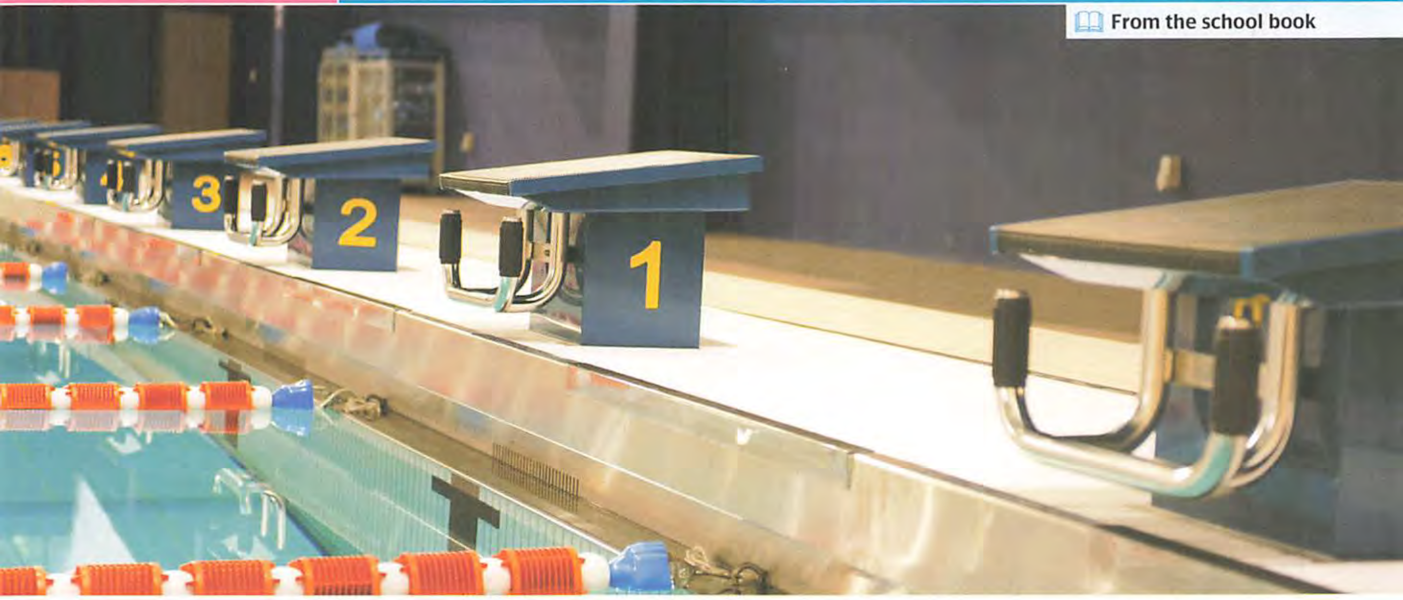
What is the product when the last rational number is $\frac{n-1}{n}$?

EXERCISE 5

Applications on Rational Numbers

Interactive test

From the school book



Remember   Understand   Apply   Problem Solving

1 **Find a rational number in the middle of the way (half-way) between :**

1. $\frac{3}{8}$, $\frac{5}{8}$
2. $\frac{2}{5}$, $\frac{4}{5}$
3. $-\frac{3}{4}$, $\frac{3}{4}$
4. $\frac{1}{2}$, $\frac{7}{8}$
5. $-\frac{1}{2}$, $-\frac{3}{4}$
6. 0.1 , $-\frac{2}{5}$
7. $-\frac{11}{9}$, $-\frac{13}{35}$
8. $-4\frac{3}{7}$, $8\frac{1}{3}$
9. zero , $\frac{2}{5}$

2 **Find a rational number lying at :**

1. One fourth of the way between $\frac{5}{7}$, $-\frac{3}{7}$ *from the side of the smaller number.*
2. One fourth of the way between $\frac{1}{3}$, 1 *from the side of the greater number.*
3. One third of the way between $-\frac{3}{5}$, $-\frac{4}{5}$ *from the side of the greater number.*
4. One third of the way between $\frac{4}{7}$, $1\frac{3}{4}$ *from the side of the smaller number.*
5. One fifth of the way between $-\frac{1}{2}$, $-\frac{2}{5}$ *from the side of the greater number.*
6. One fifth of the way between $-\frac{2}{3}$, $-\frac{3}{5}$ *from the side of the smaller number.*
7. One tenth of the way between $\frac{5}{6}$, $\frac{2}{3}$ *from the side of the smaller number.*
8. One eighth of the way between zero , $-1\frac{1}{2}$

3 **Choose the correct answer from the given ones :**

1. If $\frac{2}{3}$ lies at the middle of the way between X and $\frac{1}{2}$, then $X =$ ....

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{7}{8}$

2. If $a \times \frac{b}{2} = \frac{a}{2}$, $a \neq 0$, then b = ...

(a) 1 (b) 0 (c) a (d) $\frac{a}{2}$

3. If $\frac{X}{3} - 4 = 6$, then $\frac{X}{3} + \frac{2}{3} =$ ....

(a) 1 (b) 10 (c) $\frac{32}{3}$ (d) X

4. If $\frac{X}{y} = 1$, then $2X - 2y =$ ....

(a) 3 (b) 2 (c) 1 (d) 0

5. If $X + \frac{2}{X} = 5 + \frac{2}{5}$, then $X =$ ....

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) $\frac{5}{2}$ (d) 5

6. If 5 a = 45 and b a = 1 , then b = ...

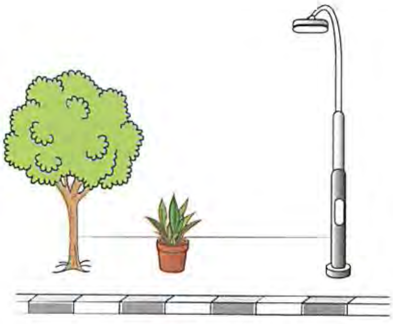
(a) $\frac{1}{45}$ (b) $\frac{1}{9}$ (c) $\frac{1}{5}$ (d) 9

7. If $\frac{3}{7} X = 42$, then $\frac{5}{7} X =$ ....

(a) 70 (b) 45 (c) 30 (d) 10

Life Application

4 In one of the projects of paving and afforesting roads , a tree was planted at a distance of 3.3 m. from the beginning of the road and a lamp post was fixed at a distance of $7\frac{1}{2}$ m. from the beginning of the road. If we want to put a flower bed at the third of the distance between them from the side of the tree , at which distance should we put the flower bed from the beginning of the road ?



« 4.7 m. »

A Research Project

On Unit One

Project aims :

- Writing the rational number in its different forms.
- Putting the rational number in the simplest form.
- Comparing and ordering the rational numbers.
- Performing mathematical operations on rational numbers.
- Associating mathematics with science.

Do a research project on the following topic :

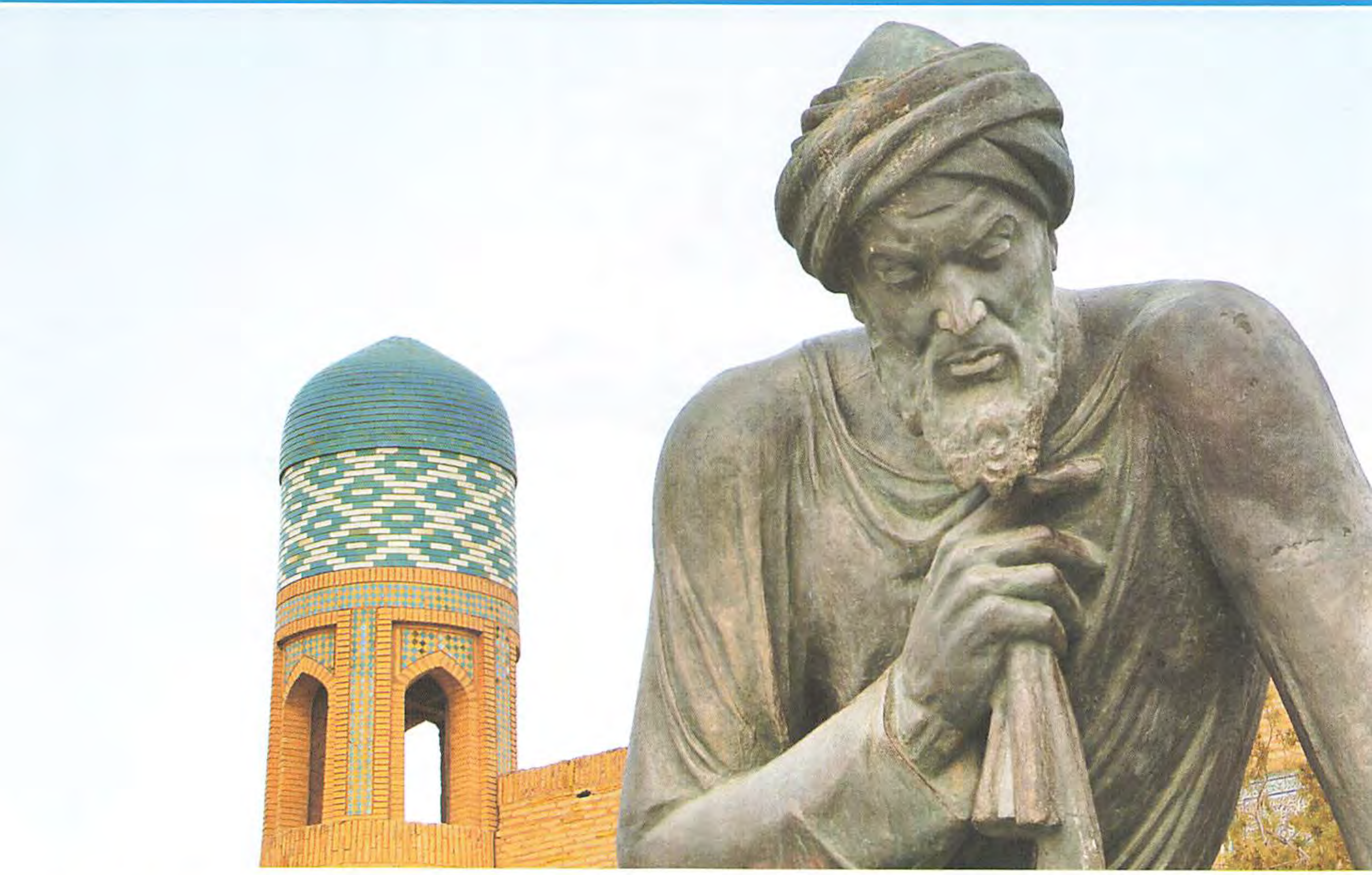
"Density is a special feature of matter. There are no two matter with the same density".

Discuss the following points using available resources :

1. Write about the density values of each of the following matter. Then write them in two different forms of rational numbers : (gold – mercury – water – aluminum – cork)
2. Arrange these matter in an ascending order according to their densities.
3. Define which of these matter float on water and which of them sink in it, giving the reasons.
4. Write the law of density and conclude the mass of an aluminum cube with a volume of 15 cm^3. and put it in the simplest form as a fraction.

UNIT

2 Algebra



Exercises of the unit :

6. Algebraic terms and algebraic expressions.

7. Like algebraic terms.

8. Adding and subtracting algebraic expressions.

9. Multiplying and dividing algebraic terms.

10. Multiplying a monomial by an algebraic expression.

11. Multiplying a binomial by an algebraic expression.

12. Dividing an algebraic expression by a monomial.

13. Dividing an algebraic expression by another one.

14. Factorization by identifying the highest common factor (H.C.F.).

A research project on unit two

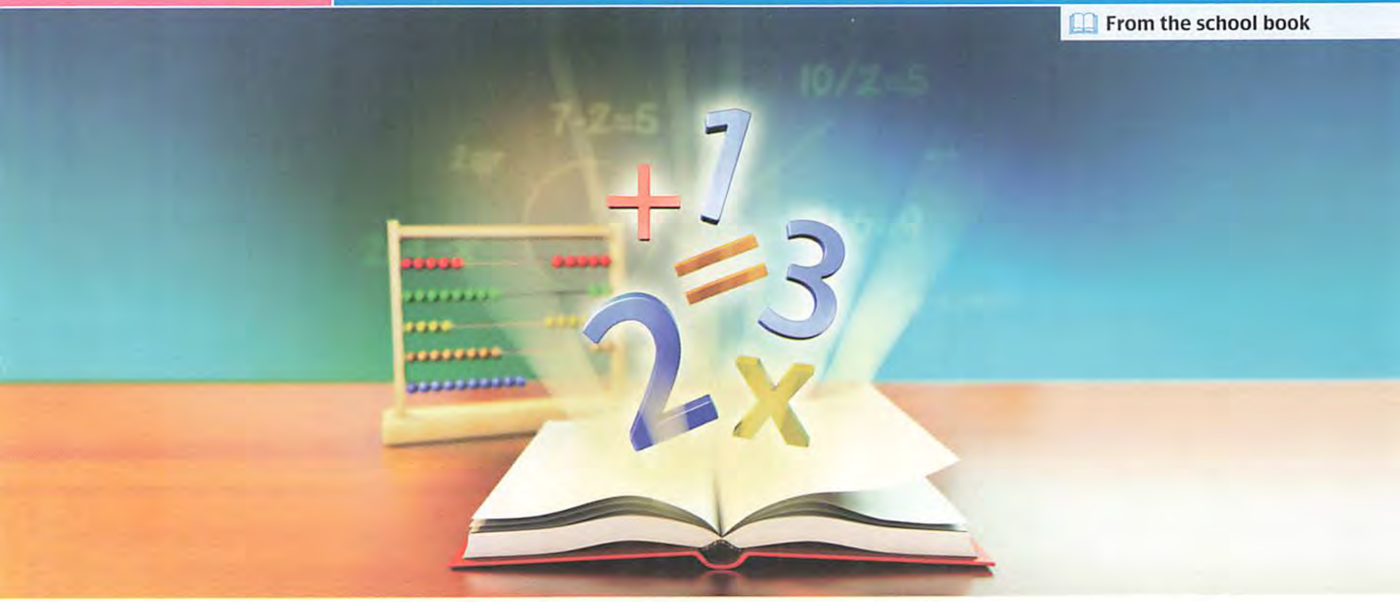
Scan the QR code to solve an interactive test on each lesson

EXERCISE 6

Algebraic Terms and Algebraic Expressions

Interactive test

From the school book



● Remember ● Understand ● Apply ● Problem Solving

1 **Complete the following table :**

Algebraic term	-7	$2ab^2$	3	$7ab^3c$	$-8x^2b$	xy^2
Coefficient	-7	2	....	....	....	....
Degree	zero	$1+2=3$	....	....	....	....

2 **Complete the following table :**

The algebraic expression	Number of terms	Name	Degree
$-3a^5b$	1	monomial	6
$3x^2+y$	2	binomial	2
$5x^3-7x+4$	....	trinomial	....
$2a^2b+3ab^2-a^2b^2$	....	....	....
$x^2y^2-3xy^4$	....	....	....
$a^2b-3ab^3+2a^3b^2+b^4$	....	....	....

3 **Complete the following :**

1. The degree of the term $3x^2y$ is .... and its coefficient is ....
2. The coefficient of the algebraic term $\frac{x^3yz^2}{2}$ is .... and its degree is ....
3. The coefficient of the algebraic term x is .... and its degree is ....

4 The degree of the absolute term in any algebraic expression is ....

5 The coefficient of the algebraic term $(-2)^3$ is .... and its degree is ....

6 $5x^2 + 3$ is an algebraic expression of the .... degree.

7 The number of terms of the algebraic expression $5y^2 - 3xy + 2x^2$ is .... and its degree is ....

4 Choose the correct answer from the given ones :

1 The degree of the algebraic term $x^4 y$ equals the degree of the algebraic term ....

(a) $x^2 y^2$ (b) $x^2 y^3$ (c) $x^4 y^2$ (d) $y^4 x^2$

2 The degree of the algebraic expression $5x^3 - 3xy + 2y^2$ equals the degree of the algebraic expression ....

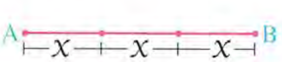
(a) $5a^2 - 2ab + 3$ (b) $2x^2y^2 - 3x^2y + 5y^3$

(c) $2x + 5x^2y + y^2$ (d) $a^3 + 2a^2b - b^4$

3 The algebraic term b^3 = ...

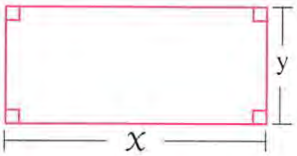
(a) $3 \times b \times b$ (b) $b + b + b$ (c) $b \times b \times b$ (d) $3 \times b$

4 The algebraic term that represents the length of $\overline{AB}$ in the opposite figure is ....



(a) x^3 (b) $3x$ (c) x (d) $\frac{x}{3}$

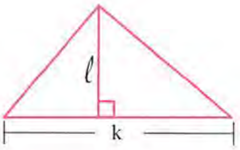
5 The algebraic term which expresses the area of the opposite figure is ....



(a) $x + y$ (b) $2x + 2y$

(c) xy (d) $x^2 y^2$

6 The algebraic term which expresses the area of the opposite figure is ....



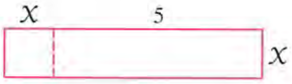
(a) $2k\ell$ (b) $\frac{1}{2}k\ell$

(c) $\frac{1}{2}k + \ell$ (d) $k\ell$

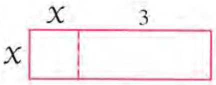
7 Which of the following represents the expression $3x + 2x$?

(a)

(b)



(c)



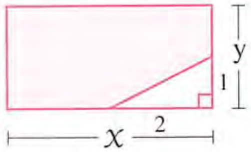
(d)

5 1 Arrange the terms of the algebraic expression $7\,a\,b + 5\,a^5\,b^3 - 3\,a^2\,b^5$ according to the descending order of the indices of a

2 Arrange the terms of the algebraic expression $5\,x + x^2 - 7 + x^3$ according to the ascending order of the indices of x

Geometric Applications

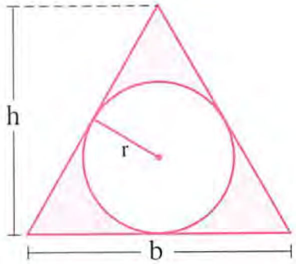
6 Write the algebraic expression which represents the area of the coloured part in the opposite figure and determine its degree.



7 **In the opposite figure :**

Write the algebraic expression which expresses the area of the coloured region , then state its degree

(The area of the circle = $\pi\,r^2$)



For excellent pupils

8 **Complete the following :**

1 If the algebraic term : $4\,x\,y^{k-1}$ is of the fifth degree , then k = ·········

2 If the two algebraic terms : $2\,a^3\,b^{m+1}$, $3\,a^n\,b^6$ are of the ninth degree , then n = ········· , m = ·········

3 If the degree of the algebraic term $y^{2\,m}$ is the degree of the algebraic term $5\,x^2\,y^4$, then m = ·········

4 If the algebraic expression $x^4 + 3\,x^{n+1} - 2\,x^2 + 5$ is arranged according to the descending order of the indices of x where $n \in \mathbb{Z}$, then n = ·········

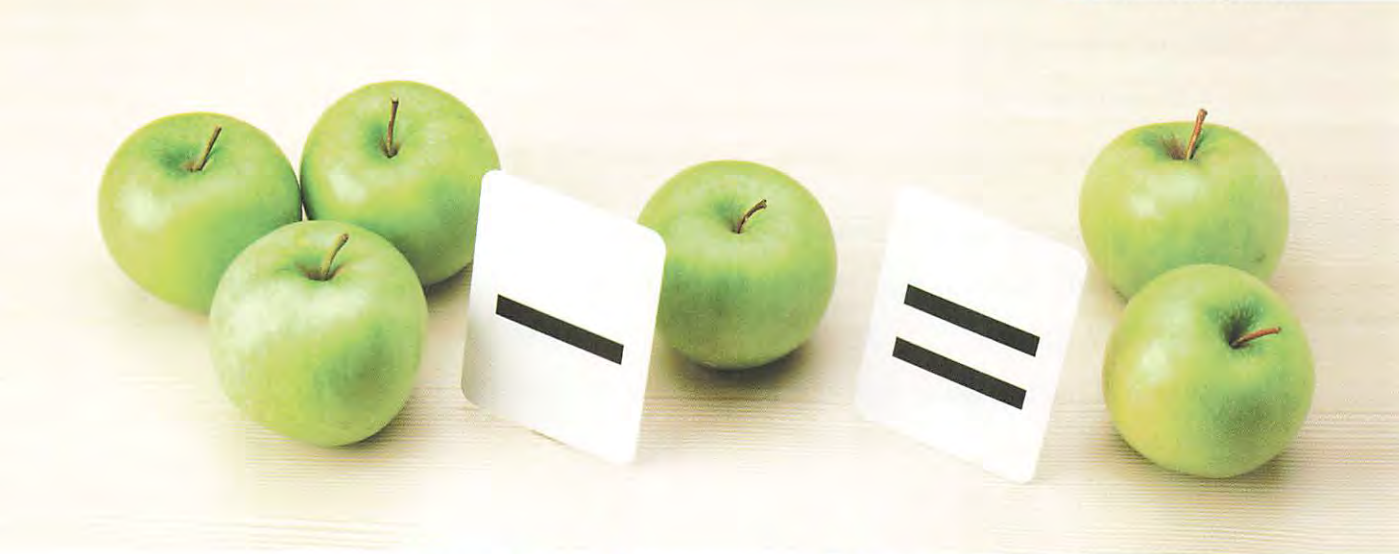
5 If the algebraic expression $2\,x\,y^2\,z^3 + 3\,x^2\,y\,z^n$ is of the sixth degree where n is a natural number , then $n \in \{$·········$\}$

EXERCISE 7 ?

Like Algebraic Terms

Interactive test

From the school book



Remember Understand Apply Problem Solving

1 **Find the result of each of the following :**

1. $3x + 2x$
2. $5x - 2x$
3. $4x - 11x$
4. $-7x - 3x$
5. $-5a^2 + 3a^2$
6. $-2x^2y + 3yx^2$
7. $2a + 3a - 4a$
8. $3ab - 2ba + 5ba - 6ab$
9. $\frac{5x}{4} + \frac{3x}{4}$
10. $\frac{3x}{7} - \frac{x}{7}$

2 **Answer each of the following :**

1. Subtract : y^2 from $-3y^2$
2. Subtract : $-6x^2y$ from $9x^2y$
3. What is the increase of : $-2x$ than $-5x$?
4. What is the increase of : $3a^2b$ than a^2b ?
5. What is the decrease of : $-3ab$ than $2ab$?
6. What is the decrease of : $6x^2y$ than $-7x^2y$?

3 **Complete each of the following :**

1. The result of subtracting $3a$ from $7a$ is ....
2. The result of subtracting $-3x^2$ from $5x^2$ is ....

3 The result of subtracting 2 m from zero is ....

4 The result of subtracting $2x$ from $-3x$ is ....

5 5 a increases 3 a by ....

6 $7x$ increases $-3x$ by ....

7 $4x$ decreases $7x$ by ....

8 $5x$ decreases $3x$ by ....

9 $2x$ decreases $4x$ by .... while $2x$ increases $4x$ by ....

4 Choose the correct answer from the given ones :

1 Which of the following are two like algebraic terms ?

(a) x^2 , $2x$ (b) $7x^2$, $2x^7$ (c) $3b^2a$, $-ab^2$ (d) $2a^2$, $2b^2$

2 Which of the following algebraic terms is like the algebraic term $2x^2y$?

(a) $2y^2x$ (b) yx^2 (c) $2x^2$ (d) x^2y^2

3 $7x^2 - 2x^2 =$ ....

(a) 5 (b) $5x^2$ (c) $5x$ (d) $9x^2$

4 $2xy - 2yx =$ ....

(a) xy (b) $2xy$ (c) $4yx$ (d) zero

5 $\frac{1}{2}x^2a + \frac{1}{2}ax^2 =$ ....

(a) $\frac{1}{4}x^2a$ (b) $\frac{1}{2}ax^2$ (c) $2ax^2$ (d) x^2a

6 $a + a + a =$ ....

(a) $3a^3$ (b) $3a$ (c) a^3 (d) $a + 3$

5 Complete each of the following :

1 .... $+ 2a^2 = 7a^2$

2 $3x^2 -$ .... $= x^2$

3 $2m^2 +$ .... $=$ zero

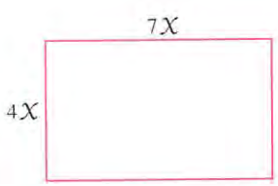
4 $5a^2b -$ .... $= 7a^2b$

5 $3a^2b + 2a^2b =$ .... $- 2a^2b$

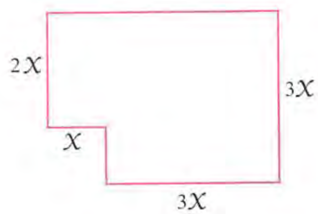
6 If $4x - y = 11$, $y = 3x$, then $x =$ ....

7 If $a = 2b$, $b = 15$, then the numerical value of the expression : $a + 2b + 5 =$ ....

8 The perimeter of the opposite rectangle equals .... length units.



9 The perimeter of the opposite figure equals .... length units.



6 If the sum of two terms is $12\,x^2 y$ and one of them is $4\,x^2 y$, find the other term.

7 **Reduce to the simplest form :**

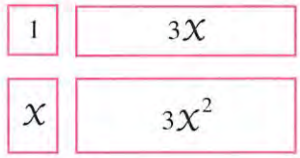
1. $3a + 2b + 5a + 4b$
2. $3x - 5y - x + 2y$
3. $2x - 4y - 9x - 3y$
4. $19m - 4n + 11m - 17n + 9n$
5. $2a + 7 - 5a - 4 - a$
6. $5a + 2b - 8a - 7b + 3a$
7. $2y - 3x - 7y - 5x - y + x$
8. $4a + 9b + 5a - 2b + 6b - 3a$

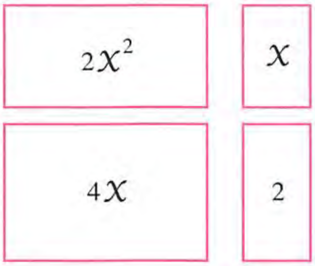
8 **Reduce each of the following algebraic expressions :**

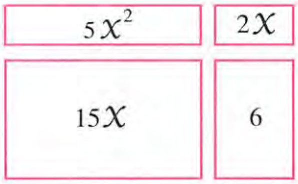
1. $5x - 3x^2 + 4 - 7x^2 - 6x - 1$
2. $6x^2 y - 3xy^2 + 2xy^2 - 5x^2 y + 2x^2 y^2$
3. $a^2 + 4a - 5 + 3a^2 - 6a + 1$
4. $5x^2 - 2x + 8 - 7x - 3 + x^2$

Geometric Applications

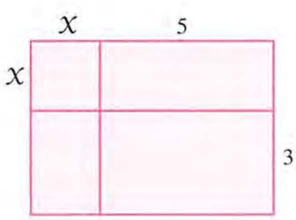
9 **Write the sum of the areas of the rectangles as an algebraic expression :**

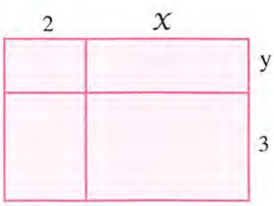
1. 

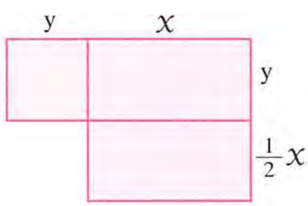
2. 

3. 

10 **Write the algebraic expression which expresses the perimeter of the coloured part in each of the following :**

1. 

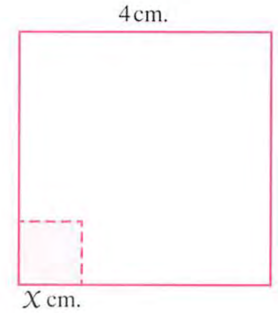
2. 

3. 

11 **In the opposite figure :**

A square whose side length is X cm. was cut from a square with side length 4 cm.

Find the perimeter of the remained part.



For excellent pupils

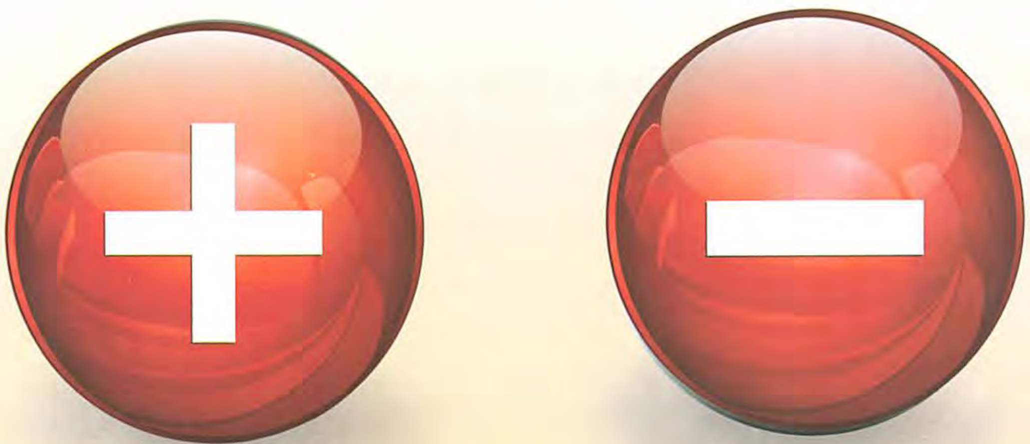
12 **Complete the following :**

1. If the two algebraic terms $2\,a^2\,b^{n+2}$ and $5\,a^2\,b^5$ are like terms , then n = ...
2. If the two algebraic terms $9\,X^m\,y^{m+n}$ and $4\,X\,y^3$ are like terms , then m = ... and n = ...
3. If $3\,X^m + 7\,X^n = 10\,X^6$ where $X \neq 0$, then m + n = ...

EXERCISE 8 Adding and Subtracting Algebraic Expressions

Interactive test

From the school book



Remember Understand Apply Problem Solving

1 **Find the sum of each of the following :**

1
$$\begin{array}{rrr} 3a & -4b & +6c \\ 5a & +6b & -2c \\ \hline \end{array}$$
....

2
$$\begin{array}{rrrr} 3a & -7b & -5c & +2 \\ -a & +4b & +c & -5 \\ 2a & & +3c & +3 \\ \hline \end{array}$$
....

3
$$\begin{array}{rrrr} 5x & +2y & -z & +2 \\ 7x & +y & -3z & +3 \\ -2x & -5y & +4z & -1 \\ \hline \end{array}$$
....

4
$$\begin{array}{rrrr} -2a^3 & +3a^2b & & -b^3 \\ & -5a^2b & +3ab^2 & -2b^3 \\ 5a^3 & & -4ab^2 & +3b^3 \\ \hline \end{array}$$
....

2 **Find the sum of each of the following :**

1 $3x-2y+5$, $x+2y-2$

2 $3\ell-4m+5n$, $4m-5n-\ell$

3 $3n^2+5n-6$, $-n^2-3n+3$

4 $5m^2+2\ell m$, $\ell^2-3m^2-2\ell m$

5 $2a^2b-3ab^2+b^3$, $-a^2b+b^3$

6 $3a^3-2ab^2+b^3$, $a^3+4a^2b-b^3$

3 **Find the sum of each of the following :**

1 $3a+2b-5$, $2a-7b+4$, $5b-4a+3$

2 $3x+3y-z$, $3x+3z-2y$, $x+2y+z$

3 $5x^2-3x+9$, x^2+2x-5 , $x-3-6x^2$

4 $3x-4x^2+2$, x^2+x-5 , $3+3x^2-4x$

5 $3x-4x^2+x^3$, $2x^2-6x+5$, $4+7x-x^3$

6 $2x^2-3xy+y^2$, $xy-2y^2+x^2$, $3xy-2x^2$

4 **Subtract :**

1. $x - 2$ **from** $2x - 5$
2. $2x + 6y - 7$ **from** $2x - 5y + 2$
3. $3x^2 - 1 - 5x$ **from** $1 - 5x + 6x^2$
4. $3ab^2 - 4a^2b - b^3$ **from** $a^3 - 2ba^2 + 2b^3$

---

5 **What is the increase of :**

1. $5a + 7b$ **than** $3a - 2b$
2. $7x + 5y + z$ **than** $2x - y + z$
3. $x^2 - 5x - 1$ **than** $3x^2 + 2x - 3$
4. $3x^2y - 5x$ **than** $3x - 4x^2y$

---

6 **What is the decrease of :**

1. $2a + 3b$ **than** $5b - 3a$
2. $3y^2 - 2xy + x^2$ **than** $3x^2 - 5xy + y^2$
3. $2a^2 - 3ab - 5b^2$ **than** $4b^2 + 3a^2 + ab$
4. $5x^2 + 2x$ **than** $7x^2 - x + 3$

---

7 **Choose the correct answer from the given ones :**

1. $2x + 3y$ increases $3y - 2x$ by ....

(a) $-6y$ (b) $-4x$ (c) $4x$ (d) $6y$

2. The result of subtracting $7a$ from $15a - 4$ is ....

(a) $-8a + 4$ (b) $8a + 4$ (c) $8a - 4$ (d) $-22a - 4$

3. The sum of $x + 2y - 3z$ and $-2y - x - 3z$ is ....

(a) $-6z$ (b) zero (c) $6z$ (d) $2x - 4y + 6z$

4. The additive inverse of $x + 2$ is ....

(a) $x - 2$ (b) $-x - 2$ (c) $2 - x$ (d) 2

5. The additive inverse of $3a - 4b + 5$ is ....

(a) $-3a + 4b + 5$ (b) $-3a - 4b - 5$

(c) $3a + 4b - 5$ (d) $4b - 3a - 5$

8 What is the expression which should be added to $2x - 3x^2 + 5$ to get $6 + x^2 - x$?

9 What is the expression which should be subtracted from $2x - 3y + 6z - \ell$ to get $5z - 4y + 3x - 2\ell$?

10 What is the expression which should be added to $3a^2 - 5ab + 2b^2$ to get zero ?

11 If the sum of two algebraic expressions is $5x - 7y + 9$ and if one of the two expressions is $2y + 3x - 4$, find the other expression.

12 Subtract $2b + 5a$ from $6a + 7b - 2$, then find the numerical value of the result when $a = 2$ and $b = 1$ « 5 »

13 Add $7x - 6y - z$ and $y - 3x - 5z$, then subtract the result from $5x + 5y - z$

14 What is the decrease of $2a - 8b - c$ than the sum of $3a - 3b + c$ and $2a - 4b - 8c$?

15 Add the expressions $3\ell - 2m + 7n$, $5m - 4\ell - 2n$ and $2\ell - 3n - m$, then subtract the result from $2\ell - 4m + 5n$

16 By what expression is $3x^2 - 5 + 2x$ increased than the sum of $x + 5x^2 + 1$ and $2x^2 - 4 - 2x$?

17 Add $3x^2 + 2xy - 5$ and $-2x^2 - 3xy + x$, then find the numerical value of the result when $x = -1$ and $y = 2$ « -3 »

18 If $x = a - 2b + c$, $y = 2a + 3b - 4c$ and $z = b - 4a + c$, find the expression $x + y - z$ in terms of a , b and c

Geometric Application

19 **In the following figure, calculate the total surface area of the two solids together :**

For excellent pupils

20 If $a + b = \frac{5}{4}$, $b + c = \frac{3}{4}$, $a + c = \frac{1}{2}$

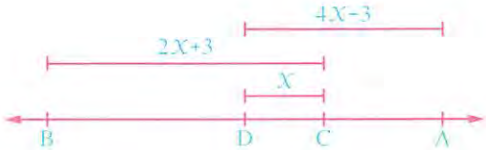
, then find the value of :

1. $a + 2b + c$ « 2 »
2. b « $\frac{3}{4}$ »

---

21 **In the opposite figure :**

Write the algebraic expression that expresses the length of $\overline{AB}$

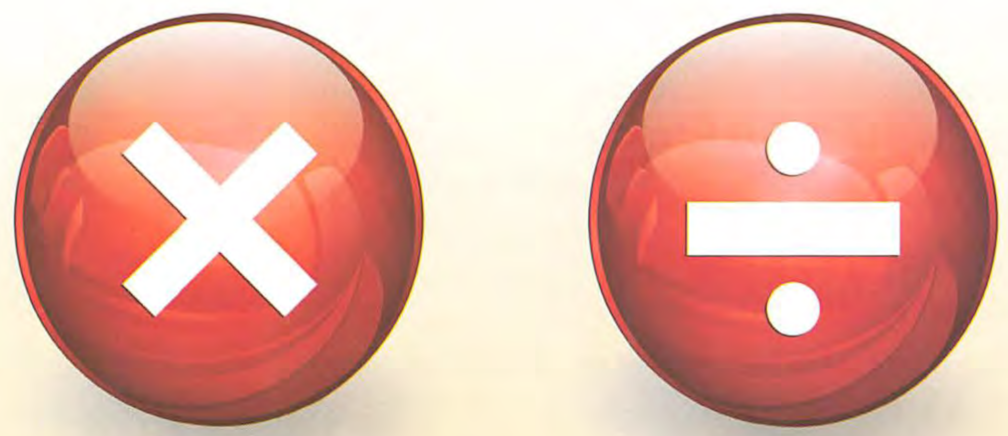


EXERCISE **9**

Multiplying and Dividing Algebraic Terms

Interactive test

From the school book



Remember   Understand   Apply   Problem Solving

1 Multiply :

1. $(5x) \times (3y)$
2. $(-3a) \times (7c)$
3. $(2x) \times (-3x)$
4. $-8y^5 \times (-7y^4)$
5. $(2xy) \times (-3x^2)$
6. $5x^3y^4 \times 2xy^2$
7. $5ab^2 \times (-2a^2b)$
8. $(x) \times (x) \times (2x)$
9. $(5) \times (-2a) \times (4a)$
10. $ab \times (-3a) \times (-2b)$
11. $(2x^3) \times (-3x^2) \times (-5x^4)$
12. $(4x^3y) \times (-2xy^2) \times (-3x^2y^5)$

2 If the symbols represent non-zero integers , find the quotient of each of the following :

1. $6a \div 2$
2. $12x \div (-x)$
3. $10c \div 2c$
4. $-14x^2 \div 7x$
5. $-25a^6 \div (-5a^2)$
6. $24c^5 \div (-24c^5)$
7. $9x^5y^4 \div 6x^3y$
8. $-32a^3b^6 \div (-4a^3b^2)$
9. $8m^4n^3 \div (-4mn^2)$
10. $-18x^5y^6z^3 \div (-6x^3y^3z^3)$

3 Simplify :

1. $\frac{2}{3}t^4 \times \frac{3}{2}t^4$
2. $\frac{2}{7}a^2 \times 21a^5$
3. $\frac{15a^3b}{2} \times \frac{8ab^2}{10}$
4. $(3x^3) \times \left(\frac{1}{6}x^2\right)$
5. $\frac{4h^3k^3}{7} \times \frac{21hk^5}{2}$
6. $4m^3 \times \frac{1}{4}m^2 \times (-7m)$

4 Choose the correct answer from the given ones :

1 $(2 x) \times (5 x) = \cdots\cdots$

(a) $10 x$ (b) $7 x$ (c) $7 x^2$ (d) $10 x^2$

2 $2 x y \div \text{zero} = \cdots\cdots$

(a) $2 x y$ (b) $x y$ (c) zero (d) meaningless.

3 $3 a^4 b \times 5 a^2 b^2 \times 2 a^3 = \cdots\cdots$

(a) $60 a^{11} b^3$ (b) $30 a^{10} b^2$ (c) $15 a^{10} b^3$ (d) $30 a^9 b^3$

4 $-6 x^3 y \div 2 x y = \cdots\cdots$

(a) $-3 x^3$ (b) $-3 x^2 y$ (c) $-3 x^4 y^2$ (d) $-3 x^2$

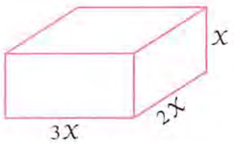
5 If 2 b is the edge length of a cube , then its volume is $\cdots\cdots$

(a) $4 b^2$ (b) $2 b^3$ (c) $4 b^3$ (d) $8 b^3$

6 If the area of a rectangle is $24 x^3$ and its length is $8 x^2$, then its width is $\cdots\cdots$

(a) $3 x^5$ (b) $3 x$ (c) $3 x^2$ (d) 3

7 The volume of the opposite cuboid equals $\cdots\cdots$



(a) $6 x^3$ (b) $6 x$

(c) $5 x^3$ (d) $6 x^2$

8 If the price of 4 shirts is x pounds , then the price of 40 shirts of the same kind equals $\cdots\cdots$ pounds.

(a) $10 x$ (b) $\frac{x}{40}$ (c) $\frac{5 x}{2}$ (d) $\frac{40}{4}$

9 You drove 200 km. in 3 hours. Which expression represents your average speed if "d" represents distance and "t" represents time ?

(a) d t (b) $\frac{d}{t}$ (c) $\frac{3 t}{200 d}$ (d) d + t

5 Complete the following if the symbols represent non-zero integers :

1 $\frac{4 y^5}{y^3} + 2 y^2 = \cdots\cdots$

2 $(6 x^3 \div 2 x) - 2 x = \cdots\cdots$

3 $(10 x^2 + 5 x^2) \div 5 x = \cdots\cdots$

4 $(5 a \div a) + \cdots\cdots = \text{zero}$

5 $81 \ell^4 \div \cdots\cdots = 27 \ell^3$

6 $\cdots\cdots \div 7 a^3 = -5 a^2$

7 $15 x^2 y^3 \div \cdots\cdots = 3 x y^2$

8 $\cdots\cdots \div (-4 x^3 y^2) = 16 x^4 y^4$

9 If $3 a \times n = 12 a^4$, then $n = \cdots\cdots$

6 Complete :

1 $36 a^5 b^8 = 12 a^3 b^2 \times \cdots\cdots$

2 $9 a^5 = 3 a \times \cdots\cdots$

3 $-4c^3d^3 = 2cd^2 \times \dots\dots$

4 $98a^7b^4 = \dots\dots \times 14a^7b$

5 $36a^8b^5 = 6ab^2 \times 3a^4b \times \dots\dots$

6 $42x^4y^5 = 3x^2y \times 2xy \times \dots\dots$

7 If $x \neq$ zero , $y \neq$ zero and n is a positive number , simplify :

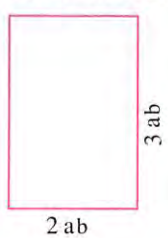
1 $\dfrac{27y^{2n+4}}{3y^{2n+3}}$

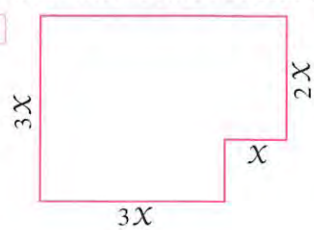
2 $\dfrac{-24x^{5n+1}y^{2n}}{36x^{5n}y^{n}}$

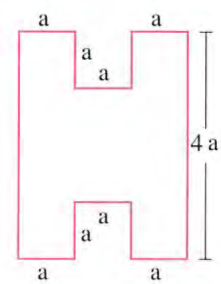
Geometric Applications

8 A cuboid of dimensions x cm. , $2x$ cm. and $4x$ cm. was melted to make small cubes with edge length x cm. for each one.
Find the maximum number of the resulted small cubes. « 8 »

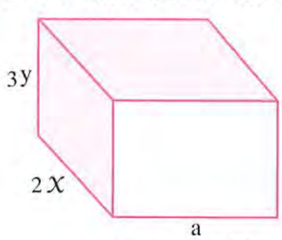
9 Calculate the perimeter and the area of each figure :

1 

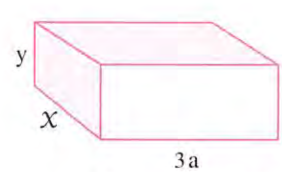
2 

3 

10 Calculate the sum of the total surface areas of the two solids :



First solid

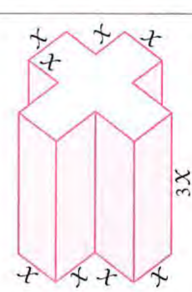


Second solid

For excellent pupils

11 Three tennis balls fit into a cuboid box where the balls touch all faces of the box. Calculate the ratio between the volume of the three balls and the volume of the box.
(Given that : the volume of the sphere = $\frac{4}{3}\pi r^3$, $\pi \simeq 3.14$) « $\frac{157}{300}$ »

12 Calculate the total surface area and the volume of the opposite solid.

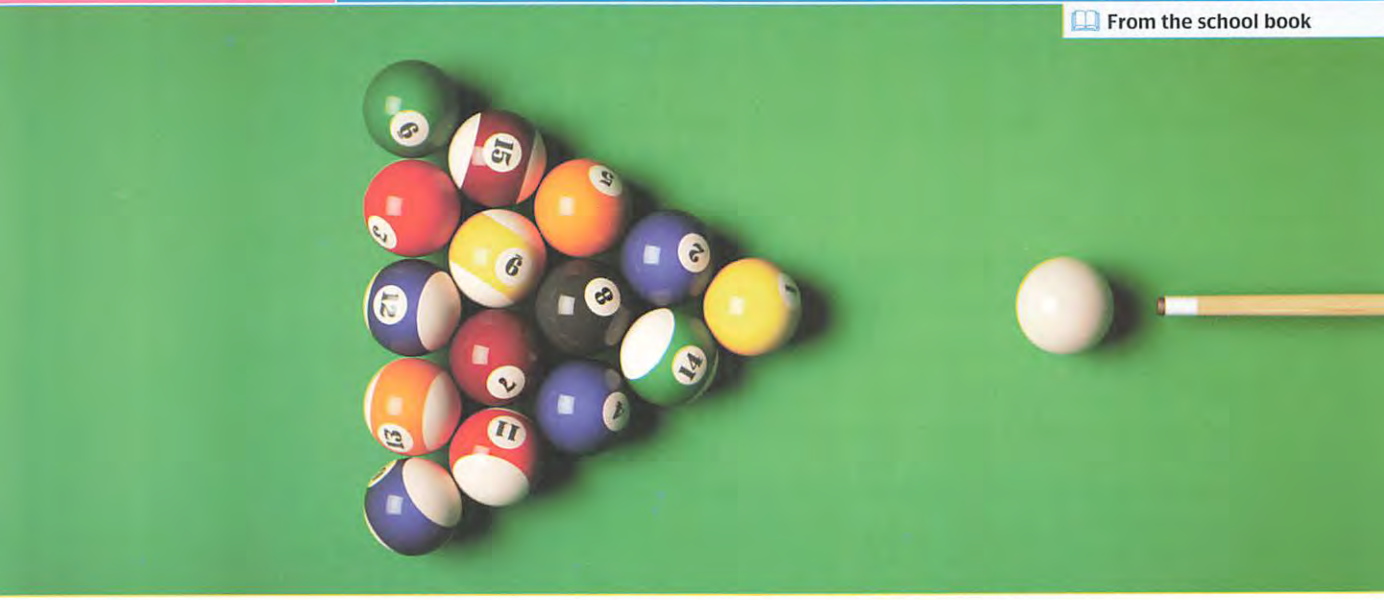


EXERCISE **10**

Multiplying a Monomial by an Algebraic Expression

Interactive test

From the school book



Remember · Understand · Apply · Problem Solving

1 **Find the following products :**

1. $a\,(a+1)$
2. $a\,(a-2)$
3. $3\,x\,(7\,y-4\,z)$
4. $-3\,(y+3)$
5. $-2\,c\,(7-3\,c)$
6. $2\,x\,(3\,x^2+4\,y^2)$
7. $-5\,x\,(2\,x+y-3\,z)$
8. $3\,x\,y\,(2\,x^2-5\,x^2\,y-4\,y^2)$
9. $\ell\,m^2\,(\ell^2-3\,m\,\ell-4\,m^2)$
10. $\frac{1}{3}\,x^2\,(6\,x^2-9\,x\,y-3\,y^2)$

2 **Complete the following :**

1.
$$\begin{array}{r} 2\,y^2-y-5 \\ \times\ 2\,y \\ \hline \dots\dots\dots\dots \end{array}$$

2.
$$\begin{array}{r} 4\,x\,y+3\,x^2-5 \\ \times\ (-\,y^2) \\ \hline \dots\dots\dots\dots \end{array}$$

3.
$$\begin{array}{r} -5\,x+4\,y-x\,y \\ \times\ 4\,x\,y \\ \hline \dots\dots\dots\dots \end{array}$$

4.
$$\begin{array}{r} -2\,x+y \\ \times\ \dots\dots \\ \hline 4\,x^2\,y+\dots\dots \end{array}$$

3 **Complete the following :**

1. $x\,(\dots\dots-2\,x)=6\,x-\dots\dots$
2. $3\,x\,(\dots\dots+5\,y)=6\,x^2+\dots\dots$
3. $2\,x\,(\dots\dots-5\,y)=8\,x^3-\dots\dots$
4. $3\,x\,(\dots\dots-4\,x\,y^2)=15\,x^3\,y-\dots\dots$
5. $-2\,a\,b\,(\dots\dots+2\,a^2\,b)=-6\,a^2\,b^3-\dots\dots$
6. $2\,x\,(3\,x-\dots\dots)=\dots\dots-10\,x$
7. $-4\,a\,(2\,a\,b-\dots\dots)=\dots\dots+8\,a\,b^2$
8. $\dots\dots\,(3\,x+y)=6\,x^2+\dots\dots$

9 $4y(\cdots\cdots + \cdots\cdots) = 20y^2 + 8xy$

10 $abc(\cdots\cdots + \cdots\cdots - \cdots\cdots) = a^2bc + ab^2c - abc^2$

11 $3xy(\cdots\cdots - \cdots\cdots - 5x^2) = 6x^2y - 12xy^2 - \cdots\cdots$

12 $\cdots\cdots(\cdots\cdots + 3m^2n) = 10m^2n^2 + 6m^3n^3$

4 **Put in the simplest form :**

1 $3a(a-b) + 4a(2a+b)$

2 $3a(4a-2) - 4a(3a-2)$

3 $3a(4a-1) + 2a(a+3) - 5a(2a-1)$

4 $2x(x+y) - y(2x-y) + 2(y^2 - x^2)$

5 Simplify : $2a(3a-1) + 3a(a+2)$, then find the value of the result when $a = 1$ « 13 »

6 Simplify : $2a(3a+b) - 3b(a+b)$, then find the value of the result when $a = b = 1$ « 2 »

7 Simplify : $x(2x-y) - 2y(x-y)$, then find the numerical value of the result when $x = 2$ and $y = -1$ « 16 »

8 Find the sum of : $2x(3x-2y)$, $y(x+y)$ and $x^2 - y^2$, then find the value of the result if $x = -2$ and $y = -1$ « 22 »

9 Simplify : $3(1-2x) - (x^2 - 5x + 3) + 2x(x+3)$, then find the numerical value of the expression when $x = -2$ « −6 »

10 Simplify : $ab(3a-2b) - 2a(ab-b^2) + b(4ab-a^2)$, then find the numerical value of the result when $a = 1$ and $b = -3$ « 36 »

11 Simplify : $2x[x - 2(y-x)] - 3y[y - 2(x-y)]$, then find the numerical value of the result when $x = y = 1$ « −1 »

12 If $a + 3b = 7$, $c = 3$, find the numerical value of the expression : $a + 3(b+c)$ « 16 »

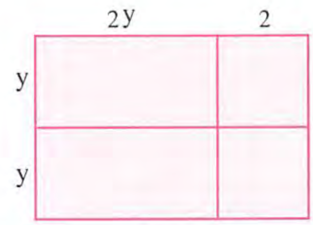
Geometric Applications

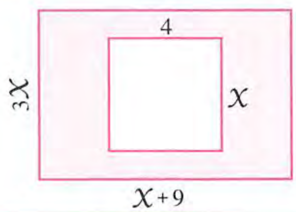
13 An equilateral triangle is of side length $(22x - 3y + 5z)$ cm. Find its perimeter.

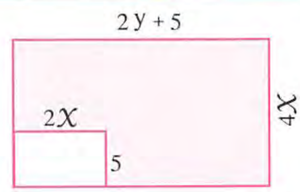
14 The dimensions of a rectangle are $(2a+b)$ cm. and $(4a-2b)$ cm. Find its perimeter.

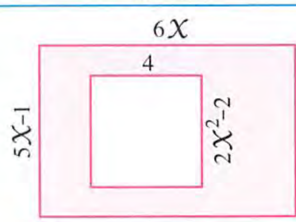
15 **Find the algebraic expression which expresses the area of the coloured part in each of the following :**

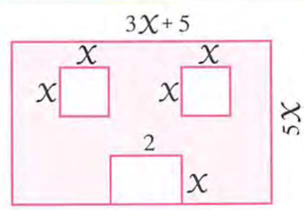
1

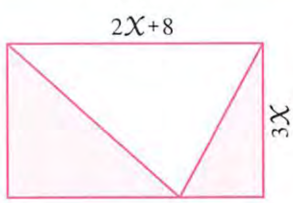

2


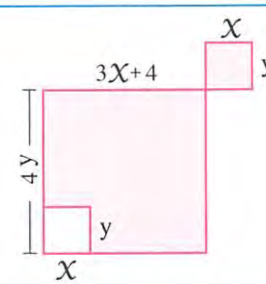
3


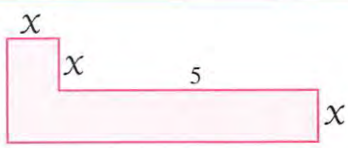
4


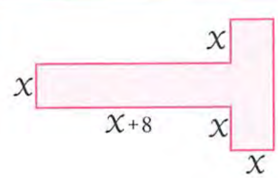
5


6


7


8


9


10


For excellent pupils

16 The width of a rectangle is X cm. and its length exceeds twice its width by 3 cm. Find its area in terms of X

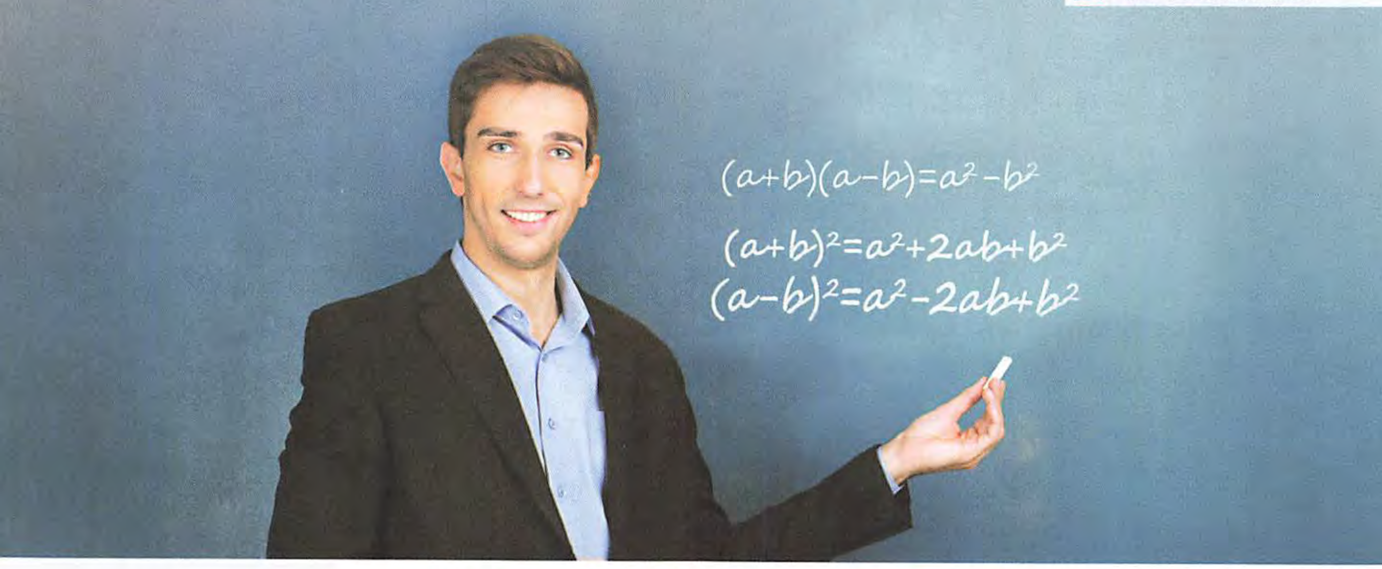
17 A cuboid of a square base of side length 3 X cm. If its height is $(2X^2 + 3)$ cm. , find its volume in terms of X

EXERCISE **11** ?

Multiplying a Binomial by an Algebraic Expression

Interactive test

From the school book



Remember · Understand · Apply · Problem Solving

1 **Write the missing terms in each of the following products :**

1. $(x + 3)(x + 2) = \cdots\cdots + 5x + 6$
2. $(x + 2)(x - 5) = x^2 \cdots\cdots - 10$
3. $(y - 4)(y + 5) = \cdots\cdots + y - \cdots\cdots$
4. $(a - 3)(a - 7) = a^2 - \cdots\cdots + \cdots\cdots$
5. $(2x - 5)(x + 7) = \cdots\cdots + \cdots\cdots - 35$
6. $(4x - 3y)(2x + 5y) = 8x^2 + \cdots\cdots - 15y^2$

2 **Find by inspection the product of each of the following :**

1. $(x + 2)(x + 4)$
2. $(y - 5)(y + 2)$
3. 📖 $(5m - 2)(6m + 1)$
4. 📖 $(4x + 1)(2x + 3)$
5. 📖 $(3a - 5b)(2a + 7b)$
6. $(2x - y)(3x + 4y)$
7. $(b^2 - 4)(b^2 + 2)$
8. $(3m^2 + 8)(2m^2 - 3)$
9. $(x - y)(7y - x)$
10. $\left(\frac{3}{2}a - 6b\right)\left(\frac{3}{2}a + 4b\right)$

3 **Find by inspection the expansion of each of the following :**

1. $(a + 3)^2$
2. $(2y + 3)^2$
3. 📖 $(4m - 7)^2$
4. 📖 $(3x + y)^2$
5. 📖 $(x - 3y)^2$
6. $(-\ell - m)^2$
7. $(-4a - 7)^2$
8. 📖 $(2x + 3y)^2$
9. $\left(4x^2 - \frac{1}{2}y^2\right)^2$

4 Find by inspection the product of each of the following :

1. $(a + 3)(a - 3)$
2. $(4m - 7)(4m + 7)$
3. $(6x - 2y)(6x + 2y)$
4. $(a^2 + 9)(a^2 - 9)$
5. $(-12m + 9)(-12m - 9)$
6. $(3x^2 - 5y^2)(3x^2 + 5y^2)$
7. $(\ell m + 6n)(\ell m - 6n)$
8. $\left(\frac{1}{2}x - \frac{1}{3}y\right)\left(\frac{1}{2}x + \frac{1}{3}y\right)$
9. $(2x - 3y)(3y + 2x)$
10. $(y - 3)(y + 3)(y^2 + 9)$
11. $(x - 2y)(x + 2y)(x^2 + 4y^2)$

5 Find the following products :

1. $(x + 3)(x^2 + x + 1)$
2. $(x + 1)(x^2 - x + 1)$
3. $(2y + 1)(y^2 + y + 5)$
4. $(2x + 3)(4x^2 - 6x + 7)$
5. $(2x - y)(2x^2 - 3xy + y^2)$
6. $(a^2 - 3b^2)(3a^4 - 2a^2b^2 + 5b^4)$
7. $(2a + a^2 - 5)(2a^2 - 1)$
8. $(4 + 2a + 3a^2)(2 - a)$
9. $(x + 4)^2(3x + 2)$
10. $(3x + 2y)^3$

6 Choose the correct answer from the given ones :

1. The middle term in the expansion of $(3x - 1)^2$ is ....
   (a) $3x$ (b) $-6x$ (c) $6x$ (d) $6x^2$
2. The middle term in the expansion of $(2a + 3b)^2$ is ....
   (a) $12ab$ (b) $-12ab$ (c) $6ab$ (d) $-6ab$
3. The coefficient of ab in the expansion of $(4a - 5b)^2$ is ....
   (a) 40 (b) 20 (c) -20 (d) -40
4. If $x = -1$, then the numerical value of the expression $(x + 1)^2$ is ....
   (a) zero (b) 1 (c) 2 (d) 3
5. If $x = \frac{4}{3}$, then $(x - 2)(x + 2) =$ ....
   (a) $\frac{4}{3} - 2$ (b) $\left(\frac{4}{3}\right)^2 - 2$ (c) $\left(\frac{4}{3}\right)^2 - 4$ (d) $\left(\frac{4}{3}\right)^2 + 4$
6. If $x - y = 3$ and $x + y = 5$, then $x^2 - y^2 =$ ....
   (a) 2 (b) -2 (c) 8 (d) 15
7. If $x^2 = 10$ and $y^2 = 7$, then $(x + y)(x - y) =$ ....
   (a) 70 (b) 17 (c) 3 (d) -3
8. If $(x + y)^2 = 26$ and $x^2 + y^2 = 20$, then $xy =$ ....
   (a) 3 (b) 6 (c) 9 (d) 12
9. If $x^2 = 16$, $y^2 = 9$ and $xy = 12$, then $(x - y)^2 =$ ....
   (a) 49 (b) 165 (c) -1 (d) 1

10 If $\mathcal{X} + y = 7$, then $\mathcal{X}^2 + 2\mathcal{X}y + y^2 = \cdots\cdots$

(a) 7 (b) 14 (c) 49 (d) 28

11 If $(2\mathcal{X} + y)^2 = 4\mathcal{X}^2 + k\mathcal{X}y + y^2$, then $k = \cdots\cdots$

(a) 2 (b) 4 (c) 8 (d) 6

12 If $(\mathcal{X} - 3)(\mathcal{X} + 3) = \mathcal{X}^2 + k$, then $k = \cdots\cdots$

(a) 9 (b) 6 (c) -9 (d) -6

13 If $(5 - \mathcal{X})(\mathcal{X} + 5) = k - \mathcal{X}^2$, then $k = \cdots\cdots$

(a) -25 (b) 5 (c) 10 (d) 25

14 If $(\mathcal{X} - y)(2\mathcal{X} + y) = 2\mathcal{X}^2 + k\mathcal{X}y - y^2$, then $k = \cdots\cdots$

(a) -1 (b) 1 (c) 3 (d) 4

15 Which of the following is equivalent to $\left(a + \frac{b}{2}\right)^2$?

(a) $a^2 + \frac{b^2}{2}$ (b) $a^2 + \frac{b^2}{4}$

(c) $a^2 + \frac{ab}{2} + \frac{b^2}{2}$ (d) $a^2 + ab + \frac{b^2}{4}$

16 $(a + b)^2 - (a - b)^2 = \cdots\cdots$

(a) $-4ab$ (b) $4ab$ (c) $2a^2$ (d) zero

7 Complete the following :

1 $(2\mathcal{X} - 1)^2 = \cdots\cdots - 4\mathcal{X} + 1$

2 $(\mathcal{X} + 7)(\mathcal{X} - 7) = \mathcal{X}^2 - \cdots\cdots$

3 $(\mathcal{X} - 5)(\cdots\cdots) = \mathcal{X}^2 - 25$

4 $(3\mathcal{X} + \cdots\cdots)(\cdots\cdots - y) = 9\mathcal{X}^2 - y^2$

5 $(\mathcal{X} + 5)(\mathcal{X} + \cdots\cdots) = \mathcal{X}^2 + \cdots\cdots + 15$

6 $(a + \cdots\cdots)^2 = \cdots\cdots + \cdots\cdots + 16$

7 $(2\mathcal{X} + 5)(3\mathcal{X} + \cdots\cdots) = \cdots\cdots + \cdots\cdots + 10$

8 $(4\mathcal{X} + \cdots\cdots)(\cdots\cdots - 5) = 8\mathcal{X}^2 - \cdots\cdots - 5$

9 $(2a + \cdots\cdots)(\cdots\cdots - 5b) = 8a^2 + \cdots\cdots - 15b^2$

10 $(\cdots\cdots + 4)(\mathcal{X} + \cdots\cdots) = \mathcal{X}^2 + 7\mathcal{X} + \cdots\cdots$

11 $(\cdots\cdots - 3b)^2 = \cdots\cdots - 24ab + \cdots\cdots$

8 **Reduce to the simplest form :**

1 $(x-3)^2-9$

2 $2a(5a+4b)(5a-4b)$

3 $3(m-5)(m+2)$

4 $(x-2)(x+2)-x(x+1)$

5 $(2x+3)(2x-3)-2(2x^2+1)$

6 $(2x+3)^2+(x-2)(x+5)$

7 $(x+1)^2-x(x+2)$

8 $(x-2)^2-(x^2-4)$

9 $(2x-y)(2x+y)-(x-2y)^2$

10 $2a(3a+b)+(a-b)^2$

11 $(5x-2y)^2-(5x+2y)^2$

12 $(7xy-3x)^2-(5xy-x)^2$

13 $2(3x-5)(2x+1)-3(4x+1)(x-7)$

9 **Multiply, then find the numerical value of the expression when $x=1$ and $y=-2$:**

1 $(x-5y)(x+5y)$

2 $(3x+y)(x+3y)$

3 $(x+4)(3x+2)$

4 $(2y+7)(3y+4)$

5 $|(x+2y)(x-2y)|$

6 $(2x+y)^2$

10 Reduce to the simplest form :

$(2x-5)(2x+5)+25$, then find the numerical value of the result when $x=2$ « 16 »

11 Reduce : $(x-y)^2+2xy$, then find the numerical value of the result when $x=-1$, $y=2$ « 5 »

12 Reduce : $(2x-2)^2+(x-2)(x+2)$, then find the numerical value of the result when $x=-1$ « 13 »

13 Reduce to the simplest form : $(x+2)(x+5)-x(6-x)$, then find the numerical value of the result when $x=-1$ « 11 »

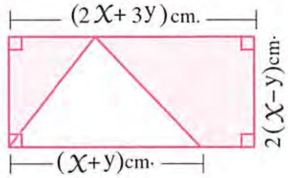
14 Subtract : $(x-3)^2$ from $(2x+1)(x+9)$

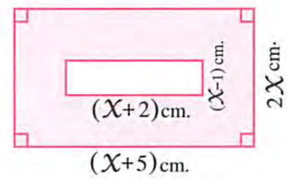
15 If $a=3x-4$, $b=x+2$ and $c=2x-3$,
find the value of the expression $ab-c^2$, when $x=$ zero « −17 »

16 If $a=4x-3$, $b=2x+1$ and $c=3x-2$,
find the value of the expression $2a^2-3b^2+bc$ in terms of x

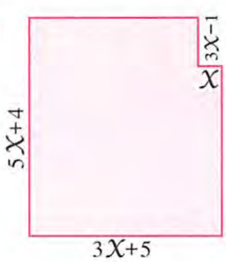
Applications on multiplying the algebraic expressions

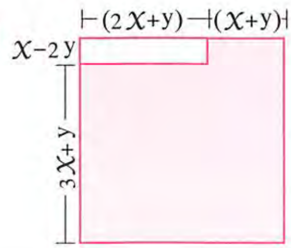
17 **Find the area of the coloured part in each of the following figures :**

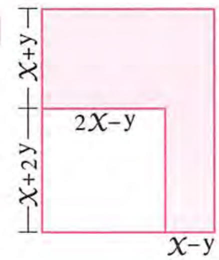
1


2


18 **Write an expression for the perimeter and area of each coloured region :**

1


2


3


19 **Use the multiplication by inspection to find the following easily :**

1 $(101)^2$ 2 $\left(10\frac{1}{2}\right)^2$ 3 $(99)^2$

4 64×56 5 98×102 6 19×21

7 201×199 8 $(49)^2$ 9 $(41)^2$

For excellent pupils

20 If $(2 - y)^3 = 8 - 12y + 6y^2 - y^3$ **, find :** $(2 - y)^4$

21 If $(X + 8)(X + 2) = 100$

, find the value of : $(X + 4)(X + 6)$ « 108 »

22 A square whose side length is $(2X + 5)$ cm. Find its area in terms of X
If two opposite sides of it increases by $(X - 1)$ cm. and the two other sides decreases by the same value , find in terms of X the area of the resulting rectangle.

EXERCISE 12 ? Dividing an Algebraic Expression by a Monomial

Interactive test

From the school book

Remember · Understand · Apply · Problem Solving

1 **If the symbols in the following expressions represent integers not equal to zero , find the quotient in each case :**

1. $5a - 10$ **by** 5
2. $12x + 15y$ **by** -3
3. $4a^2 + 6a$ **by** $2a$
4. $24x^3 - 18x^2$ **by** $-6x^2$
5. $12a^2b + 20ab^2$ **by** $4ab$
6. $16a^3b^2 - 24a^2b^2$ **by** $4a^2b$
7. $42x^3 + 12x^2 - 24x$ **by** $6x$
8. $3a^2b - 6ab^2 + 12ab$ **by** $-3ab$
9. $2a^3b^2 - 4a^2b^3 + 6a^2b^2$ **by** $2b^2a^2$

2 **If the symbols represent non-zero integers , find the quotient of each of the following :**

1. $\dfrac{26x^2 + 14x^4}{2x}$
2. $\dfrac{18m^4 + 32m^2}{-2m^2}$
3. $\dfrac{48x^3 - 80x^2}{8x^2}$
4. $\dfrac{9\ell^3m^4 - 18\ell m^2}{3\ell m^2}$
5. $\dfrac{32x^5 - 48x^3 + 72x^7}{-8x^3}$
6. $\dfrac{15x^3y^2 + 6xy^2 - 3xy}{-9xy}$
7. $\dfrac{5\ell^2m^3n - 20m^2n^3 - 15mn^5}{-5n}$
8. $\dfrac{18x^4y^2 - 42x^5y^4 + 30x^6y^5}{-6x^2y^2}$

3 Choose the correct answer from those given :

1. $(x^2 + x) \div x = \cdots\cdots$ $(x \neq 0)$
   (a) zero (b) x (c) $2x + 1$ (d) $x + 1$
2. $(15a + 5) \div 5 = \cdots\cdots$
   (a) $3a$ (b) $10a$ (c) $3a + 1$ (d) $4a$
3. $(4a^3 - 2a) \div (-2a) = \cdots\cdots$ $(a \neq 0)$
   (a) $-2a^2$ (b) $-2a^2 + 1$ (c) $2a^2 + 1$ (d) -1
4. $(15x^4 + 5x^3) \div 5x^3 = \cdots\cdots$ $(x \neq 0)$
   (a) $3x^2 + x$ (b) $5x^2 + 1$ (c) $3x + 1$ (d) $4x^4$
5. $(10x^2 - 15x^2y) \div 5x = \cdots\cdots$ $(x \neq 0)$
   (a) $2x - 3y$ (b) $2x - 3xy$ (c) $2x + 3y$ (d) $2x - 3$
6. $(3x^2y - \cdots\cdots) \div 3xy = x - 2y$ $(xy \neq 0)$
   (a) $6x$ (b) $6xy^2$ (c) $6y^2$ (d) $-6xy^2$
7. If $(6x^2y^3 + kxy) \div 6x = xy^3 - 12y$ where $(x \neq 0)$, then $|k| = \cdots\cdots$
   (a) -72 (b) -2 (c) 2 (d) 72

4 Complete the following :

1. $\dfrac{15n^3 - 9m^4n^2}{-3n^2} = \dfrac{15n^3}{-3n^2} + \dfrac{-9m^4n^2}{-3n^2} = \cdots\cdots + \cdots\cdots$
2. $(4a^2 + 2a) \div 2a = \cdots\cdots$
3. $\dfrac{4x^2y - 2xy^2}{-2xy} + 2x - y = \cdots\cdots + 2x - y = \cdots\cdots$
4. $\dfrac{16x^4y^2 - 12x^3y^3 + 24x^2y^4}{8x^2y} = \dfrac{16x^4y^2}{8x^2y} - \dfrac{\cdots\cdots}{8x^2y} + \dfrac{\cdots\cdots}{8x^2y}$
   $= \cdots\cdots - \cdots\cdots + \cdots\cdots$
5. $(12a^3b^3 - 3a^5b^4) \div \cdots\cdots = 4ab - \cdots\cdots$
6. $\dfrac{\cdots\cdots - 6ab^2 + 12ab}{\cdots\cdots} = ab + 2b - 4$
7. If $\dfrac{4x^2y - 8xy^4}{-4xy} = kx + \ell y^3$, then $|k| + \ell = \cdots\cdots$
8. If $x = -1$, then $\left|\dfrac{-4x^2 + 20x^3}{-4x}\right| = \cdots\cdots$
9. If $\dfrac{a^2 - ab - 3a}{a} = 2$, $a + b = 3$, then $a^2 - b^2 = \cdots\cdots$

5 **Multiply :** $4\,x^2$ **by** $(3\,x^3 y^3 - 6\,x^2 y^3)$, then divide the result by $12\,x^4 y^2$

6 Add the quotient of : $(x^3 y - 3\,x y + 2\,x^2 y^2 - 4\,x y)$ **by** $-x y$

to the expression $2\,x y - 5\,x^2 + 3\,y^2$

7 **Divide :** $(12\,y^3 - 8\,y^2)$ **by** $4\,y$, then find the absolute value of the result when $y = \frac{1}{2}$ « $\frac{1}{4}$ »

8 **Divide :** $(12\,x^3 y^2 - 4\,x^2 y^3)$ **by** $4\,x^2 y^2$,

then find the numerical value of the result when $x = 1$ and $y = -1$ « 4 »

9 **Divide :** $(16\,x^3 + 8\,x - 12\,x^2)$ **by** $4\,x$, then add the result to $3\,x - x^2 + 7$,

and find the numerical value of the result when $x = 1$ « 12 »

Geometric Applications

10 The area of a rectangle is $(24\,x^3 + 18\,x^2 + 42\,x)$ cm.^{2} and its width is $6\,x$ cm.

Find the length of the rectangle in terms of x

11 The area of a rectangle is $(8\,a^4 b^3 + 12\,a^3 b^4 - 8\,a^2 b^2)$ cm.^{2} and its length is $(4\,a^2 b^2)$ cm.

Find its width if $a = 1$ and $b = 2$ « 14 cm. »

12 The area of a triangle is $(12\,x^2 + 9\,x)$ cm.^{2} and the length of its base is $3\,x$ cm.

Find the height of the triangle corresponding to this base.

For excellent pupils

13 The volume of a cuboid is $(12\,x^3 + 8\,x^2 y)$ cm.^{3} and its base is a square of side length

$2\,x$ cm. Find its height when $x = 1$ and $y = 2$ « 7 cm. »

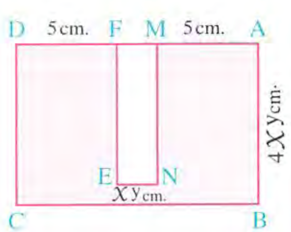
14 **In the opposite figure :**

ABCD and MNEF are two rectangles.

Use the given data on the figure to find

the length of $\overline{FE}$ given that the area of the

coloured part is $(3\,x^2 y^2 + 35\,x y)$ cm.^{2}



EXERCISE 13 ?

Dividing an Algebraic Expression by Another One

Interactive test

From the school book

Remember Understand Apply Problem Solving

1 Find the quotient of dividing each of the following expressions , given the divisor in each $\neq 0$:

1. $x^2 + 5x + 6$ by $x + 2$
2. $y^2 - 9y + 20$ by $y - 4$
3. $x^2 - 5x - 14$ by $x - 7$
4. $2x^2 + 13x + 15$ by $x + 5$
5. $3x^2 + 2x - 8$ by $3x - 4$
6. $x^2 - 6 - x$ by $x + 2$
7. $14 - 17x - 6x^2$ by $7 + 2x$
8. $8x^2 + 6xy - 9y^2$ by $4x - 3y$
9. $4x^2 - 16xy + 16y^2$ by $2x - 4y$
10. $x^2 - 1$ by $x + 1$
11. $16y^2 - 4x^2$ by $4y - 2x$

2 Find the quotient of dividing each of the following expressions , given the divisor in each $\neq 0$:

1. $x^3 + 5x^2 + 7x + 2$ by $x^2 + 3x + 1$
2. $6x^3 + 7x^2 - 18x + 5$ by $3x^2 - 4x + 1$
3. $2x^3 - 43x - 9x^2 - 20$ by $x^2 - 4 - 7x$
4. $3x^2 + x^3 - x - 3$ by $x^2 - 1$
5. $8x^3 - 20x^2 - 10 + 4x$ by $4x^2 + 2$
6. $x^4 + 3x^2 + 2$ by $x^2 + 1$
7. $x^3 - x$ by $x - 1$
8. $8x^3 - 1$ by $4x^2 + 2x + 1$

3 **Find the quotient of dividing each of the following expressions, given the divisor in each $\neq 0$:**

1. $x^3 + 5x^2 + 7x + 2$ **by** $x + 2$
2. $x^3 - x^2 - 9x - 12$ **by** $x - 4$
3. $6x^3 - 5x^2 - 14x + 12$ **by** $2x - 3$
4. $9x + 6x^3 + 10 - 5x^2$ **by** $2 + 3x$
5. $15 - 7x^2 + 3x - 4x^3$ **by** $5 - 4x$
6. $3x^3 - 4x + 1$ **by** $x - 1$
7. $x^3 - 27$ **by** $x - 3$
8. $27a^3 - 8$ **by** $3a - 2$
9. $x^4 + 49 - 18x^2$ **by** $2x - 7 + x^2$
10. $37x^2 - 4 - 9x^4$ **by** $3x^2 - 2 + 5x$

4 **Find the quotient of dividing each of the following expressions, given the divisor in each $\neq 0$:**

1. $13xy + 6(x^2 + y^2)$ **by** $2x + 3y$
2. $a^4 - 16ab^3 - 6a^2b^2 - 15b^4$ **by** $a^2 - 2ab - 5b^2$

5 If $x + 3$ is one factor of $2x^2 + 3x - 9$, find the other factor.

6 If $x^2 + 3x + 3$ is one of the factors of $x^3 - x^2 - 9x - 12$, find the other factor.

7 Find the sum of $3x^3 - 5x^2 + 7x + 1$ and $3x^3 - x + 7$, then divide the result by $3x + 2$

8 Find the quotient of dividing $2x^3 - x^2 - 2x + 6$ by $2x + 3$, then find the numerical value of the quotient when $x = 1$ « 1 »

9 Find the value of m that makes the expression $2x^2 - 7x + m$ divisible by $x - 2$ « 6 »

10 Find the value of k that makes the expression $x^3 - 3x^2 - 25x + k$ divisible by $x^2 + 4x + 3$ « – 21 »

11 Find the value of k that makes the expression $6x^3 - 13x^2 - 13x + k$ divisible by $3x - 5$ « 30 »

12 What is the expression that if multiplyed by $x^2 + x + 2$ the result will be $x^3 + 2x^2 + 3x + 2$?

Geometric Applications

13 If the area of a rectangle is $(15x^2 + 11x - 14)$ cm^2 and its width is $(3x - 2)$ cm. , calculate its length $\left(x > \frac{2}{3}\right)$

14 If the area of a rectangle is $(2x^2 + 7x - 15)$ square units and its length is $(x + 5)$ length units, then find its width and calculate its perimeter when $x = 3$ « 3 , 22 »

For excellent pupils

15 Find the value of k that makes the expression $x^2 - kx + 12$ divisible by $x - 4$ « 7 »

16 Find the number that if added to the expression $6x^2 - 11x - 17$, the result will become divisible by $2x - 5$ « 7 »

17 ABC is a triangle , if its area is $(6x^2 + 7x + 2)$ cm^2 and the length of $\overline{BC}$ equals $(2x + 1)$ cm. , find the corresponding height to the side $\overline{BC}$

EXERCISE **14**

Factorization by Identifying the (H.C.F.)

Interactive test

From the school book

Remember · Understand · Apply · Problem Solving

1 **Factorize each of the following by identifying the H.C.F. :**

1. $5a + 5b$
2. $3x - 3y$
3. $5y - 10$
4. $8y^3 - 4x^2$
5. $7xy + 7yz$
6. $5ab - 15bc$
7. $3x^2 + 6x$
8. $35a + 10a^2$
9. $6a^3 - 4a^2b^2$
10. $49b^2 - 7b^3$
11. $35x^3y + 5xy^2$
12. $15a^3b - 5a^2b^2$

2 **Factorize each of the following by identifying the H.C.F. :**

1. $5a - 5b + 5c$
2. $6a + 8b + 10c$
3. $3x^2 + 12x - 6$
4. $8a^3 - 4a^2 + 6a$
5. $2x^2y + 6xy^2 - 2y$
6. $9m^4n^2 - 6m^3n^3 + 12m^2n^4$
7. $-2x^5 + 4x^2 - 6x + 2x^3$
8. $32x^3y^3 + 16x^2y^2 + 8xy$
9. $18a^2bc - 6abc + 30abc^2 - 24ab^2c^2$
10. $15a^3b^4 + 6a^5b^3 - 3a^2b^2$

3 **Factorize each of the following by identifying the H.C.F. :**

1. $3x(a + b) + 7(a + b)$
2. $a(a + 3) + b(a + 3)$
3. $(x + 4)x^2 + (x + 4)y^2$
4. $14a(x + y) - 21b(x + y)$
5. $6a^2(x - 1) - 8a(x - 1)$
6. $12x^2(x + 1) - 8xy(x + 1)$
7. $24a^2b^3(a - 2) - 36a^3b^2(2 - a)$
8. $3x^2(x - 7) + 2x(x - 7) + 5(x - 7)$
9. $4m^2(2x + y) - 3m(2x + y) - 7(2x + y)$
10. $16a^2b^2(a + b + 2) - 8a^2b(a + b + 2)$

4 Find the result by identifying the H.C.F. :

1. $48 \times 45 + 48 \times 55$
2. $52 \times 43 - 52 \times 33$
3. $7 \times 123 + 7 \times 35 - 7 \times 18$
4. $15 \times 17 + 15 \times 13 - 15 \times 30$
5. $12 \times 5 + 12 \times 4 + 12$
6. $35 + 14 \times 35 - 5 \times 35$
7. $\frac{5}{18} \times 11 + \frac{35}{18}$
8. $(58)^2 + 58 \times 42$
9. $(256)^2 - 256 \times 156$
10. $6 \times (15)^2 + 18 \times 15 - 8 \times 15$
11. $5 \times (48)^2 + 7 \times 48 + 53 \times 48$
12. $(31)^2 + 31 \times 23 - 31 \times 54$
13. $(51 \times 17 + 51 \times 33) + (49 \times 21 + 49 \times 29)$
14. $(49)^2 + 49 + (50)^2 + 50$

5 Complete the following :

1. $6a^2 + 12ab = 3a$ (.......... + ...)
2. $a^2 b + b^2 a =$ .... $(a + b)$
3. $12 x^2 y - 16 x y^2 =$ .... $(3x -$ ....$)$
4. $x(a + b) + y(a + b) =$ (.......... + ...) $(a + b)$
5. $3(a - b) - 4(b - a) =$ .... $(a - b)$
6. $x(a + 1) - y(a + 1) = (a + 1)$ (.......... – ....)
7. If $a + b = 3$, then $5a + 5b =$ ....
8. If $7x - 7y = 21$, then $x - y =$ ....
9. $20 x^2 + \frac{15 x^2}{3 x} = 5x$ (.......... + ...) , $(x \neq 0)$
10. If $x + y = 5$, then $x(x + y) + y(x + y) =$ ....
11. $7 + 7^2 + 8 + 8^2 =$ .... $\times 8$

6 Choose the correct answer from those given :

1. $3x - 9x^2 =$ ....
   (a) $12x$ (b) $-6x$ (c) $-6x^2$ (d) $3x(1 - 3x)$
2. $7x^2 + 14y^2 = 7$ (................)
   (a) $x^2 + y^2$ (b) $x^2 + 2y^2$ (c) $7x^2 + y^2$ (d) $x + 2y$
3. $4x^2 y^2 - 2x y^2 + 4x^2 y =$ .... $(2xy - y + 2x)$
   (a) $4xy$ (b) $2xy$ (c) $2x$ (d) $2y$
4. The factorization of $6x^2 y - 4x$ by identifying the H.C.F. is ....
   (a) $3xy(x + y)$ (b) $2xy(3y - 2)$ (c) $2xy(3x - 2)$ (d) $2x(3xy - 2)$

5 $(75)^2 + 75 \times 25 = \cdots\cdots$

(a) 75 (b) 750 (c) 7500 (d) 75000

6 The highest common factor of the expression $12 x^3 y^4 + 8 x^2 y^3$ is $\cdots\cdots$

(a) $2 x^2 y^3$ (b) $4 x^2 y^3$ (c) $4 x^3 y^4$ (d) $12 x^3 y^4$

7 If $x - y = 4$ and $x + y = 10$, then $x (x - y) + y (x - y) = \cdots\cdots$

(a) 4 (b) 6 (c) 14 (d) 40

8 If $2 a^2 b - a b = a b (2 a + k)$, then $| k | = \cdots\cdots$

(a) zero (b) 1 (c) -1 (d) 2

7 If $2 a + b = 3$, find using the factorizing by identifying the H.C.F. the numerical value of the expression $2 a (2 a + b) + b (2 a + b)$ « 9 »

8 If $a + c = -3$, find using the factorizing by identifying the H.C.F. the absolute value of the expression $2 a (a + c) + 2 c (a + c)$ « 18 »

9 If $x + y = 3$ and $b - a = 4$, find the numerical value of the expression $a (x + y) - b (x + y)$ « -12 »

10 **Use the factorization by identifying the H.C.F. to find the value of the following easily :**

1 $\dfrac{(19)^2 - 2 \times 19 + 19}{9}$ 2 $\dfrac{5 \times (9)^2 + 11 \times 9 - 9}{45}$ 3 $\left| \dfrac{(36)^2 \times 5 - 3 \times (36)^2}{-2 \times (36)^2} \right|$

11 If $3 a^2 b^2$ is one of the factors of the expression : $12 a^2 b^2 c - 6 b^3 a^2 c^3 + 9 a^2 b^2$, find the other factor.

Geometric Application

12 **In the opposite figure :**

Write in two different ways the algebraic expression which expresses the area of the whole figure.

For excellent pupils

13 If $b = 8$, find the value of : $b (x - 1) + b (c - 2 x) - b (c - x)$ « -8 »

14 If $2 x + 3 y = 2$ and $m (4 x + 6 y) + 2 n (2 x + 3 y) = 16$, find the value of $m + n$ « 4 »

15 If $a b c = 12$ and $a + b + c = 8$, find the numerical value of the expression $a^2 b c + a b^2 c + a b c^2 - a b c$ « 84 »

A Research Project

On Unit Two

Project aims :

- Identifying algebraic expressions.
- Performing mathematical operations on algebraic expressions.
- Multiplying an algebraic term by an algebraic expression.
- Using algebra in solving life problems.
- Associating algebra with history.

Do a research project on the following topic :

"Practicing sports is the start of the road of a more healthy life. Ancient Egyptians had many sports that were the origin of many recent sporting games".

Discuss the following points using available resources :

1. Mention some sorts of sports that Ancient Egyptians practiced for thousands of years in the ancient pharaonic civilization.

2. If you run around a rectangular playground which is (x) metres long and (y) metres wide for five successive days, one time on the first day, twice on the second, and three times on the third and so on until the fifth day :
   - In a table, record the algebraic expressions that show the distance covered on each day and then write one algebraic expression that shows the distance covered over the five days.
   - Supposing that the length of the playground was 40 metres and its width was 20 metres, find the distance covered in metres every day , and then the distance covered over the five days.

UNIT 3 Statistics

Exercises of the unit :

15. The arithmetic mean.

16. The median.

17. The mode.

✪ Activities from the school book.

A research project on unit three

Scan the QR code to solve an interactive test on each lesson

EXERCISE 15

The Arithmetic Mean

Interactive test

From the school book

Remember   Understand   Apply   Problem Solving

1 **Find the arithmetic mean for each set of the following :**

1. 4 , 6
2. 3 , 5
3. 3 , 4
4. 2 , 4 , 6
5. 1 , 3 , 5
6. 1 , 2 , 3 , 4 , 5
7. 6 , 10
8. $\frac{1}{2}$, 1
9. 35 , 50 , 60 , 55

2 If the heights of 5 students in grade 1st prep. in cm. are 124 , 130 , 122 , 126 and 128 , calculate the arithmetic mean of the heights of these students.

3 If the marks of Sherif in 3 consecutive months in maths tests are as the following : 89 , 91 and 96 , calculate the mean of the monthly marks for this student.

4 If the temperature degrees for a week in one of the cities in December are : 25° , 27° , 31° , 23° , 22° , 22° and 18° , calculate the arithmetic mean of these degrees.

5 If the number of goals registered by Al Zamalek in 6 matches are 3 , 2 , zero , 6 , 1 and 6 , calculate the arithmetic mean of the number of goals.

6 **If the numbers of studying hours for one of the students during 5 consecutive days are as the following :**

The day	Saturday	Sunday	Monday	Tuesday	Wednesday
Number of studying hours	$3\frac{1}{2}$	3	$2\frac{1}{2}$	3	4

Find the mean of the daily number of studying hours.

7 **Complete the following :**

1. The arithmetic mean of the values 18 , 35 , 24 , 6 is ....
2. The arithmetic mean of the values $2 - a$, 4 , 1 , 5 , $3 + a$ is ....
3. The arithmetic mean of the values $X + y$, $9 - y$, $-X$ is ....
4. If the arithmetic mean of the numbers 3 , 5 , X is 4 , then $X =$ ....
5. If the sum of five numbers is 30 , then the arithmetic mean of these numbers is ....

8 **Choose the correct answer from the given ones :**

1. The arithmetic mean of the values X , $X - y$, $y - X$ is ....
   (a) $X y$ (b) $\frac{y}{2}$ (c) $\frac{X}{2}$ (d) $\frac{X}{3}$
2. If the arithmetic mean of the numbers 9 , 4 , 5 , X is 5 , then $X =$ ....
   (a) 2 (b) 3 (c) 4 (d) 5
3. If the arithmetic mean of the values 3 , 4 , 8 , a , a + 2 is 15 , then a = ...
   (a) 29 (b) 58 (c) 75 (d) 17
4. If the arithmetic mean of the values $X - 1$, X , $X + 1$ is 6 , then $X =$ ....
   (a) 18 (b) 9 (c) 15 (d) 6
5. If the arithmetic mean of the marks of 5 students is 20 marks , then the sum of their marks equals .... marks.
   (a) 4 (b) 15 (c) 25 (d) 100
6. If the arithmetic mean of the ages of Hanan and Wesam is 7 years and the age of Hanan is 8 years , then the age of Wesam is .... years.
   (a) 6 (b) 7 (c) 8 (d) 15
7. If the arithmetic mean of the side lengths of a triangle is 8 cm. , then the perimeter of the triangle is ....
   (a) 8 cm. (b) 18 cm. (c) 24 cm. (d) 15 cm.

9 **Find the rational number which lies at the middle of the distance between each of the following two numbers :**

1 $\frac{1}{3}, \frac{2}{3}$ 2 $-\frac{3}{5}, -\frac{1}{5}$ 3 $1\frac{1}{2}, 2$

For excellent pupils

10 If the arithmetic mean of the marks of Youssif in three tests in a subject is 16 marks and the arithmetic mean of the marks of the next two successive tests in the same subject is 18 marks , what is the arithmetic mean of his marks in the 5 tests ? « 16.8 marks »

11 If the arithmetic mean of the marks of Magdi in 4 tests is 16 marks , what is the mark which Magdi should obtain in the 5^{th} test to make the arithmetic mean of his marks in all tests 18 marks ? « 26 marks »

12 **The following table shows the distribution of marks of 30 students in an examination :**

Mark	6	9	12	15	17	Total
Number of students	4	7	8	5	6	30

Find the arithmetic mean of these marks. « 12 marks »

EXERCISE 16

The Median

Interactive test

From the school book

Remember Understand Apply Problem Solving

1 Choose the correct answer from those given :

1 The median of the values 4 , 8 , 3 is ....

(a) 3 (b) 4 (c) 5 (d) 8

2 The median of the values 6 , 5 , 9 , 8 is ....

(a) 5 (b) 6 (c) 7 (d) $7\frac{1}{2}$

3 The median of the values 4 , 8 , 3 , 5 , 7 is ....

(a) 3 (b) 4 (c) 5 (d) 7

4 The median of the values 3 , 7 , 2 , 9 , 5 , 11 is ....

(a) 5 (b) 6 (c) 7 (d) 12

5 The median of the marks 25 , 32 , 28 , 40 , 50 , 58 , 50 is ....

(a) 40 (b) 45 (c) 50 (d) 58

6 The order of the median of the values 6 , 2 , 5 , 4 , 1 is ....

(a) 1 (b) 2 (c) 3 (d) 4

7 If the order of the median for a set of ordered values is the fourth , then the number of these values equals ....

(a) 3 (b) 5 (c) 7 (d) 9

8 If the order of the median for a set of ordered values is the fourth and the fifth , then the number of these values equals ....

(a) 4 (b) 5 (c) 8 (d) 9

9 If the median of the values a + 3 , a + 2 , a + 4 is 8 , then a = ...

(a) 2 (b) 3 (c) 4 (d) 5

10 If the median of the values a − 1 , a + 1 , a − 2 , a + 2 , a + 4 is 6 , then a = ...

(a) 2 (b) 4 (c) 5 (d) 7

2 Find the median of each set of the following :

1 − 2 , 0 , − 1 , 1 , 5

2 10 , − 2 , − 2 , 8 , − 12 , 18

3 $\frac{1}{2}$, $\frac{1}{4}$, 1

4 $\frac{5}{6}$, $\frac{3}{10}$, $\frac{7}{15}$, $\frac{2}{5}$

5 2.3 , 3.2 , 2.8 , 0.2 , 2.9

6 0.8 , $\frac{3}{5}$, $\frac{1}{2}$, 0.4 , $\frac{5}{25}$, 0.25

3 The following table shows the weekly absence of one of studying classes :

Day	Sunday	Monday	Tuesday	Wednesday	Thursday
Number of pupils	6	7	10	8	6

Find the median of the number of absent pupils.

4 The following table shows the number of daily studying hours of two friends in 1^{st} preparatory in six days :

Sally	3	2	4.5	7	3.5	5
Basma	4	3	6	2	4.5	3

Find the median of the number of studying hours for each friend.

5 The following table shows the heights of a group of 20 pupils in 1^{st} preparatory in cm. Find the median height of these pupils.

128	121	116	120
122	124	123	131
125	118	127	126
120	133	128	135
134	135	133	117

6 **The following table shows the marks of Gehad in maths tests in 6 months :**

The month	October	November	December	February	March	April
The mark	41	35	47	37	44	48

Find :

1 The median of the previous marks.

2 The mean of the previous marks.

For excellent pupils

7 **Complete each of the following :**

1 If 3 , 7 and 2 X are three values such that : $3 < 2X < 7$

and the median of these values is 4 , then $X =$ ....

2 If 9 , 10 , 5 and X are four values such that : $5 < X < 9 < 10$

and the median of these values is 8 , then $X =$ ....

EXERCISE **17**

The Mode

Interactive test

From the school book

Remember Understand Apply Problem Solving

1 Complete each of the following :

1. The mode of a set of data is ....
2. The mode of the values 2 , 3 , 8 , 2 , 9 is ....
3. The mode of the values 14 , 11 , 12 , 11 , 14 , 15 , 11 is ....
4. The mode of the values 8 , 11 , 5 , 8 , 4 , 5 , 4 , 11 , 4 is ....
5. The mode of the colours red , yellow , red , white , black , red , white is the .... colour.
6. The mode of the tools pen , ruler , pen , rubber , ruler , pen , rubber , rubber , pen , pen is ....
7. If the mode of the values 4 , a , 5 , 3 is 3 , then a = ...
8. If the mode of the values $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{5}$, $\frac{1}{7}$ is $\frac{1}{X}$, then X = ...
9. If the mode of the values 15 , 9 , $X + 1$, 9 , 15 is 9 , then X = ...
10. If the mode of the values a + 2 , a + 1 , a + 3 , a + 2 equals 12 , then a = ...

2 The following frequency table represents the marks of 40 pupils in an examination :

The mark	15	16	17	18	19	20
Number of pupils (frequency)	4	5	8	12	7	4

Find the mode mark.

3 **The following frequency table shows the number of studying hours of 30 pupils in a week :**

The number of studying hours	25	26	27	28	29	30
The number of pupils (frequency)	3	5	12	6	3	1

Find the mode number of studying hours.

4 **The following frequency table shows the maximum temperature degrees registered in some Arabic capitals in one day :**

Temperature degree	18	19	20	21	22	23
No. of capitals	3	2	4	6	2	1

Find the mode of temperature degrees.

5 **Find the mean , the median and the mode for each of the following sets :**

1 2 , 5 , 8 , 12 , 13 , 5 , 4

2 5 , 4 , 10 , 3 , 3 , 4 , 7 , 4 , 6 , 5

For excellent pupils

6 **The following table shows the marks of pupils in a class in a mathematics examination. (The full mark is 10 marks)**

Mark	5	6	7	8	9	10
Frequency	4	8	10	6	3	2

1 How many pupils did obtain a mark more than the mode mark ? « 11 »

2 How many pupils did obtain a mark less than the mode mark ? « 12 »

Activities

on unit three from the school book

1 Which of the following numbers is the arithmetic mean for the other numbers ?

(a) 26 (b) 28 (c) 29 (d) 30 (e) 37

2 If the mean of Karem's marks in five tests is 84 and the mean of his marks in the first three tests is 80 , then what is the mean of his marks in the last two tests ?

3 **Calculate the mean and the median of each set of the following sets of numbers :**

1. 1 , 2 , 3 , ... , 8 , 9 , 10
2. 1 , 2 , 3 , ... , 9 , 10 , 11
3. 1 , 2 , 3 , ... , 99 , 100
4. 1 , 2 , 3 , ... , 100 , 101
5. 0 , 2 , 4 , 6 , 8 , 10
6. 1 , 3 , 5 , ... , 99

* Does each of the previous sets have a mode or not ?

A Research Project

On Unit Three

Project aims :

- Calculating the arithmetic mean of a set of values.
- Finding the median of a set of values.
- Finding the mode of a set of values.
- Associating mathematics with science.
- Appreciating the role of statistics in practical life.

Do a research project on the following topic :

"Statisticans use measuring tools to measure central tendency like arithmetic mean, median and mode".

Discuss the following points using available resources :

1. Define the arithmetic mean, median and mode.
2. Find out the average age in years of each mammal of the following :

   Lion – Horse – African elephant – Giraffe – Tiger – Rhinoceros – Chimpanzee – Bear – Camel – Donkey

   Then find the arithmetic mean, median and mode of these animals ages.
3. Measure the heights of your mates in your class. Record these heights in a table which has the name and height of each one. Then find the arithmetic mean, median and mode of those heights.

TIMSS Problems

Accumulative basic skills

1 Complete the following :

1. If half of a number is 30 , then $\frac{3}{4}$ of this number equals ....
2. If $X \in \mathbb{Z}$, $-2 < 2X < 2$, then the S.S. = ...
3. The smallest number whose prime factors are 2 , 5 and 7 is ....
4. Three consecutive natural numbers , the smallest is $X - 1$, then their sum is ....
5. Two consecutive even numbers , the greater is $X + 3$, then the smaller is ....
6. A number , if it is added to its double , the result is 12 , then the number is ....
7. If the ratio between the length and the width of a rectangle is 2 : 1 , then the ratio between its length and its perimeter is ....
8. If 15% of a number equals 30 , then the number is ....
9. There are 54 kg. of apple in two boxes. If the second box weighs 12 kg. more than the first , then the number of kilograms in each box is ....
10. The value of X which makes the two numbers X , $X + 41$ prime numbers is ....
11. If $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, ... , then the next term in this pattern is .... , and the term whose order is 50 is ....
12. 1 , 1 , 2 , 3 , 5 , 8 , ... (in the same pattern)

2 Choose the correct answer from the given ones :

1. The value of 3 in the number 0.1432 is ....

(a) $\frac{3}{10}$ (b) $\frac{3}{100}$ (c) $\frac{3}{1000}$ (d) $\frac{3}{10000}$

2 Hany is taller than Gamal 8 cm. Hossam is shorter than Hany 12 cm. If the height of Gamal is 125 cm. , then the height of Hossam is .... cm.

(a) 105 (b) 113 (c) 121 (d) 129

3 A baker makes 8 cakes using 2 kg. of butter , 3 kg. of sugar and 4 kg. of flour. How many cakes of the same kind can he make if he has 14 kg. of butter, 15 kg. of sugar and 16 kg. of flour ?

(a) 32 (b) 40 (c) 44 (d) 56

4 $\frac{3}{X-3}$ is the additive inverse of the rational number .... where $X \neq 3$

(a) $\frac{3}{X+3}$ (b) $\frac{-3}{X+3}$ (c) $\frac{3}{3-X}$ (d) $\frac{-3}{3-X}$

5 Half of $99\frac{1}{2}$ is ....

(a) $45\frac{1}{4}$ (b) $45\frac{3}{4}$ (c) $49\frac{1}{4}$ (d) $49\frac{3}{4}$

6 Which of the following is the closest to $(11)^2 + (9)^2$?

(a) 20 + 20 (b) 20 + 80 (c) 120 + 20 (d) 120 + 80

7 If k is a negative number , which of the following is positive ?

(a) k^2 (b) k^3 (c) 2 k (d) $\frac{k}{2}$

8 Sound moves through air with velocity 330 metres each second approximately.

An explosion sound takes 28 seconds to reach someone.

Which of the following is the best estimation of the distance between this person and the explosion place ?

(a) 12000 m. (b) 9000 m. (c) 8000 m. (d) 6000 m.

9 Quarter of 4^{20} equals ....

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

10 The smallest fraction of the following is ....

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{5}{8}$ (d) $\frac{7}{16}$

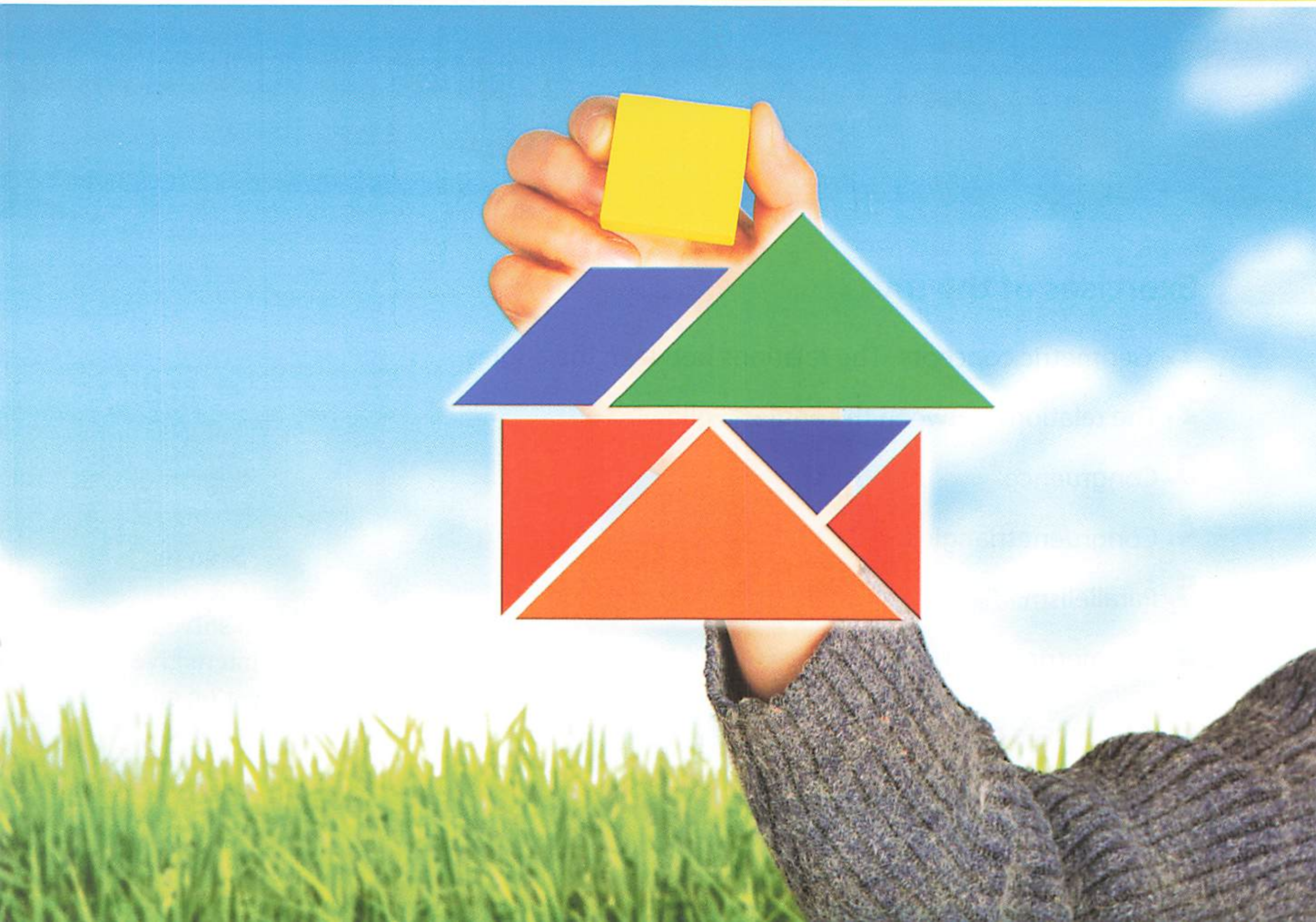
11 Which of the following is the best estimation of the result of $\frac{32 \times 2.7}{14.7}$?

(a) 0.6 (b) 3 (c) 6 (d) 60

12 The next number in the pattern $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, ... is ....

(a) 0 (b) 1 (c) 10 (d) 100

Second Geometry



UNIT

4 Geometry and Measurement

Exercises of the unit :

1. Geometric concepts - The relations between the angles.
2. The relations between the angles "Follow".
3. Congruence.
4. Congruent triangles.
5. Parallelism.
6. Geometric constructions.

A research project on unit four

Scan the QR code to solve an interactive test on each lesson

EXERCISE 1?

Geometric Concepts - The Relations between the Angles

Interactive test

From the school book

Remember Understand Apply Problem Solving

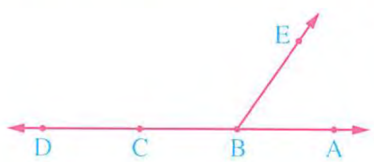
1 **In the opposite figure :**

A , B , C and D are points lying on one line ,

$\overleftrightarrow{AD} \cap \overrightarrow{BE} = \{B\}$

Complete each of the following by using

$\in$, $\notin$, $\subset$ or $\not\subset$:



1. A .... $\overrightarrow{DC}$
2. D .... $\overline{AC}$
3. C .... $\overleftrightarrow{AB}$
4. A .... $\angle$ EBC
5. $\overline{DC}$ .... $\overleftrightarrow{AB}$
6. $\overline{BC}$ .... $\overrightarrow{BA}$
7. $\overrightarrow{BA}$ .... $\overleftrightarrow{DC}$
8. $\overline{AC}$ .... $\overline{AD}$

2 **Mention the type of the angle whose measure is as the following :**

1. 57°
2. 117°
3. 90°
4. 200°
5. 180°
6. $43\frac{1}{2}^\circ$
7. $89^\circ\ 60'$
8. $179^\circ\ 62'$

3 **Write the measure of the angle which complements each of the angles whose measures are as follows :**

1. 60°
2. 45°
3. 📖 37°
4. 📖 48°
5. 📖 $22\frac{1}{2}^\circ$
6. 90°
7. $25^\circ\ 60'$
8. 0°

4 **Write the measure of the angle which supplements each of the angles whose measures are as follows :**

1 $10°$	2 $90°$	3 $82°$	4 $117°$
5 $92\frac{1}{2}°$	6 $0°$	7 $180°$	8 $141°\ 60'$

5 **Complete the following :**

1 The angle is ....

2 The measure of the straight angle = ... ° and the measure of zero angle = ... °

3 The measure of the right angle = ... °

4 The acute angle is the angle whose measure is less than .... ° and more than .... °

5 The two complementary angles are the two angles whose sum of measures is .... °

6 The two supplementary angles are the two angles whose sum of measures is .... °

7 The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are ....

8 If the two outer sides of two adjacent angles are perpendicular , then these two adjacent angles are ....

9 If the two outer sides of two adjacent angles are on the same straight line, then these two adjacent angles are ....

10 If the two adjacent angles are supplementary , then their outer sides are ....

11 If $m(\angle A) = 50°$, then m (reflex $\angle$ A) = ... °

12 If m (reflex $\angle X) = 237°$, then $m(\angle X)$ = ... °

13 The measure of the angle which is equivalent to two right angles equals .... ° and it is called .... angle.

14 The angle whose measure is $50°$ complements an angle of measure .... ° and supplements an angle of measure .... °

15 The angle whose measure is .... ° complements the angle whose measure is $30°$ and supplements the angle whose measure is .... °

16 The angle whose measure is .... ° complements the angle whose measure is .... ° and supplements the angle whose measure is $150°$

17 The acute angle complements .... angle and supplements .... angle.

18 Zero angle is complemented by .... angle and is supplemented by .... angle.

19 The right angle is complemented by .... angle and is supplemented by .... angle.

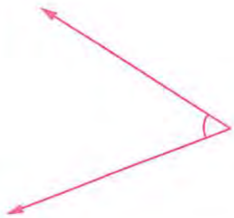
20 The obtuse angle supplements .... angle.

6 Draw the angles whose measures are as follows showing the type of each of them :

1 115°　2 80°　3 195°　4 245°　5 180°

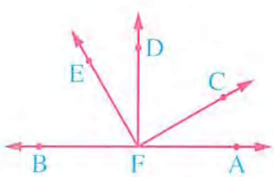
7 For each of the following angles , write the closest measure from the following 80° , 120° , 240°

1 

2 

3 

8 In the opposite figure :



$F \in \overleftrightarrow{AB}$, $\overrightarrow{FD} \perp \overleftrightarrow{AB}$ and m (∠ CFE) = 90°

Complete the following :

1 $\overrightarrow{FA} \cup \overrightarrow{FC}$ = ...

2 $\overrightarrow{FC} \cup \overrightarrow{FB}$ = ...

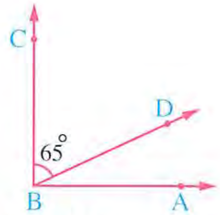
3 ∠ AFC supplements ∠ ....

4 ∠ DFC complements each of ∠ .... and ∠ ....

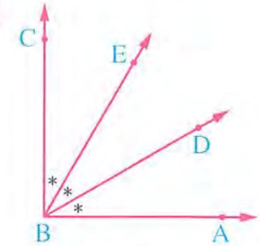
5 ∠ AFB is .... angle , and ∠ DFB is .... angle.

6 m (∠ DFE) = m (∠ ....) because each one of them complements ∠ ....

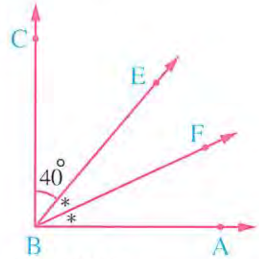
9 In each of the following figures , if $\overrightarrow{BA} \perp \overrightarrow{BC}$, find the measure of the required angle under each figure :

1 

m (∠ ABD) = ...°

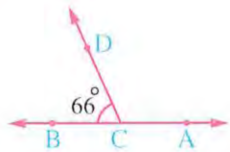
2 

m (∠ DBC) = ...°

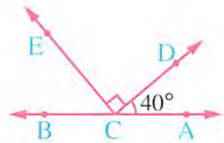
3 

m (∠ ABF) = ...°

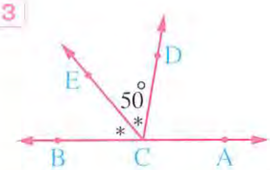
10 **In each of the following figures, if $C \in \overleftrightarrow{AB}$, find the measure of the required angle under each figure :**

1


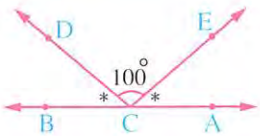
m (∠ ACD) = ...°

2


m (∠ ECB) = ...°

3


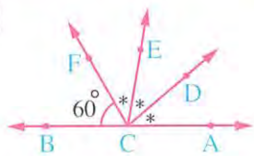
m (∠ ACD) = ...°

4


m (∠ DCB) = ...°

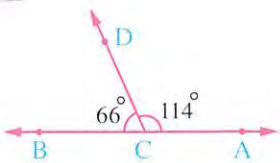
5

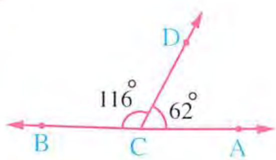

m (∠ FCB) = ...°

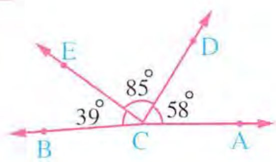
6


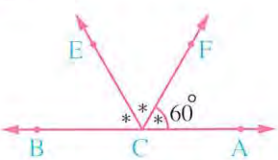
m (∠ DCB) = ...°

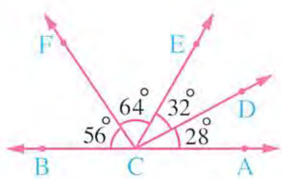
11 **In each of the following figures, state if $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line or not, and why :**

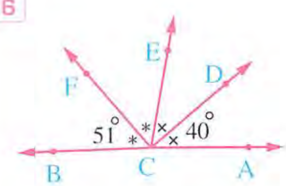
1


2


3


4


5


6


12 **Choose the correct answer from the given ones :**

1 Between any two distinct points we can draw .... straight line passing through them.

(a) zero (b) 1 (c) 2 (d) 3

2 $\overline{AB}$ .... $\overrightarrow{AB}$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

3 If m (∠ A) + m (∠ B) = 180°, then ∠ A and ∠ B are ....

(a) equal in measure. (b) complementary.

(c) supplementary. (d) adjacent.

4 If $\overrightarrow{BA} \perp \overrightarrow{BC}$, then m ($\angle$ ABC) = ...

(a) 40° (b) 90° (c) 180° (d) 360°

5 If $\angle$ A supplements $\angle$ B, $\angle$ A supplements $\angle$ C, then $\angle$ B and $\angle$ C are ....

(a) equal in measure. (b) complementary.
(c) supplementary. (d) adjacent.

6 If m ($\angle$ X) = 15°, then the two angles whose measures are 2 m ($\angle$ X), 4 m ($\angle$ X) are ....

(a) complementary. (b) supplementary.
(c) equal in measure. (d) obtuse angles.

7 If m ($\angle$ A) = m ($\angle$ B), $\angle$ A supplements $\angle$ B, then m ($\angle$ B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

8 The angle of measure x ° complements an angle of measure ....°

(a) 180° – x (b) 90° – x (c) 180° + x (d) 90° + x

9 m ($\angle$ A) + m (reflex $\angle$ A) = the measure of ....

(a) one right angle. (b) two right angles.
(c) three right angles. (d) four right angles.

10 If m ($\angle$ X) = 2 m ($\angle$ Y) and $\angle$ Y is an obtuse angle, then $\angle$ X is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

13 Complete the following :

1 If m ($\angle$ A) = 30°, $\angle$ A complements $\angle$ B, then m (reflex $\angle$ B) = ...°

2 The measure of each angle of two complementary and equal in measure angles is ....°

3 If $\angle$ A and $\angle$ B are two supplementary angles and m ($\angle$ A) = 2 m ($\angle$ B), then m ($\angle$ B) = ... °

4 If m ($\angle$ X) = $\frac{1}{2}$ m ($\angle$ Y), m ($\angle$ X) = 30°, then the two angles X and Y are ....

5 If $\angle$ A complements $\angle$ B and m ($\angle$ A) = $\frac{2}{3}$ m ($\angle$ B), then m ($\angle$ B) = ...°

6 If the ratio between the measures of two supplementary angles is 2 : 7, then the measure of the greater angle equals .... °

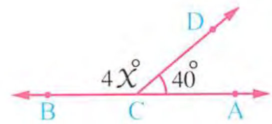
7 If m ($\angle$ A) = $\frac{1}{2}$ m ($\angle$ B), m ($\angle$ C) = $\frac{1}{2}$ m ($\angle$ D), $\angle$ B supplements $\angle$ D, then m ($\angle$ A) + m ($\angle$ C) = ... °

8 If $\angle$ A complements $\angle$ B and $\angle$ B supplements $\angle$ C , m ($\angle$ A) = 32°

, then m ($\angle$ C) = ... °

9 **In the opposite figure :**

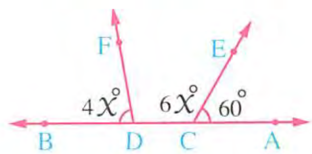
If C $\in \overleftrightarrow{AB}$, then x = ...



10 **In the opposite figure :**

If C $\in \overline{AB}$, D $\in \overline{AB}$

, then m ($\angle$ FDC) = ...°



For excellent pupils

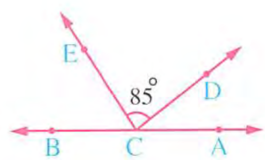
14 **In the opposite figure :**

If C $\in \overleftrightarrow{AB}$, m ($\angle$ DCE) = 85° ,

m ($\angle$ ACD) : m ($\angle$ ECB) = 2 : 3

, find : **1** m ($\angle$ ACE)

2 m ($\angle$ DCB)

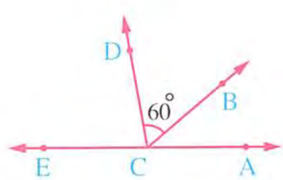


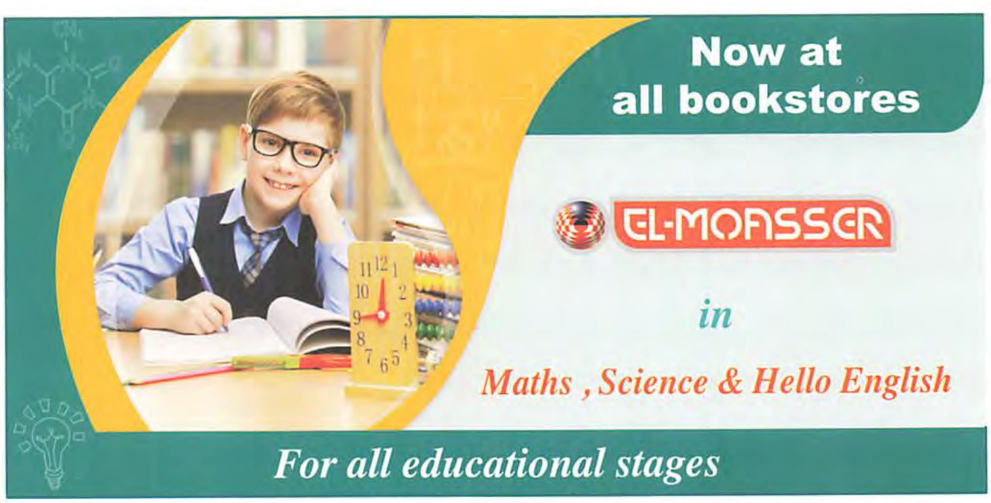
15 **In the opposite figure :**

m ($\angle$ DCB) = 60°

and m ($\angle$ ACB) : m ($\angle$ BCD) : m ($\angle$ DCE) = 2 : 3 : 4

Are $\overrightarrow{CA}$ and $\overrightarrow{CE}$ on the same straight line or not ? Why ?





EXERCISE 2 ?

The Relations between the Angles (Follow)

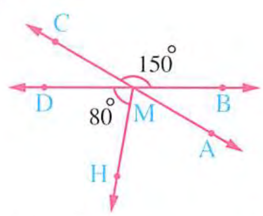
Interactive test

From the school book

Remember · Understand · Apply · Problem Solving

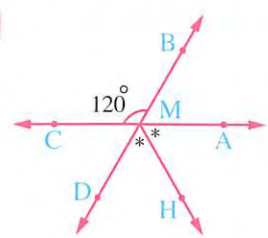
1 In each of the following figures , find the measure of the required angle under each figure :

1



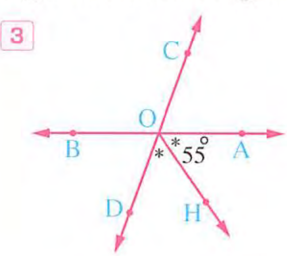
m (∠ AMH) = ...°

2



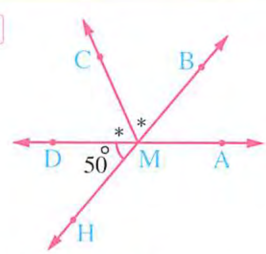
m (∠ HMD) = ...°

3



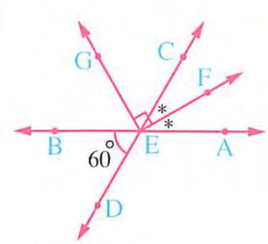
m (∠ COB) = ...°

4



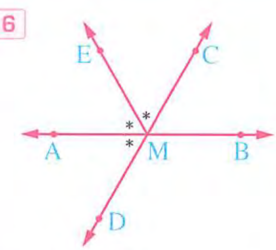
m (∠ AMC) = ...°

5



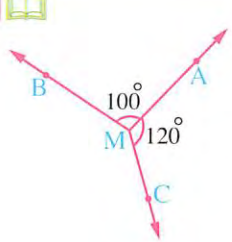
m (∠ GEB) = ...°

6



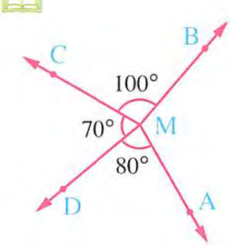
m (∠ DMB) = ...°

7



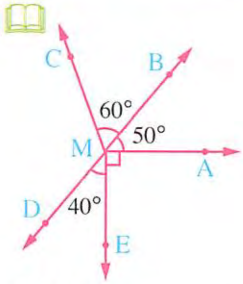
m (∠ BMC) = ...°

8



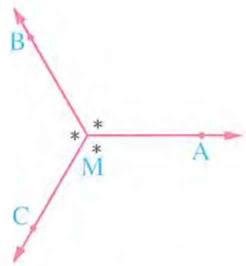
m (∠ AMB) = ...°

9



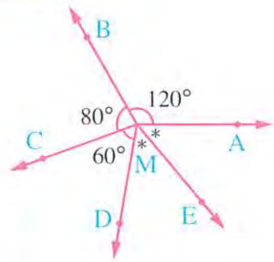
m (∠ CMD) = ...°

10



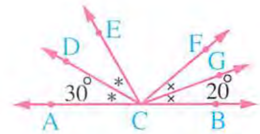
m (∠ AMC) = ...°

11



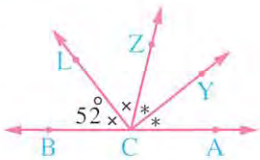
m (∠ EMD) = ...°

12



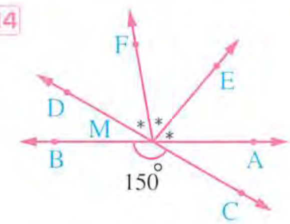
m (∠ FCE) = ...°

13



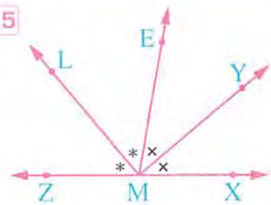
m (∠ YCA) = ...°

14



m (∠ CMF) = ...°

15



m (∠ YML) = ...°

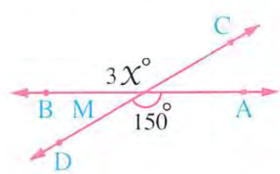
2 Complete the following :

1. If two straight lines intersect , then each two vertically opposite angles are ....

2. The sum of the measures of the accumulative angles at a point equals .... °

3. **In the opposite figure :**

   If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, then $x =$ .... °

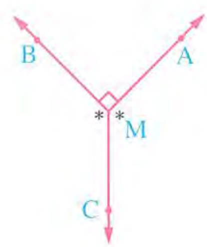


4. **In the opposite figure :**

   If $\overrightarrow{MB} \perp \overrightarrow{MA}$

   and $\overrightarrow{MC}$ bisects the reflexed angle AMB

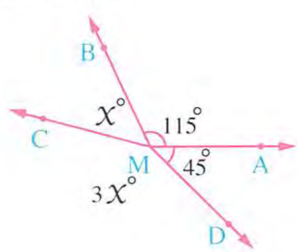
   , then m (∠ AMC) = ... °



5. If $\overrightarrow{BD}$ bisects ∠ ABC and m (∠ ABD) = 35° , then m (∠ ABC) = ... °

6. **In the opposite figure :**

   $x =$ .... °



3 Choose the correct answer from the given ones :

1 The angle of measure 60° is vertically opposite to an angle of measure ....

(a) 30° (b) 60° (c) 90° (d) 180°

2 The sum of measures of the accumulative angles at a point equals the sum of measures of .... angles.

(a) 2 right (b) 3 right (c) 4 right (d) 5 right

3 The sum of measures of 4 accumulative angles at a point .... the sum of measures of 5 accumulative angles at a point.

(a) = (b) < (c) > (d) ≠

4 The two bisectors of two adjacent supplementary angles ....

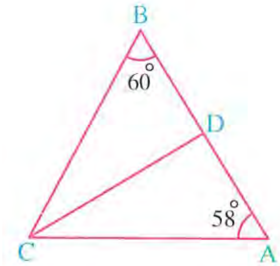
(a) are perpendicular. (b) are parallel.

(c) are coincident. (d) included an acute angle between them.

5 If the two vertically opposite angles are complementary angles , then the measure of each angle is ....

(a) 180° (b) 90° (c) 50° (d) 45°

6 **In the opposite figure :**

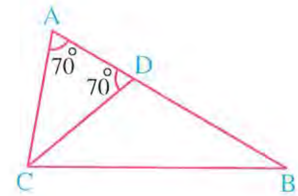


If ABC is a triangle in which $\overrightarrow{CD}$ bisects ∠ ACB , m (∠ A) = 58°, m (∠ B) = 60° , then m (∠ ADC) = ...

(a) 62° (b) 89°

(c) 91° (d) 130°

7 **In the opposite figure :**

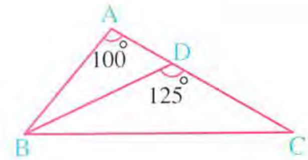


If $\overrightarrow{CD}$ bisects ∠ BCA , m (∠ A) = m (∠ ADC) = 70° , then m (∠ B) = ...

(a) 70° (b) 30°

(c) 80° (d) 40°

8 **In the opposite figure :**

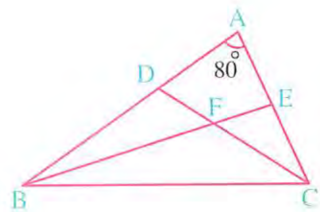


ABC is a triangle , D $\in \overline{AC}$ and $\overrightarrow{BD}$ is the bisector of $\angle$ B , what is the measure of $\angle$ C ?

(a) 25° (b) 30°

(c) 45° (d) 55°

9 **In the opposite figure :**



m ($\angle$ A) = 80° , $\overrightarrow{BE}$ is the bisector of $\angle$ B ,

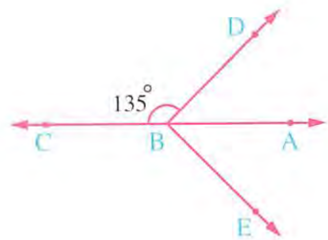
$\overrightarrow{CD}$ is the bisector of $\angle$ C

What is the measure of $\angle$ BFC ?

(a) 80° (b) 100°

(c) 120° (d) 130°

4 **In the opposite figure :**



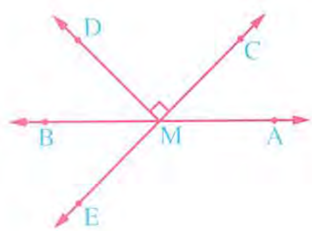
If B $\in \overleftrightarrow{AC}$, m ($\angle$ DBC) = 135°

and $\overrightarrow{BA}$ bisects $\angle$ DBE

, find each of :

m ($\angle$ ABD) , m ($\angle$ DBE) , m ($\angle$ CBE)

5 **In the opposite figure :**

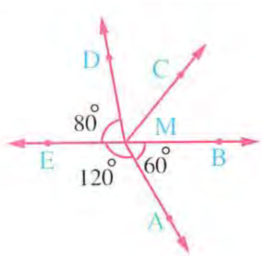


If $\overleftrightarrow{AB} \cap \overleftrightarrow{CE} = \{M\}$, $\overrightarrow{MD} \perp \overleftrightarrow{CE}$ and $\overrightarrow{MB}$ bisects $\angle$ DME

, find the measures of the following angles :

$\angle$ BME , $\angle$ DME , $\angle$ AMC and $\angle$ AME

6 **In the opposite figure :**



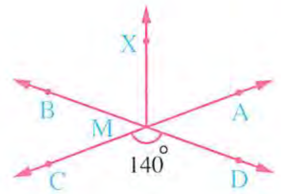
m ($\angle$ AMB) = 60° , m ($\angle$ AME) = 120° ,

m ($\angle$ EMD) = 80° and $\overrightarrow{MC}$ bisects $\angle$ BMD

Find :

1 m ($\angle$ CMD) 2 m ($\angle$ AMC)

7 **In the opposite figure :**



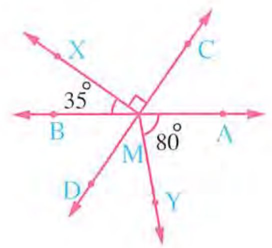
$\overleftrightarrow{AC} \cap \overleftrightarrow{BD} = \{M\}$, $\overrightarrow{MX}$ bisects ∠ AMB

and m (∠ CMD) = 140°

Find : m (∠ DMX)

---

8 **In the opposite figure :**



$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, m (∠ CMX) = 90° ,

m (∠ XMB) = 35° and m (∠ AMY) = 80°

Find :

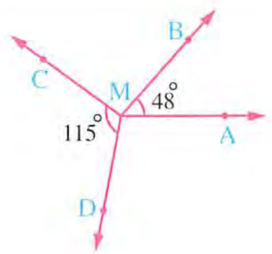
1 m (∠ AMD)

2 m (∠ DMY)

3 m (∠ BMY)

---

9 **In the opposite figure :**



m (∠ BMC) = 2 m (∠ AMB) ,

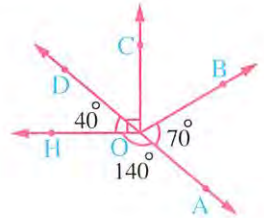
m (∠ AMB) = 48°

and m (∠ DMC) = 115°

Find : m (∠ AMD)

---

10 **In the opposite figure :**



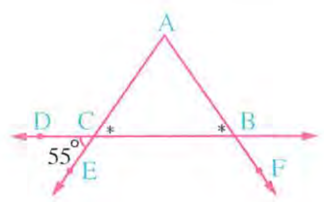
$\overrightarrow{OC} \perp \overrightarrow{OH}$

Are $\overrightarrow{OA}$ and $\overrightarrow{OD}$ on the same straight line or not ? Why ?

, then find : m (∠ BOC)

---

11 **In the opposite figure :**



D ∈ $\overrightarrow{BC}$, E ∈ $\overrightarrow{AC}$, F ∈ $\overrightarrow{AB}$

, m (∠ ABC) = m (∠ ACB)

and m (∠ ECD) = 55°

Find : m (∠ FBC)

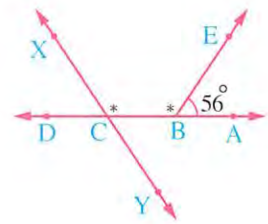
12 **In the opposite figure :**

A , B , C and D are collinear ,

$\overleftrightarrow{XY} \cap \overleftrightarrow{BD} = \{C\}$, m (∠ ABE) = 56°

and m (∠ EBC) = m (∠ BCX)

Find : m (∠ DCY)

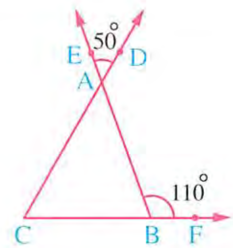


13 **In the opposite figure :**

m (∠ DAE) = 50°

and m (∠ ABF) = 110°

Find : The measures of the angles of the triangle ABC



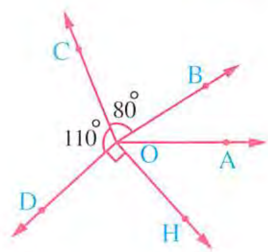
14 **In the opposite figure :**

m (∠ BOC) = 80° , m (∠ COD) = 110° ,

m (∠ DOH) = 90°

and m (∠ AOB) : m (∠ AOH) = 2 : 3

Find : m (∠ AOB) and m (∠ AOH)



For excellent pupils

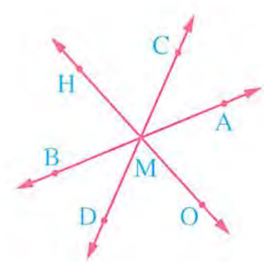
15 **In the opposite figure :**

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} \cap \overleftrightarrow{HO} = \{M\}$,

m (∠ AMO) + m (∠ BMH) = 140°

and m (∠ AMC) : m (∠ DMO) = 2 : 3

Find : m (∠ CMH)

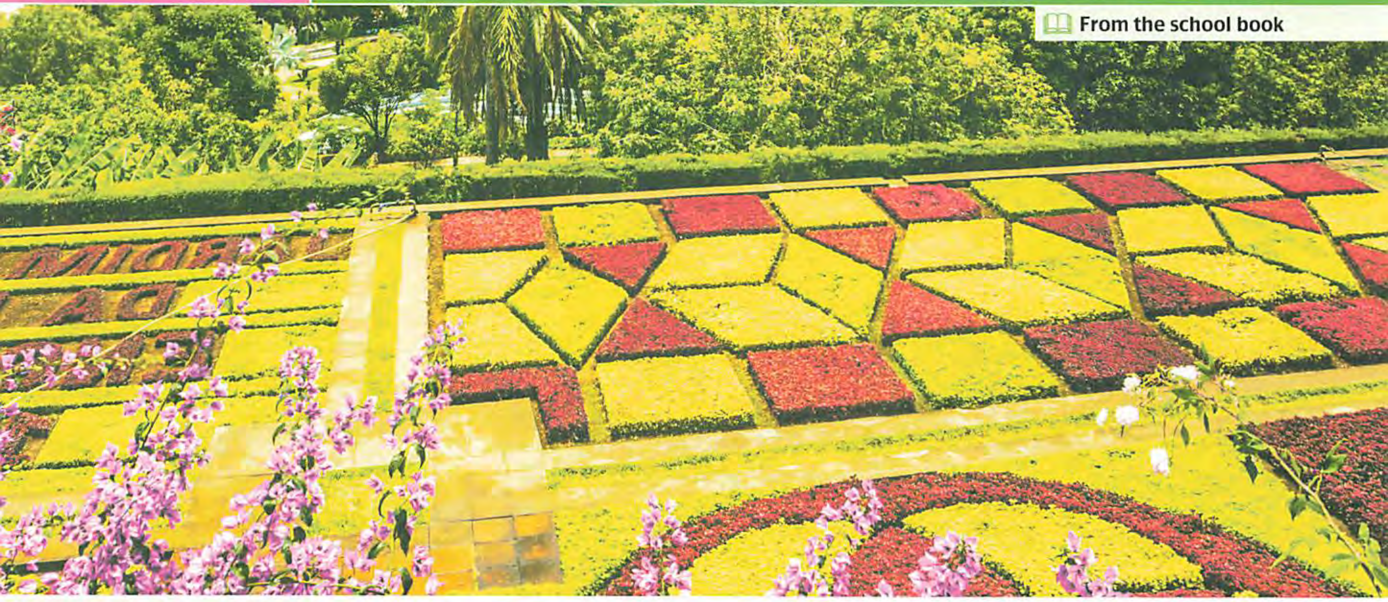


EXERCISE **3** ?

Congruence

Interactive test

From the school book



Remember   Understand   Apply   Problem Solving

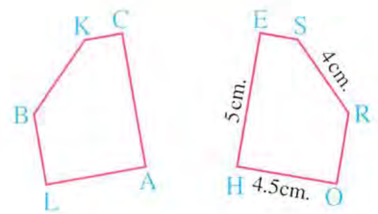
1 Complete the following :

1. Two line segments are congruent if ....
2. Two angles are congruent if ....
3. Two polygons are congruent if their corresponding angles are .... , their corresponding sides are ....
4. The axis of symmetry of a polygon divides it into two .... polygons.
5. If $\overline{AB} \equiv \overline{CD}$, AB = 5 cm. , then AB + CD = ... cm.
6. If $\overline{AB} \equiv \overline{XY}$, then AB – XY = ...
7. If $\overline{XY} \equiv \overline{XZ}$, then $\frac{XZ}{XY}$ = ...
8. If ∠ A ≡ ∠ B and m (∠ A) = 50° , then m (∠ B) = ...°
9. If $\overline{AB} \equiv \overline{CD}$, AB = 20 cm. , then $\frac{1}{4}$ CD = ... cm.
10. If m (∠ X) + m (∠ Y) = 120° , ∠ X ≡ ∠ Y , then m (∠ X) = ...°
11. If ∠ A supplements ∠ B and ∠ A ≡ ∠ B , then m (∠ B) = ...°
12. If ∠ A complements ∠ B and ∠ A ≡ ∠ B , then m (∠ A) = ...°
13. If C is the midpoint of $\overline{AB}$, then $\overline{AC}$ .... $\overline{BC}$
14. If the polygon ABCD ≡ the polygon XYZL , then $\overline{DA}$ ≡ .... , m (∠ BCD) = m (∠ ....)
15. If ABCD is a rectangle , then $\overline{BC}$ ≡ ....
16. The two squares are congruent if .... are equal in length , and the two rectangles are congruent if .... are equal.
17. The square whose side length equals 5 cm. is congruent to the square whose perimeter is .... cm.

2 **In the opposite figure :**

The two pentagons shown are congruent.

Complete :



1 B corresponds to ....

2 The polygon BLACK is congruent to the polygon ....

3 KB = ... cm.

4 m (∠ E) = m (∠ ....)

5 CA = ... cm.

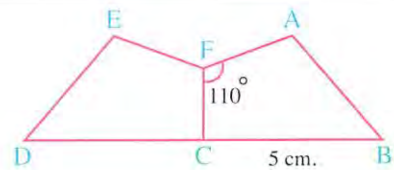
6 m (∠ A) = m (∠ ....)

3 **In the opposite figure :**

If $C \in \overleftrightarrow{BD}$, m (∠ AFC) = 110° , BC = 5 cm.

and the polygon ABCF ≡ the polygon EDCF

, complete the following :



1 AB = ...

2 AF = ...

3 m (∠ E) = m (∠ ....)

4 m (∠ B) = m (∠ ....)

5 m (∠ FCD) = m (∠ ....)

6 m (∠ EFC) = ... °

7 BD = ... cm.

8 m (∠ FCD) = ... °

9 m (∠ AFE) = ... °

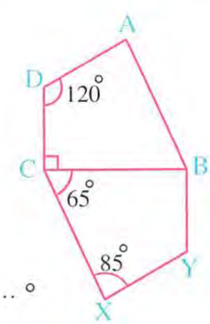
10 The axis of symmetry of the polygon ABDEF is ....

4 **In the opposite figure :**

If $\overline{DC} \perp \overline{BC}$, m (∠ ADC) = 120° , m (∠ BCX) = 65° , m (∠ X) = 85°

and the polygon ABCD ≡ the polygon XCBY

, complete the following :



1 AB = ...

2 XY = ...

3 CD = ...

4 $\overline{BC}$ is .... side.

5 m (∠ Y) = ... °

6 m (∠ A) = ... °

7 m (∠ ABC) = ...°

8 m (∠ YBC) = ... °

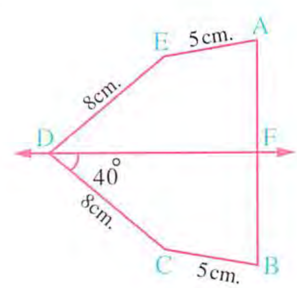
5 **In the opposite figure :**

If m (∠ A) = m (∠ B) , m (∠ C) = m (∠ E) , $\overleftrightarrow{FD}$ bisects ∠ EDC ,

$\overleftrightarrow{FD}$ is the axis of symmetry of $\overline{AB}$, AE = BC = 5 cm. ,

CD = ED = 8 cm. , AB = 12 cm. and m (∠ CDF) = 40°

, complete the following :



1 m (∠ AFD) = ...°

2 m (∠ CDE) = ...°

3 The length of $\overline{BF}$ = ... cm.

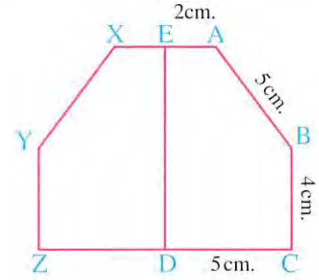
4 The two figures .... and .... are congruent.

6 **In the opposite figure :**

If $D \in \overleftrightarrow{CZ}$ and the figure ABCDE ≡ the figure XYZDE ,

AE = 2 cm. , BC = 4 cm. and AB = CD = 5 cm.

, **find :** The perimeter of the figure ABCZYX

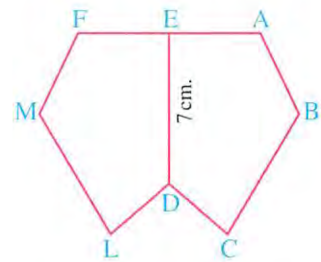


7 **In the opposite figure :**

If $E \in \overline{AF}$, the perimeter of the figure ABCDE = 27 cm. ,

DE = 7 cm. and the polygon ABCDE ≡ the polygon FMLDE

, **find :** The perimeter of the figure ABCDLMF

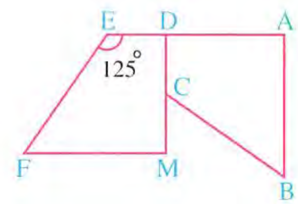


8 **In the opposite figure :**

If the figure ABCD ≡ the figure MFED ,

C is the midpoint of $\overline{MD}$ and MC = 3 cm. and m (∠ E) = 125°

, **complete the following :**



1. m (∠ A) = m (∠ ....)
2. m (∠ ADC) = m (∠ ....)
3. m (∠ B) = m (∠ ....)
4. m (∠ BCD) = ... °
5. m (∠ BCM) = ... °
6. AE = ... cm.

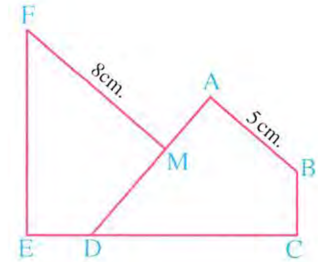
For excellent pupils

9 **In the opposite figure :**

If $D \in \overline{CE}$

and the figure ABCD ≡ the figure MDEF

, **complete the following :**



1. AM = ... cm.
2. m (∠ B) + m (∠ F) = ...°

EXERCISE 4 ?

Congruent Triangles

Interactive test

From the school book

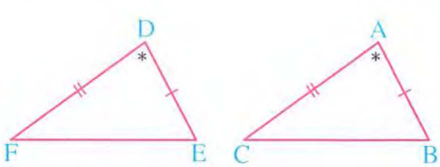
● Remember ● Understand ● Apply ● Problem Solving

1 **Complete the following :**

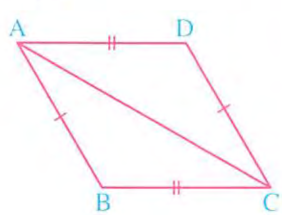
1. Any two triangles are congruent if two sides and ....
2. Any two triangles are congruent if two angles and .... in one of the triangles are congruent to their corresponding elements in the other.
3. Any two triangles are congruent if each .... is congruent to its corresponding .... in the other triangle.
4. Any two right-angled triangles are congruent if ....
5. The diagonal of the rectangle divides its surface into two .... triangles.
6. If $\Delta ABC \equiv \Delta XYZ$, then AB = ... and m ($\angle$ Z) = m ($\angle$ ....)
7. If AB = LM , BC = MN and m ($\angle$ B) = m ($\angle$ M) , then the two triangles .... and .... are congruent.

2 **In each of the following figures , show if the two triangles are congruent or not. If they are congruent , name the case of congruence. If they are not congruent , give reason "Given that the similar signs denote the congruence of the shown elements labelled by these signs".**

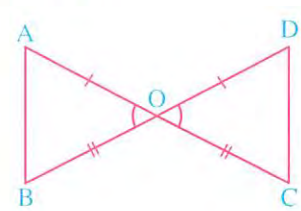
1



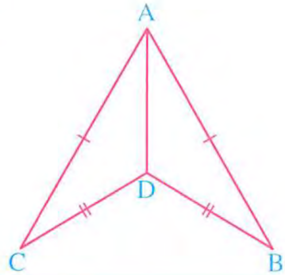
2



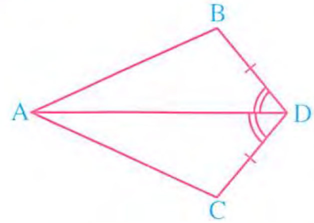
3



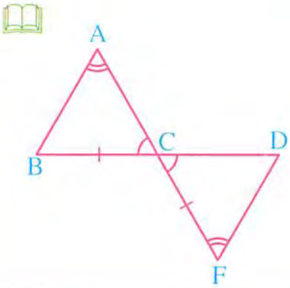
4



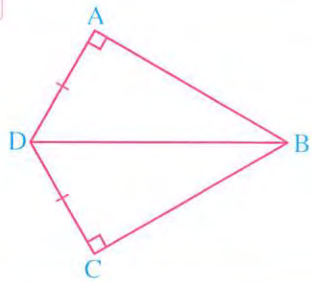
5



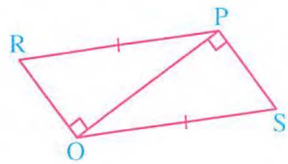
6



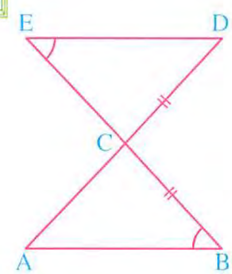
7



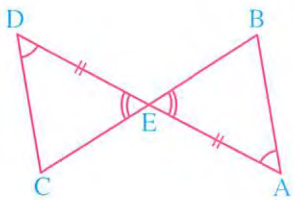
8



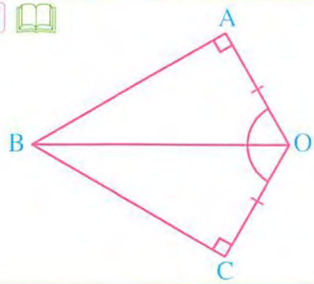
9



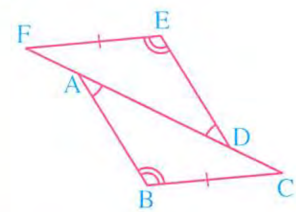
10



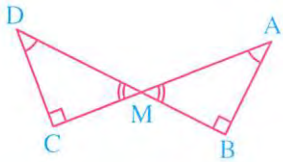
11



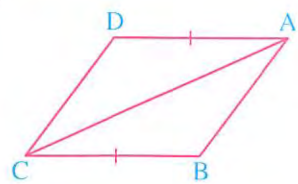
12



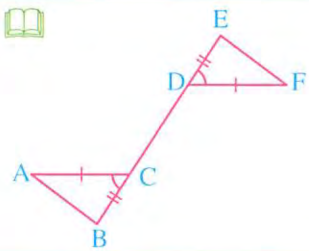
13



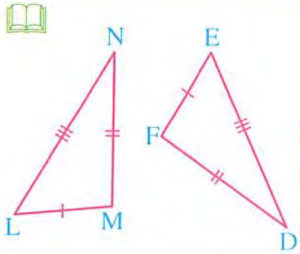
14



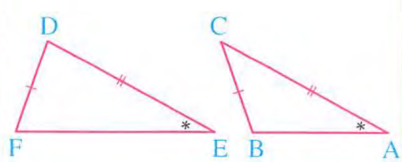
15



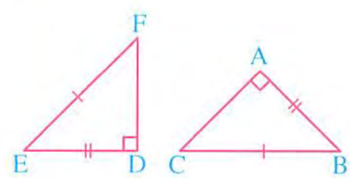
16



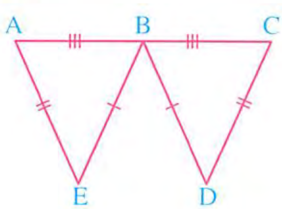
17



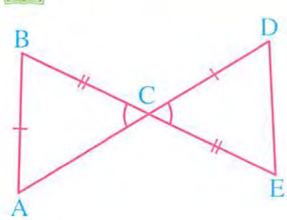
18



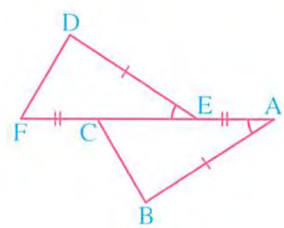
19

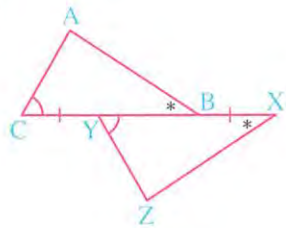


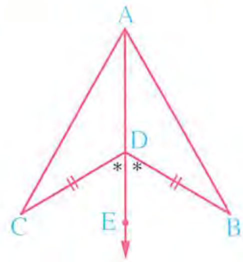
20

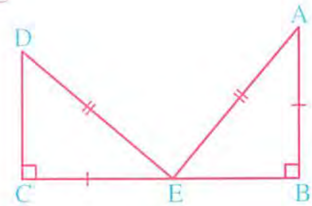


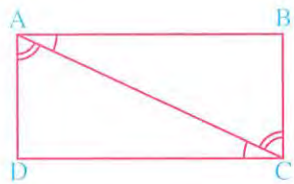
21

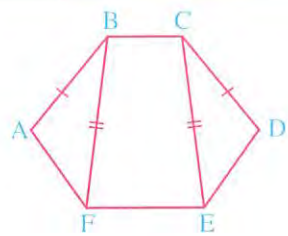


22


23


24


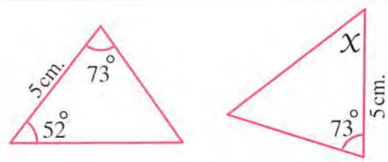
25


26


3 **In the opposite figure :**

These triangles are congruent

Complete : X = ...°



4 **In the opposite figure :**

If AB = AD , BC = 7 cm. , m (∠ BAC) = m (∠ DAC) = 25°

and m (∠ B) = 30°

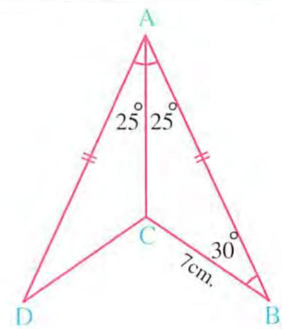
, complete the following :

1 Δ ACB ≡ Δ ....

2 m (∠ D) = ...°

3 CD = ... cm.

4 m (∠ ACD) = ...°

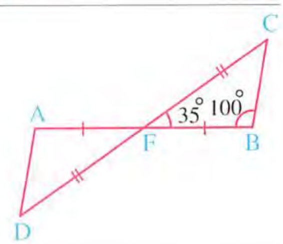


5 **In the opposite figure :**

If $\overline{CD} \cap \overline{BA} = \{F\}$, FA = FB , CF = FD ,

m (∠ CFB) = 35° and m (∠ B) = 100° ,

then complete : m (∠ D) = ...°



6 **In the opposite figure :**

If BC = FD , m (∠ A) = m (∠ E) = 95° ,

m (∠ B) = 35° , m (∠ D) = 50° and FE = 7 cm.

, complete the following :

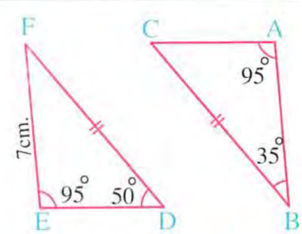
1 m (∠ C) = ...°

2 m (∠ F) = ...°

3 Δ ABC ≡ ....

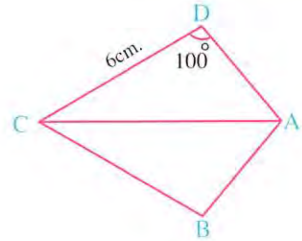
4 $\overline{AC}$ ≡ ....

5 AB = ... cm.



7 **In the opposite figure :**

If $\overleftrightarrow{AC}$ bisects $\angle$ DCB , $\angle$ DAB , m ($\angle$ D) = 100° and DC = 6 cm. **, complete the following :**



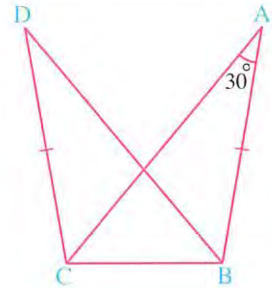
1 Δ ADC $\equiv \Delta$ ....

2 m ($\angle$ B) = ...°

3 BC = ... cm.

8 **In the opposite figure :**

If AB = DC , AC = DB and m ($\angle$ A) = 30° **, complete the following :**



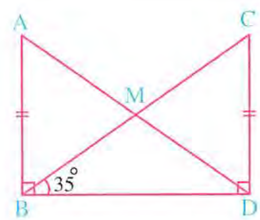
1 Δ ABC $\equiv \Delta$ ....

2 m ($\angle$ D) = ...°

3 m ($\angle$ DBC) = m ($\angle$ ....)

9 **In the opposite figure :**

If AB = CD , m ($\angle$ DBC) = 35° , $\overline{AB} \perp \overline{BD}$ and $\overline{DC} \perp \overline{DB}$ **, then complete the following :**



1 m ($\angle$ A) = ...°

2 m ($\angle$ ADC) = ...°

3 m ($\angle$ DMB) = ...°

10 **Choose the correct answer from the given ones :**

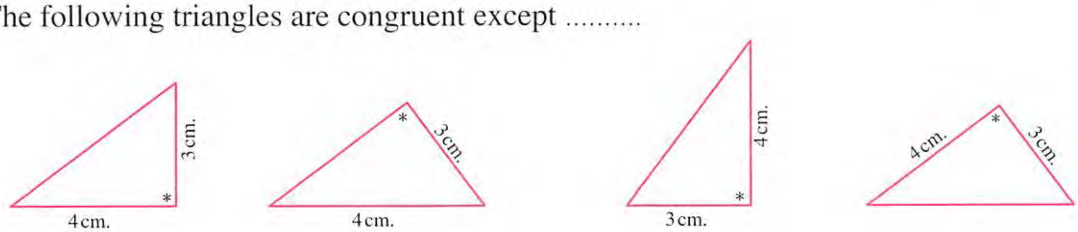
1 The following triangles are congruent except ....

(a) (b) (c) (d)

2 The following triangles are congruent except ....

(a) (b) (c) (d)

3 The following triangles are congruent except ....



(a) (b) (c) (d)

4 Which pair of the following triangles are congruent ?

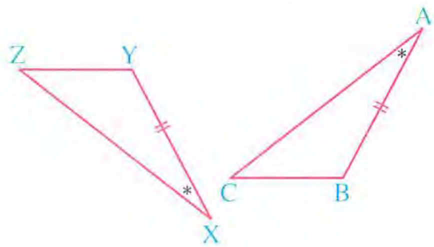
(a) (b) (c) (d)

5 **In the opposite figure :**

The necessary and enough condition which makes the two triangles ABC and XYZ be congruent is ....

(a) BC = YZ (b) AC = XZ

(c) m (∠ C) = m (∠ Z) (d) m (∠ B) = m (∠ Z)



---

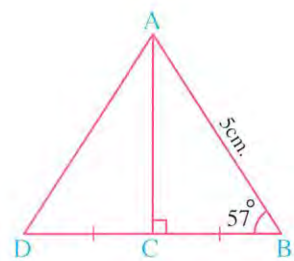
11 **In the opposite figure :**

C is the midpoint of $\overline{BD}$, $\overline{AC} \perp \overline{BD}$,

AB = 5 cm. and m (∠ B) = 57°

Find : 1 The length of $\overline{AD}$

2 m (∠ DAC)



---

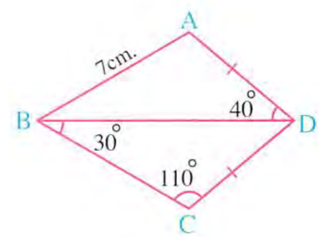
12 **In the opposite figure :**

AD = DC , m (∠ ADB) = 40° , m (∠ DBC) = 30° ,

m (∠ BCD) = 110° and AB = 7 cm.

Find : 1 The length of $\overline{BC}$

2 m (∠ BAD)



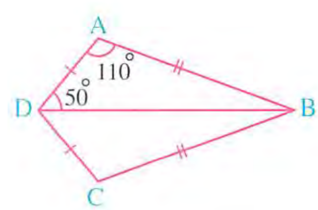
---

13 **In the opposite figure :**

BA = BC , DA = DC ,

m (∠ ADB) = 50° and m (∠ BAD) = 110°

Find : m (∠ ABC)



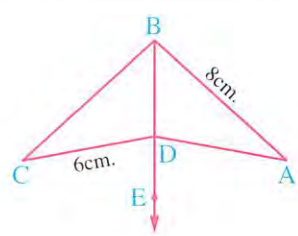
---

14 **In the opposite figure :**

$\overrightarrow{BE}$ bisects ∠ ADC , ∠ ABC , DC = 6 cm. and AB = 8 cm.

Find : 1 The length of $\overline{CB}$

2 The length of $\overline{AD}$

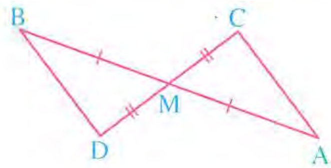


15 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{M\}$, AM = BM

and CM = DM

Is Δ AMC $\equiv \Delta$ BMD ? Why ?

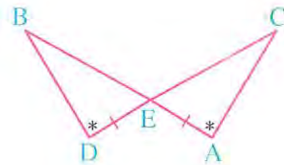


16 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$, AE = ED and $\angle$ A $\equiv \angle$ D

Is Δ ACE $\equiv \Delta$ DBE ? Why ?

Then prove that : CE = EB

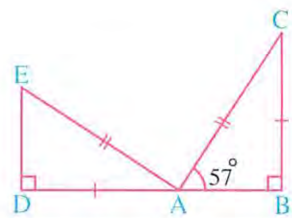


17 **In the opposite figure :**

BC = AD , AC = AE

and m ($\angle$ CAB) = 57°

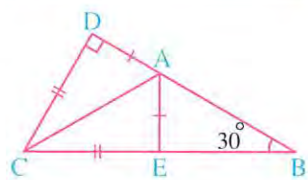
Find the measures of the unknown angles in Δ ADE



18 **In the opposite figure :**

AD = AE , DC = CE , m ($\angle$ ADC) = 90° and m ($\angle$ B) = 30°

Find : m ($\angle$ BAE)

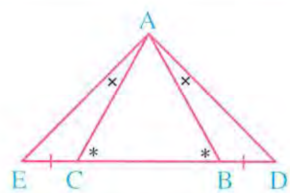


19 **In the opposite figure :**

BD = CE , m ($\angle$ ABC) = m ($\angle$ ACB)

and m ($\angle$ BAD) = m ($\angle$ CAE)

Is AD = AE ? Why ?



20 **Complete each of the following :**

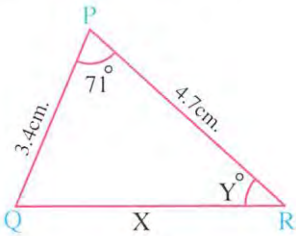
1. If Δ ABC $\equiv \Delta$ XYZ , m ($\angle$ A) = 50° and m ($\angle$ B) = 60° , then m ($\angle$ Z) = ...°
2. If Δ ABC $\equiv \Delta$ LMN , m ($\angle$ L) = 40° and m ($\angle$ B) = 90° , then m ($\angle$ C) = ...°
3. If Δ ABC $\equiv \Delta$ XYZ and m ($\angle$ A) + m ($\angle$ B) = 120° , then m ($\angle$ Z) = ...°
4. If Δ ABC $\equiv \Delta$ DEF and m ($\angle$ C) = 90° , then m ($\angle$ D) + m ($\angle$ E) = ...°
5. If Δ ABC $\equiv \Delta$ XYZ , m ($\angle$ A) + m ($\angle$ Y) = 100° , then m ($\angle$ C) + m ($\angle$ Z) = ...°
6. If Δ ABC $\equiv \Delta$ XYZ , the perimeter of Δ ABC = 12 cm. , XY = 4 cm. and YZ = 5 cm. , then AC = ...

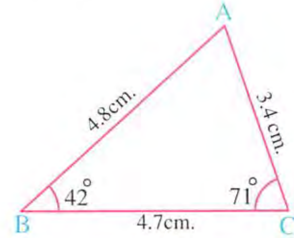
21 a. Use a protractor to draw a triangle whose angles have measures 50° , 60° and 70°

b. Can you draw another triangle whose angles have measures 50° , 60° and 70° but it is not congruent to the first triangle ?

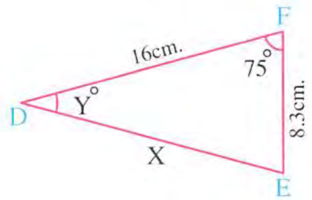
22 **Study these figures and calculate the values of X and Y :**

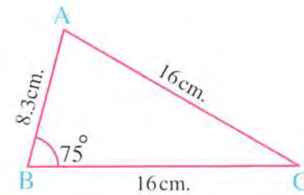
1





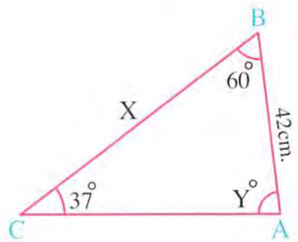
2

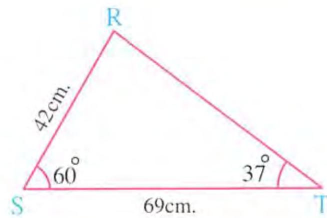




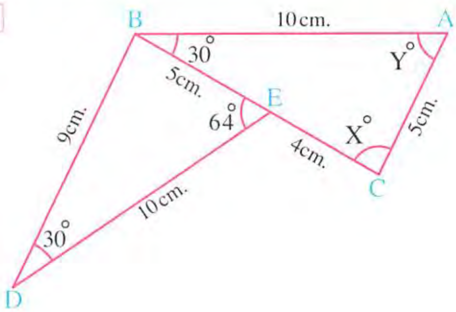
[Hint : The two angles of the base in the isosceles triangle are equal in measure]

3

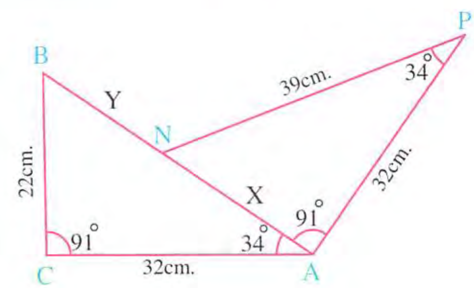




4



5

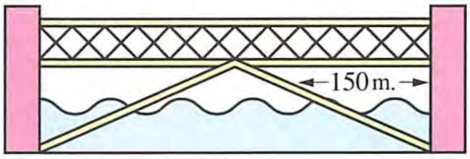


23 **Study the data for Δ ABC and Δ XPG Are these triangles congruent ? Write if applicable , a correct statement of congruence and state the test used.**

1. AB = PX , AC = XG , ∠ A ≡ ∠ X
2. BC = PG , BA = XP , ∠ B ≡ ∠ G
3. AB = PG , BC = PX , AC = XG
4. AB = XP , CA = GX , ∠ B ≡ ∠ P
5. ∠ B ≡ ∠ G , ∠ C ≡ ∠ X , BC = XG
6. ∠ A ≡ ∠ X , ∠ B ≡ ∠ P , AC = PG

Life Applications

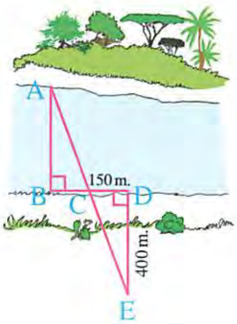
24 **In the opposite figure :**



A horizontal bridge is built above a part of the river on two vertical pillars equal in length and two sloping carriers equal in length.

Using the drawing , find the length of the bridge showing the steps of solution.

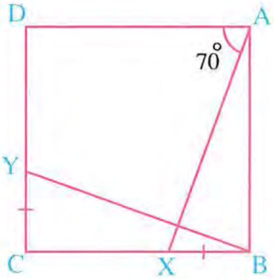
25 To find the width of the river AB , put the point C on the shore of the river , then measure the distance between B and C and move the same distance to the point D , then walk perpendicularly to reach the point E such that the points A , C and E are collinear and measure the length of $\overline{DE}$



Using the above-mentioned method , and the data on the opposite illustrative drawing , find the width of the river AB

For excellent pupils

26 **In the opposite figure :**

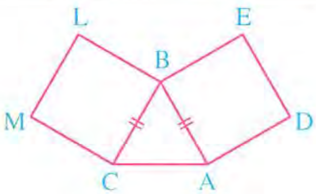


ABCD is a square , BX = CY

and m (∠ XAD) = 70°

Find : m (∠ YBC) with showing the steps of the solution.

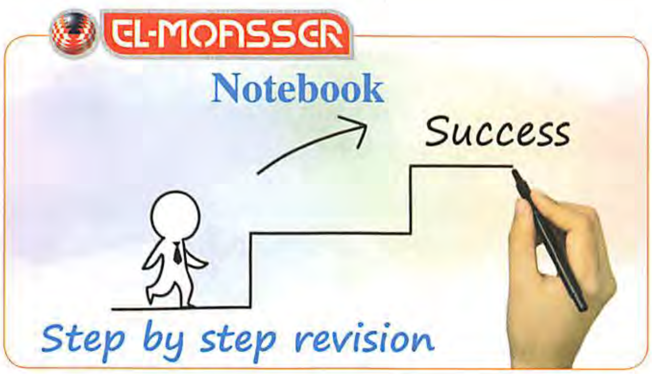
27 **In the opposite figure :**



ABC is an isosceles triangle

, ABED , CBLM are two squares

Explain that : CE = AL



EXERCISE 5 ?

Parallelism

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 Complete the following :

1 The straight line which is perpendicular to one of two parallel straight lines is .... to the other straight line in the plane.

2 If two straight lines are parallel to a third straight line , then they are ....

3 If two straight lines are perpendicular to a third straight line , then these two straight lines are ....

4 If a straight line cuts two parallel straight lines , then each two alternate angles are ....

5 If a straight line cuts two parallel straight lines , then each two corresponding angles are ....

6 If a straight line cuts two parallel straight lines , then each two interior angles in the same side of the transversal are ....

7 If a straight line cuts two straight lines and there are two corresponding angles having the same measure , then the two straight lines are ....

8 If a straight line cuts two straight lines and there are two alternate angles having the same measure , then the two straight lines are ....

9 If a straight line cuts two straight lines and there are two interior angles in the same side of the transversal are supplementary , then the two straight lines are ....

10 If a straight line cuts several parallel lines and the intercepted parts of this transversal between these parallel straight lines are equal in length , then the intercepted parts for any transversal are ....

2 **In each of the following figures , the straight line L // the straight line M and the straight line K is a transversal to them. Find the measures of the angles marked by « ? »**

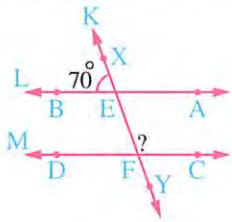


Fig. (1)

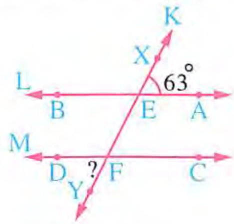


Fig. (2)

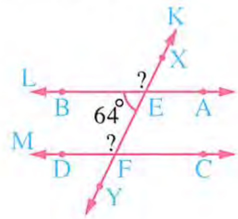


Fig. (3)

3 **In each of the following figures , if $\overrightarrow{AC}$ // $\overline{BD}$ and $\overline{AB}$ // $\overrightarrow{DE}$, find the measures of the angles marked by « ? »**

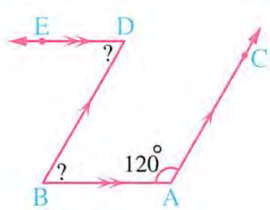


Fig. (1)

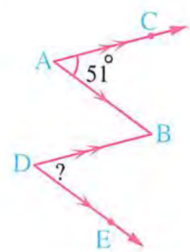


Fig. (2)

4 **Complete , using the data shown in each figure :**

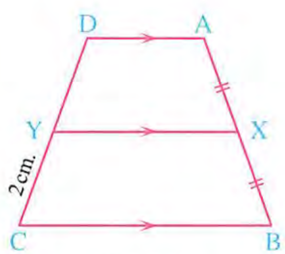


Fig. (1)

DY = ... cm.

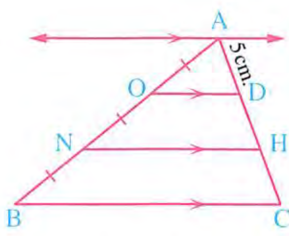


Fig. (2)

AC = ... cm.

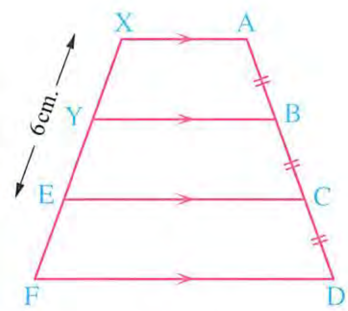


Fig. (3)

YF = ... cm.

5 **In each of the following figures , if $\overleftrightarrow{MN}$ intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ at E and F respectively , prove that : $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$**

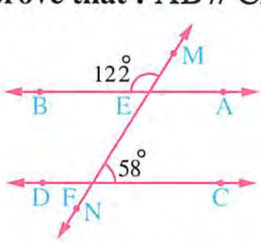


Fig. (1)

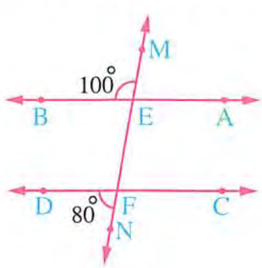


Fig. (2)

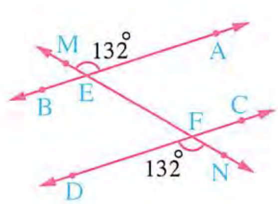


Fig. (3)

6 **In each of the following figures , show with reasons why is $\overline{AD} \parallel \overline{BC}$:**

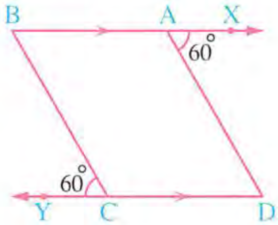


Fig. (1)

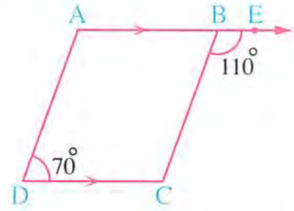


Fig. (2)

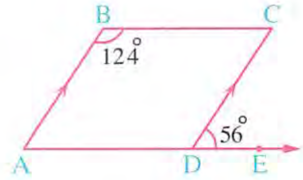


Fig. (3)

7 **Choose the correct answer from those given :**

1 If L_1 and L_2 are two coplanar straight lines where $L_1 \cap L_2 = \varnothing$, then L_1 and L_2 are ....

(a) intersecting. (b) perpendicular.

(c) parallel. (d) coincident.

2 The two straight lines parallel to a third one are ....

(a) perpendicular. (b) coincident.

(c) parallel. (d) intersecting.

3 If L_1 , L_2 and L_3 are three coplanar straight lines , $L_1 \perp L_3$ and $L_2 \perp L_3$, then ....

(a) $L_1 \parallel L_2$ (b) $L_1 \perp L_2$

(c) L_1 is coincides L_2 (d) L_1 intersects L_2

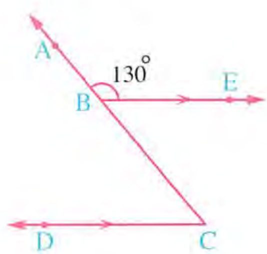
4 If L_1 , L_2 and L_3 are three coplanar straight lines , $L_1 \parallel L_3$ and $L_2 \parallel L_3$, then ....

(a) $L_1 \perp L_2$ (b) $L_1 \perp L_3$ (c) $L_1 \parallel L_2$ (d) $L_2 \perp L_3$

5 If L_1 , L_2 and L_3 are three coplanar straight lines , $L_1 \perp L_2$ and $L_1 \parallel L_3$, then L_2 .... L_3

(a) $\perp$ (b) $\parallel$ (c) coincides. (d) bisects.

6 **In the opposite figure :**



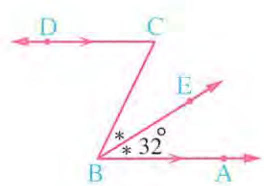
$B \in \overline{AC}$, $\overrightarrow{BE} \parallel \overrightarrow{CD}$ and m ($\angle$ ABE) = 130°

, then m ($\angle$ C) = ...

(a) 130° (b) 40°

(c) 50° (d) 90°

7 **In the opposite figure :**



$\overrightarrow{BE}$ bisects $\angle$ ABC , $\overrightarrow{BA} \parallel \overrightarrow{CD}$ and

m ($\angle$ ABE) = 32° , then m ($\angle$ C) = ...

(a) 32° (b) 64°

(c) 60° (d) 80°

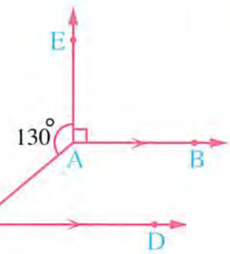
8 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, m (∠ EAC) = 130°

and m (∠ EAB) = 90° , then m (∠ C) = ...

(a) 90° (b) 130°

(c) 140° (d) 40°



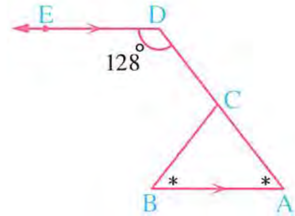
9 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{DE}$, m (∠ D) = 128° ,

m (∠ A) = m (∠ B) and C ∈ $\overline{AD}$, then m (∠ B) = ...

(a) 64° (b) 128°

(c) 52° (d) 26°



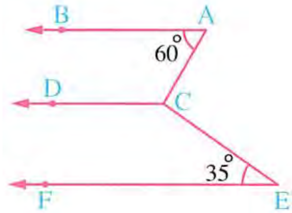
10 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AB} \parallel \overrightarrow{EF}$, m (∠ A) = 60° and

m (∠ E) = 35° , then m (∠ ACE) = ...

(a) 60° (b) 35°

(c) 95° (d) 85°



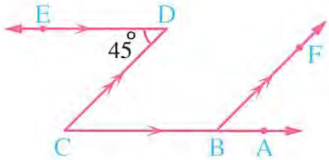
11 **In the opposite figure :**

m (∠ D) = 45° , $\overrightarrow{DE} \parallel \overrightarrow{CA}$ and

$\overrightarrow{CD} \parallel \overrightarrow{BF}$, then m (∠ ABF) = ...

(a) 45° (b) 90°

(c) 135° (d) 40°



12 **In the opposite figure :**

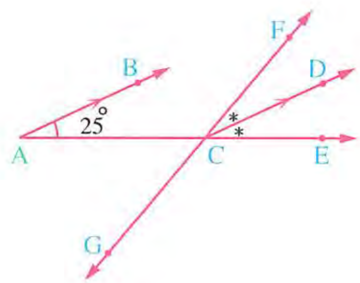
$\overleftrightarrow{FG} \cap \overrightarrow{AE} = \{C\}$, $\overrightarrow{CD}$ bisects ∠ FCE ,

$\overrightarrow{CD} \parallel \overrightarrow{AB}$ and m (∠ A) = 25°

, then m (∠ GCA) = ...

(a) 25° (b) 50°

(c) 130° (d) $12\frac{1}{2}$°



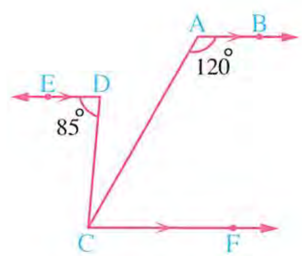
13 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CF} \parallel \overrightarrow{DE}$, m (∠ A) = 120°

and m (∠ D) = 85° , then m (∠ ACD) = ...

(a) 60° (b) 85°

(c) 25° (d) 120°



14 **In the opposite figure :**

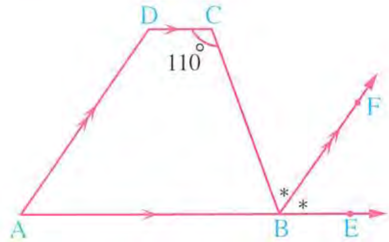
$\overline{CD} // \overrightarrow{AB}$, m (∠ C) = 110° ,

$\overline{AD} // \overrightarrow{BF}$ and $\overrightarrow{BF}$ bisects ∠ CBE

where E ∈ $\overleftrightarrow{AB}$, then m (∠ A) = ...

(a) 55° (b) 110°

(c) 70° (d) 160°

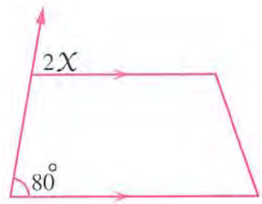


15 **In the opposite figure :**

What is the value of X ?

(a) 40° (b) 60°

(c) 80° (d) 100°

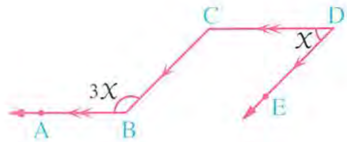


16 **In the opposite figure :**

$\overline{CD} // \overrightarrow{BA}$, $\overrightarrow{DE} // \overline{CB}$, then X = ...

(a) 60° (b) 45°

(c) 120° (d) 90°



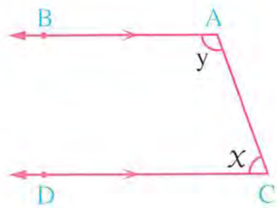
17 **In the opposite figure :**

If $\overrightarrow{AB} // \overrightarrow{CD}$ and $\frac{X}{y} = \frac{7}{11}$

, then X = ...

(a) 60° (b) 70°

(c) 100° (d) 110°

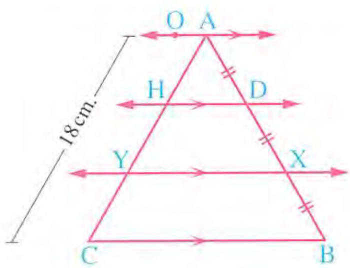


8 **In the opposite figure :**

$\overleftrightarrow{AO} // \overleftrightarrow{HD} // \overleftrightarrow{YX} // \overline{CB}$, AD = DX = XB

and AC = 18 cm.

Find the length of $\overline{AY}$

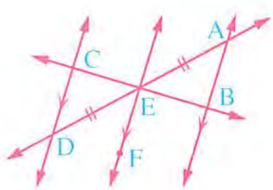


9 **In the opposite figure :**

$\overleftrightarrow{AD} \cap \overleftrightarrow{BC} = \{E\}$, $\overleftrightarrow{AB} // \overleftrightarrow{EF} // \overleftrightarrow{CD}$, AE = DE

and BC = 8 cm.

Find the length of $\overline{BE}$

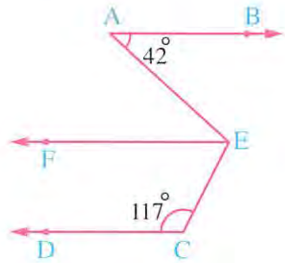


10 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{EF} \parallel \overrightarrow{CD}$

, m ($\angle$ A) = 42° and m ($\angle$ C) = 117°

Find : m ($\angle$ AEC)

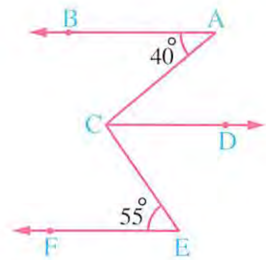


11 **In the opposite figure :**

m ($\angle$ A) = 40° , m ($\angle$ E) = 55°

, $\overrightarrow{AB} \parallel \overrightarrow{EF}$ and $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Find : m ($\angle$ ACE)

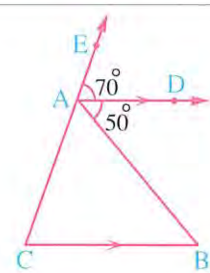


12 **In the opposite figure :**

$\overrightarrow{AD} \parallel \overline{BC}$, E $\in \overrightarrow{CA}$,

m ($\angle$ DAE) = 70° and m ($\angle$ DAB) = 50°

Find the measures of the angles of the triangle ABC

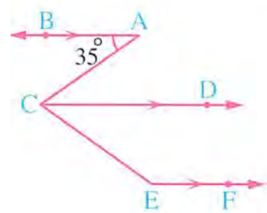


13 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD} \parallel \overrightarrow{EF}$, m ($\angle$ A) = 35° and

$\overrightarrow{CD}$ bisects $\angle$ ACE

Find : 1 m ($\angle$ DCE) 2 m ($\angle$ CEF)

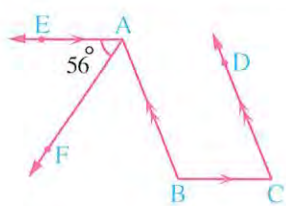


14 **In the opposite figure :**

$\overrightarrow{AE} \parallel \overline{CB}$, $\overline{BA} \parallel \overrightarrow{CD}$,

$\overrightarrow{AF}$ bisects $\angle$ BAE and m ($\angle$ EAF) = 56°

Find : m ($\angle$ C)

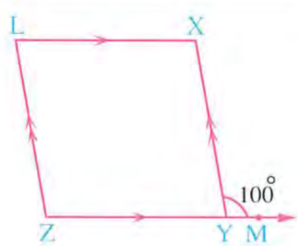


15 **In the opposite figure :**

$\overline{XL} \parallel \overline{YZ}$, $\overline{XY} \parallel \overline{LZ}$ and m ($\angle$ XYM) = 100°

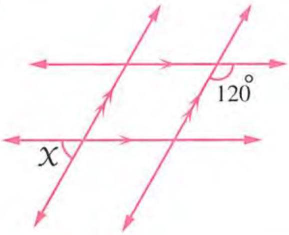
, where M $\in \overrightarrow{ZY}$

Find : 1 m ($\angle$ X) 2 m ($\angle$ Z) 3 m ($\angle$ L)

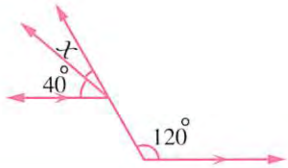


16 **Find the value of x in each figure :**

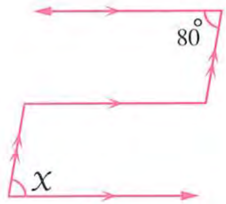
1



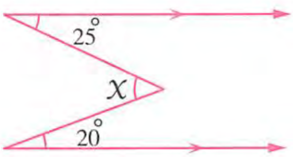
2



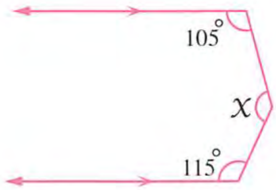
3



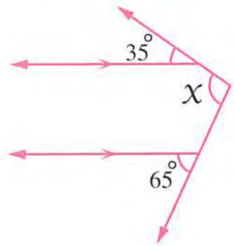
4



5



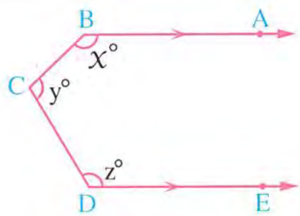
6



17 **In In the opposite figure :**

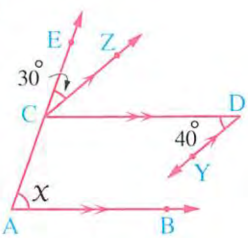
$\overrightarrow{BA} \mathbin{/\!/} \overrightarrow{DE}$

Find the value of the expression : $x + y + z$

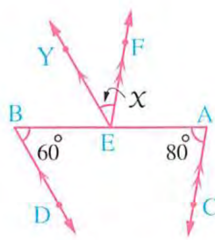


18 **Find the value of x in each of the following figures :**

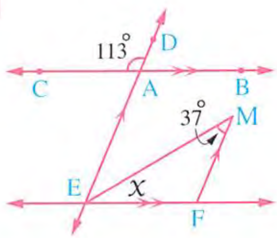
1



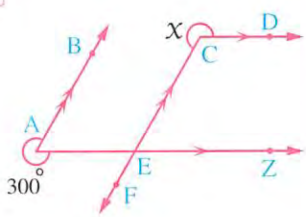
2



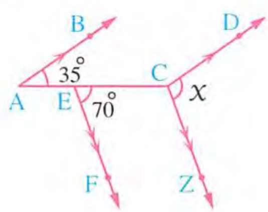
3



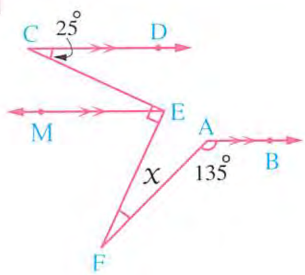
4



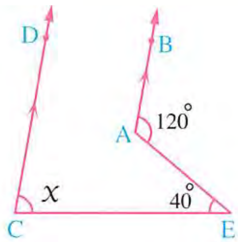
5



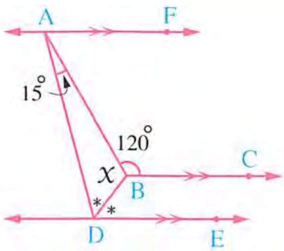
6



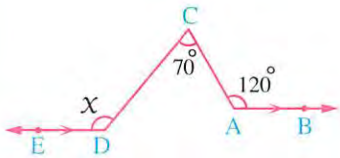
7



8

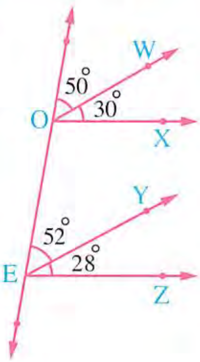


9

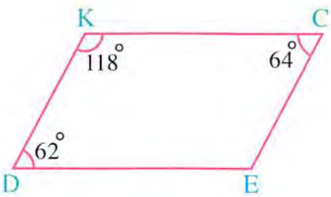


19 **Find the pairs of parallel lines in each figure :**

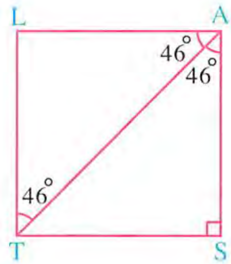
1



2



3

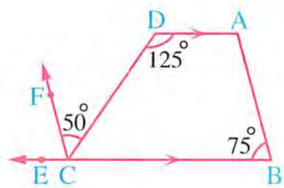


20 **In the opposite figure :**

$\overrightarrow{AD} \parallel \overrightarrow{BC}$, $E \in \overrightarrow{BC}$,

m (∠ B) = 75° , m (∠ D) = 125° and

m (∠ DCF) = 50° Is $\overleftrightarrow{AB} \parallel \overrightarrow{CF}$? Why ?

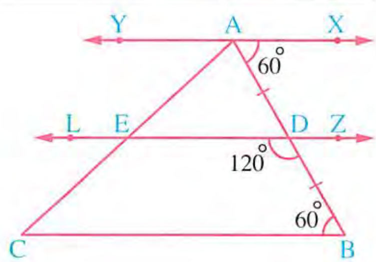


21 **In the opposite figure :**

m (∠ XAD) = m (∠ B) = 60°

, m (∠ EDB) = 120° , AD = DB and AC = 18 cm.

Find the length of $\overline{AE}$ giving the reason.

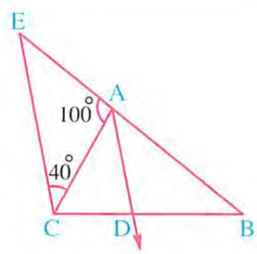


22 **In the opposite figure :**

$A \in \overline{BE}$, $\overrightarrow{AD}$ bisects ∠ BAC

, m (∠ EAC) = 100° and m (∠ ACE) = 40°

Is $\overrightarrow{AD} \parallel \overline{CE}$? Why ?

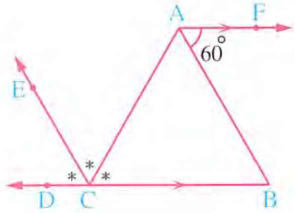


23 **In the opposite figure :**

$m(\angle FAB) = 60°$, $\overrightarrow{AF} // \overrightarrow{BD}$,

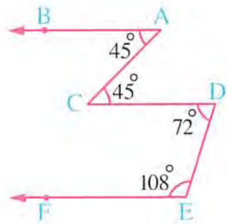
$C \in \overline{BD}$ and $m(\angle ACB) = m(\angle ACE) = m(\angle ECD)$

Is $\overline{AB} // \overline{CE}$? Why ?



24 **In the opposite figure :**

Is $\overrightarrow{AB} // \overline{DC} // \overrightarrow{EF}$? Why ?



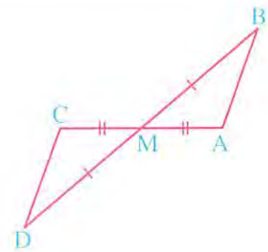
25 **In the opposite figure :**

$\overline{BD} \cap \overline{AC} = \{M\}$

, MB = MD and MA = MC

1 Is $\Delta AMB \equiv \Delta CMD$? Why ?

2 Is $\overline{AB} // \overline{CD}$? Why ?

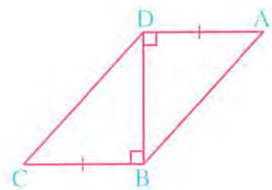


26 **In the opposite figure :**

ABCD is a quadrilateral in which

AD = CB and $m(\angle ADB) = m(\angle CBD) = 90°$

Is $\overline{AB} // \overline{CD}$? Why ?

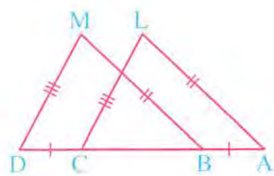


27 **In the opposite figure :**

$B \in \overline{AD}$ and $C \in \overline{AD}$

such that : AB = CD , AL = BM and LC = MD

Is $\overline{AL} // \overline{BM}$, $\overline{CL} // \overline{DM}$? Why ?

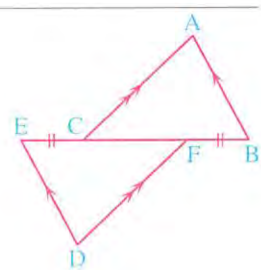


28 **In the opposite figure :**

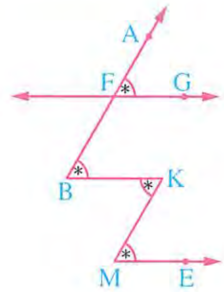
$\overline{AB} // \overline{ED}$, $\overline{AC} // \overline{FD}$

and $\overline{BF} \equiv \overline{CE}$

Is $\overline{AB} \equiv \overline{DE}$? Why ?



29 **In the opposite figure :**

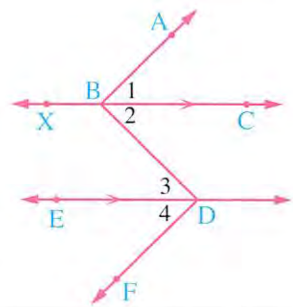


m (∠ AFG) = m (∠ B) = m (∠ K) = m (∠ M)

Write the four pairs of parallel lines.

Give your reasons.

30 **In the opposite figure :**



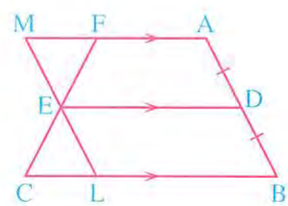
m (∠ 1) = m (∠ 4)

and $\overleftrightarrow{BC} \,//\, \overleftrightarrow{ED}$

Does $\overrightarrow{BA} \,//\, \overrightarrow{DF}$?

Give reason.

31 **In the opposite figure :**



$\overleftrightarrow{AM} \,//\, \overleftrightarrow{DE} \,//\, \overleftrightarrow{BC}$, AD = DB , F $\in \overline{AM}$

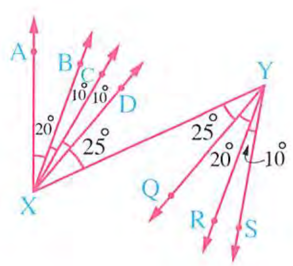
, L $\in \overline{BC}$, $\overline{ML} \cap \overline{FC} = \{E\}$

Is FM = LC ? Why ?

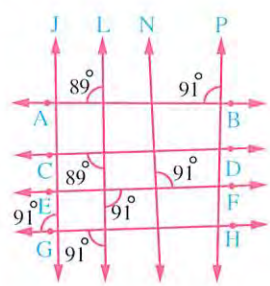
For excellent pupils

32 **In each of the following figures , name the pairs of parallel lines :**

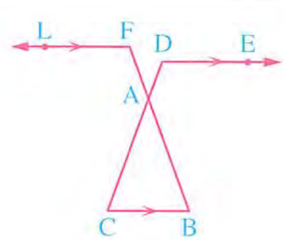
1



2



33 **In the opposite figure :**



If $\overline{DE} \,//\, \overline{BC} \,//\, \overline{FL}$

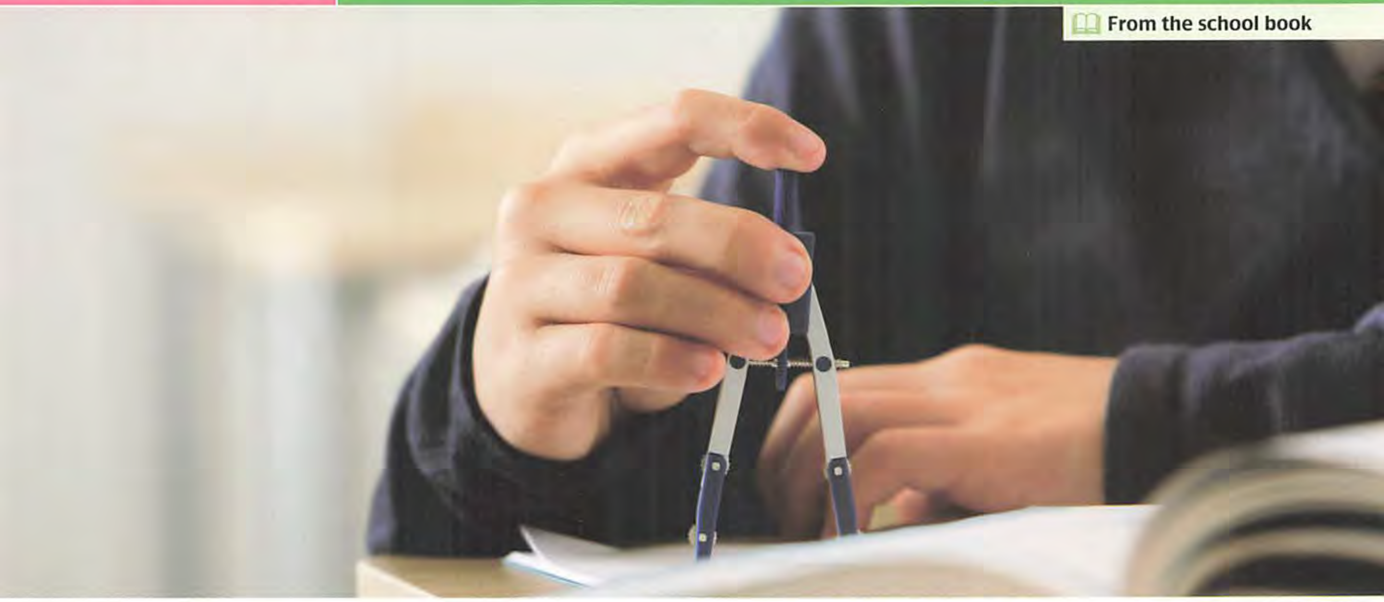
, m (∠ D) + m (∠ F) = 220°

Find : m (∠ BAC)

EXERCISE 6 ?

Geometric Constructions

From the school book



● Remember ● Understand ● Apply Problem Solving

First Constructing a perpendicular from a given point to a straight line

1 Using the ruler and the compasses, draw Δ ABC in which AB = AC = 5 cm. , BC = 6 cm. , then draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$
Then find by measuring the length of $\overline{AD}$ (Don't remove the arcs). «4 cm.»

2 Using the geometric tools, draw the equilateral triangle ABC of side length 5 cm. , then draw $\overline{AD} \perp \overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$ (Don't remove the arcs).

3 Draw Δ ABC in which AB = 6 cm. , m ($\angle$ A) = 50° , m ($\angle$ B) = 70° , then draw $\overline{CD} \perp \overline{AB}$ to cut it at D , then find the length of $\overline{CD}$ by measuring and calculate the area of Δ ABC (Don't remove the arcs). «5 cm. , 15 cm².»

4 Draw the equilateral triangle ABC in which the length of each side is 4 cm. , then draw $\overrightarrow{CD} \perp \overleftrightarrow{CB}$ that intersects $\overrightarrow{BA}$ at D , find by measuring the length of $\overline{DA}$ « 4 cm. »

5 **Using the geometric instruments , draw a triangle , then draw its altitudes if the triangle is :**

1 an acute-angled triangle.
2 a right-angled triangle.
3 an obtuse-angled triangle.

Are the straight lines that contain the altitudes concurrent.
Where is the position of the intersection point with respect to the triangle ?
Is it inside or outside the triangle or belongs to one of its sides ?

Second Bisecting a given line segment "Constructing the symmetry axis of a given line segment"

6 Using the ruler and the compasses , draw the line segment $\overline{BC}$ of length 7 cm. , then draw the straight line L as an axis of symmetry of it. (Don't remove the arcs)

7 Draw $\overline{AB}$ of length 6 cm. , using compasses and a ruler , draw the straight line L that is the axis of symmetry of $\overline{AB}$, where $\overline{AB} \cap L = \{C\}$

Label the point $D \in L$ such that CD = 4 cm. Measure the lengths of $\overline{DA}$ and $\overline{DB}$

(Don't remove the arcs) « 5 cm. »

8 Draw $\overline{BC}$ with a suitable length. Using compasses and the unscaled ruler, bisect $\overline{BC}$ at D and from D, draw $\overline{DA}$ perpendicular to $\overline{BC}$, then draw $\overline{AB}$ and $\overline{AC}$

Compare the lengths of $\overline{AB}$ and $\overline{AC}$ using the compasses. What do you observe ?

9 Draw the isosceles triangle ABC in which AB = AC Using the compasses , bisect $\overline{BC}$ at D, draw $\overline{AD}$, is $\overline{AD} \perp \overline{BC}$?

10 Using the geometric instruments, draw Δ XYZ in which m ($\angle$ Y) = 90° ,

XY = YZ = 4 cm., then bisect $\overline{XZ}$ at L , then draw $\overline{YL}$

Find by measuring m ($\angle$ XLY) (Don't remove the arcs) « 90° »

11 Draw the triangle ABC in which AB = AC = 4 cm. , BC = 6 cm. Bisect $\overline{AB}$ at D and $\overline{AC}$ at E , then draw $\overline{DE}$, and find its length. (Don't remove the arcs) « 3 cm. »

12 Draw the triangle ABC in which m ($\angle$ B) = 90° , AB = 8 cm. , BC = 6 cm. Bisect $\overline{AC}$ at D , is BD = $\frac{1}{2}$ AC ?

13 Draw the triangle ABC in which AB = 4 cm. , BC = 5 cm. and AC = 6 cm.

Construct the bisector altitudes of the sides of the triangle. What do you notice ?

14 **Using the geometric instruments, draw a triangle, then draw the axes of symmetry of its sides if the triangle is :**

1 an acute-angled triangle. 2 a right-angled triangle. 3 an obtuse-angled triangle.

Are the symmetry axes of the sides of the triangle concurrent ?

15 Draw Δ ABC using an unscaled ruler and compasses , bisect $\overline{AB}$ and $\overline{AC}$ at D and E respectively. Draw $\overline{DE}$

1 Using the compasses , measure the length of $\overline{DE}$ and check that BC = 2 DE

2 Does $\angle$ ABC $\equiv$ $\angle$ ADE ? Does $\overline{DE}$ // $\overline{BC}$?

16 Draw the right-angled triangle XYZ at Y using the compasses and the ruler only. Bisect $\overline{XZ}$ at M. Draw $\overline{YM}$ Are MX = MY = MZ ?
Draw other right-angled triangles and repeat the same construction.
Are MX = MY = MZ ?

Third Constructing the bisector of a given angle

17 Using the geometric instruments, draw an angle of measure 120° and bisect it (Don't remove the arcs)

18 Using the geometric tools, draw an angle of measure 75° and bisect it (Don't remove the arcs)

19 Draw an angle whose vertex is A and its measure is 130° , use a ruler and a compasses to divide the angle A into 4 equal angles in measure. (Don't remove the arcs)

20 Using the ruler and the compasses, draw Δ ABC in which AB = AC = 3 cm. , BC = 5 cm. , then bisect $\angle$ A by the bisector $\overrightarrow{AD}$ where D $\in$ $\overline{BC}$ (Don't remove the arcs)

21 Using a ruler and the compasses , draw a triangle ABC in which AB = AC = 7 cm. , BC = 6 cm. Bisect $\angle$ B and $\angle$ C by two bisectors which intersect at M
Is MB = MC ? (Don't remove the arcs)

22 Using the geometric tools, draw Δ ABC in which AB = 3 cm. , BC = 4 cm. , AC = 5 cm. , then bisect $\angle$ B by the bisector $\overrightarrow{BD}$ to cut $\overline{AC}$ at D Find the length of $\overline{BD}$ by measuring. (Don't remove the arcs) «2.4 cm.»

23 Draw the equilateral triangle ABC of side length 4 cm. using the compasses and the ruler bisect each of $\angle$ ABC and $\angle$ ACB , If the two bisectors intersect at M , find by measuring m ($\angle$ BMC) (Don't remove the arcs) «120°»

24 **Using the geometric instruments , draw a triangle and bisect each of its angles if the triangle is :**

1 an acute-angled triangle. 2 a right-angled triangle. 3 an obtuse-angled triangle.

What do you notice about the three bisectors of the angles of the triangle ?

Fourth Constructing an angle to be congruent to a given angle and drawing a straight line from a given point parallel to a given straight line

25 Draw $\angle$ A of measure 100° , then using the ruler and the compasses , draw another angle B such that angle B equals in measure angle A , and bisect it.

26 Using the protractor , draw $\angle$ ABC of measure 70° and on the other side of $\overline{BA}$, draw using the ruler and the compasses $\overrightarrow{AE} \parallel \overrightarrow{BC}$ (Don't remove the arcs)

27 Draw Δ ABC in which AB = 6 cm. , m ($\angle$ A) = 50° , m ($\angle$ B) = 70° using the compasses and ruler draw $\overleftrightarrow{XY}$ passing through A and parallel to $\overleftrightarrow{BC}$ (Don't remove the arcs)

28 **Using the compasses and the ruler , draw the triangle ABC in which AB = 5 cm. , BC = 6 cm. and CA = 7 cm. , D $\in \overrightarrow{CB}$ and D $\notin \overline{CB}$:**

1. Draw $\angle$ DBE congruent to $\angle$ A such that the ray $\overrightarrow{BE}$ is lying between the two rays $\overrightarrow{BA}$ and $\overrightarrow{BD}$
2. **Complete :** m ($\angle$ ABE) = m ($\angle$ ....)

29 Draw Δ ABC in which AB = 6 cm. , BC = 5 cm. and AC = 4 cm. , then bisect $\overline{BC}$ at D , then draw $\overrightarrow{DE} \parallel \overline{AB}$ to cut $\overline{AC}$ at E , then draw $\overrightarrow{EF} \parallel \overline{CB}$ to cut $\overline{AB}$ at F
Find by measuring the length of each of $\overline{ED}$ and $\overline{EF}$, then write the name of the figure DEFB and find its perimeter. « ED = 3 cm. , EF = 2.5 cm. , The perimeter = 11 cm.»

For excellent pupils

30 Without using the protractor, draw an angle of measure $22\frac{1}{2}^\circ$

31 Draw $\angle$ ABC with measure 60° using the ruler and the compasses , bisect $\angle$ ABC , from C , draw $\overrightarrow{CE} \parallel \overrightarrow{BA}$ to meet the bisector of the angle at E , from E , draw $\overrightarrow{EF} \perp \overrightarrow{BA}$ where $\overleftrightarrow{EF} \cap \overleftrightarrow{BA} = \{F\}$ Does m ($\angle$ ABC) = m ($\angle$ FEB) ? Why ? (Don't remove the arcs)

A Research Project

On Unit Four

Project aims :

- Classifying angles according to their types.
- Using geometric constructions to make desings.
- Associating mathematics with geography.

Do a research project on the following topic :

"The earth rotates around its axis in front of the sun from west to east once every 24 hours and orbits the sun once every 365 $\frac{1}{4}$ days".

Discuss the following points using available resources :

1. Find an interpretation of day and night phenomenon and write how they follow each other.
2. Why does temperature vary from one season to the other all over the year ?
3. Use geometric constructions to make an accurate desing for a face of a clock ensuring that distances between the signs that show hours and minutes are equal.
4. Use a hand watch and set it to show the following times. Each time, mention the type of the angle between the two hands :

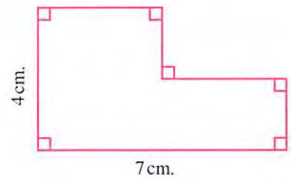
   Three o'clock – six o'clock – nine o'clock – twelve o'clock – a quarter past eight – five to ten

TIMSS Problems

Accumulative basic skills

1 Complete the following :

1 The perimeter of the opposite figure is .... cm.

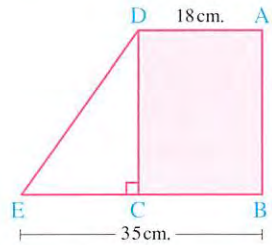


2 **In the opposite figure :**

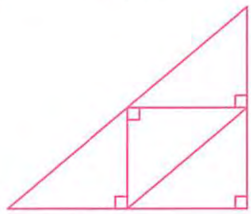
ABCD is a rectangle whose area is 360 cm^2.

, AD = 18 cm. and BE = 35 cm.

, then the area of Δ DCE = ... cm^2.



3 The number of right-angled triangles in the opposite figure equals ....



4 If A = (3 , – 4) , B = (– 2 , – 4) , then AB = ...

5 The ratio between the perimeter of the square and its side length equals ....

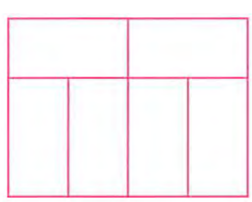
6 The image of the point (– 3 , 5) by translation 3 units in the negative direction of the y-axis is ....

7 If the sum of measures of two angles in a triangle is $\frac{5}{6}$ the sum of measures of all its angles , then the measure of the third angle is ....°

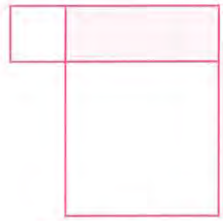
8 The ratio between the diameter of a circle and its circumference is ....

9 **In the opposite figure :**

A rectangle of area 48 cm^2. is divided into 6 congruent rectangles , then its perimeter equals .... cm.



10 **In the opposite figure :**



If the sum of perimeters of the two squares is 28 cm.

, then the perimeter of the coloured rectangle is .... cm.

11 **In the opposite figure :**

A circle is inscribed in a square whose side length is 14 cm. , then the area of the coloured region is .... cm^2. ($\pi = \frac{22}{7}$)

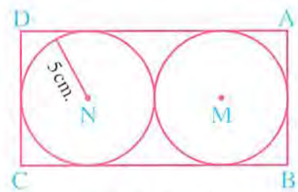
12 **In the opposite figure :**

A circle is inscribed in a square whose side length is 10 cm. , then the perimeter of the coloured region is .... cm. ($\pi = 3.14$)

---

2 Choose the correct answer from the given ones :

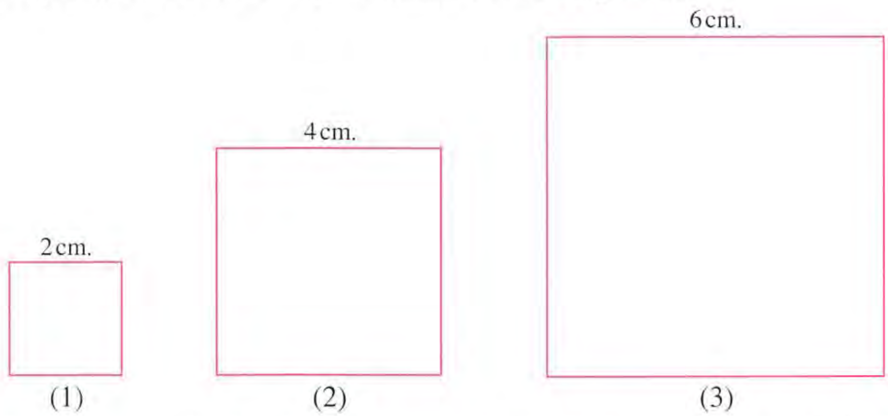
1 **In the opposite figure :**



Two circles in a rectangle , the radius length of each one is 5 cm. What is the area of the rectangle ?

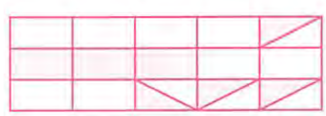
(a) 200 cm^2. (b) 100 cm^2. (c) 60 cm^2. (d) 50 cm^2.

2 What is the perimeter of the 6^{th} square in the following pattern ?



(a) 32 cm. (b) 40 cm. (c) 48 cm. (d) 56 cm.

3 **In the opposite figure :**



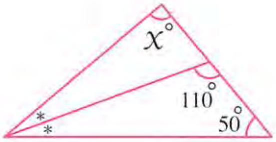
The area of the coloured part = ... the area of the whole figure.

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{7}{15}$

4 The best unit to measure the area of a room is ....

(a) mm^2. (b) cm^2. (c) m^2. (d) km^2.

5 **In the opposite figure :**



$x =$ ....

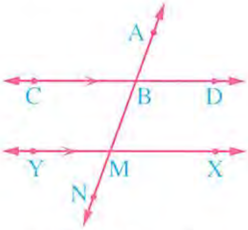
(a) 50° (b) 80°

(c) 90° (d) 100°

6 Which of the following figures shows that $\frac{2}{3}$ the square is coloured ?

(a) (b) (c) (d)

7 **In the opposite figure :**



If $\overleftrightarrow{DC} \parallel \overleftrightarrow{XY}$, then ∠ ABC and ∠ XMN are ....

(a) complementary. (b) supplementary.

(c) congruent. (d) adjacent.

8 Which of the following sentences is wrong for all rectangles ?

(a) Opposite sides are parallel. (b) Opposite sides are equal in length.

(c) All angles are right. (d) The diagonals are perpendicular.

9 The small squares in the figures (a) and (b) are congruent. If the perimeter of fig. (a) is 48 cm. , then the perimeter of fig. (b) is .... cm.

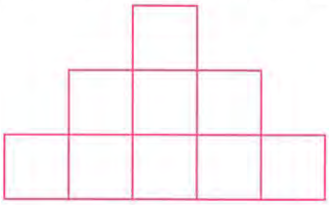


Fig. (a)

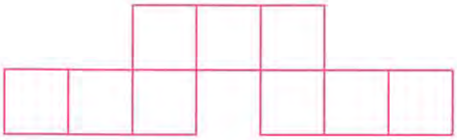


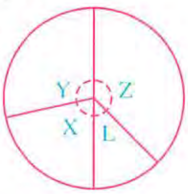
Fig. (b)

(a) 48 (b) 57 (c) 60 (d) 63

10 A square its side length is a whole number , then its perimeter can be ....

(a) 33 cm. (b) 44 cm. (c) 55 cm. (d) 66 cm.

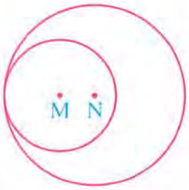
11 Which of the angles in the opposite figure has the closest measure to 45° ?



(a) X (b) Y

(c) Z (d) L

12 The number of axes of symmetry of the opposite figure is ....



(a) 1 (b) 2

(c) 3 (d) an infinite number.

EL-MOASSER

By a group of supervisors

NOTEBOOK

- Accumulative Tests
- Important Questions
- Final Revision
- Final Examinations

1st PREP. 2023

FIRST TERM

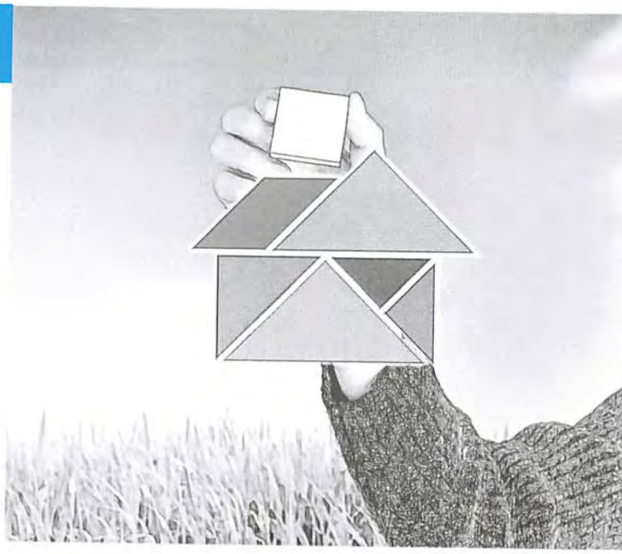
Maths

CONTENTS

First Algebra and Statistics

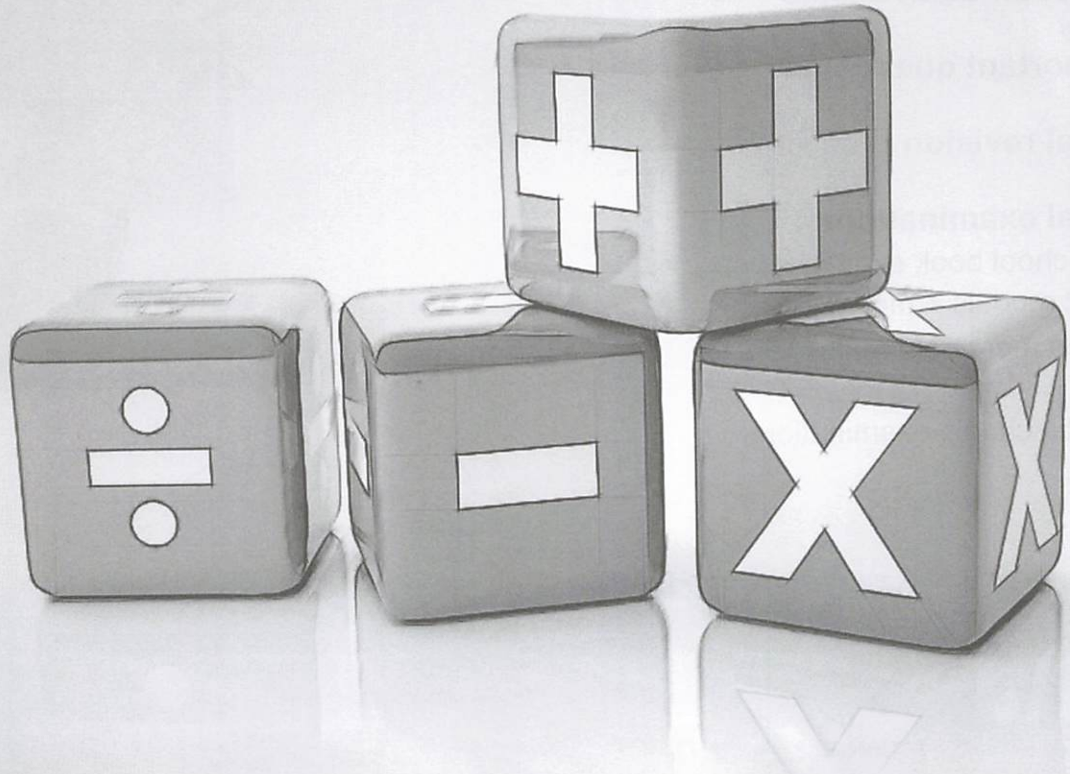
- 17 Accumulative tests
- Important questions
- Final revision
- Final examinations :
  - School book examinations (2 models + model for the merge students)
  - 15 schools examinations

Second Geometry



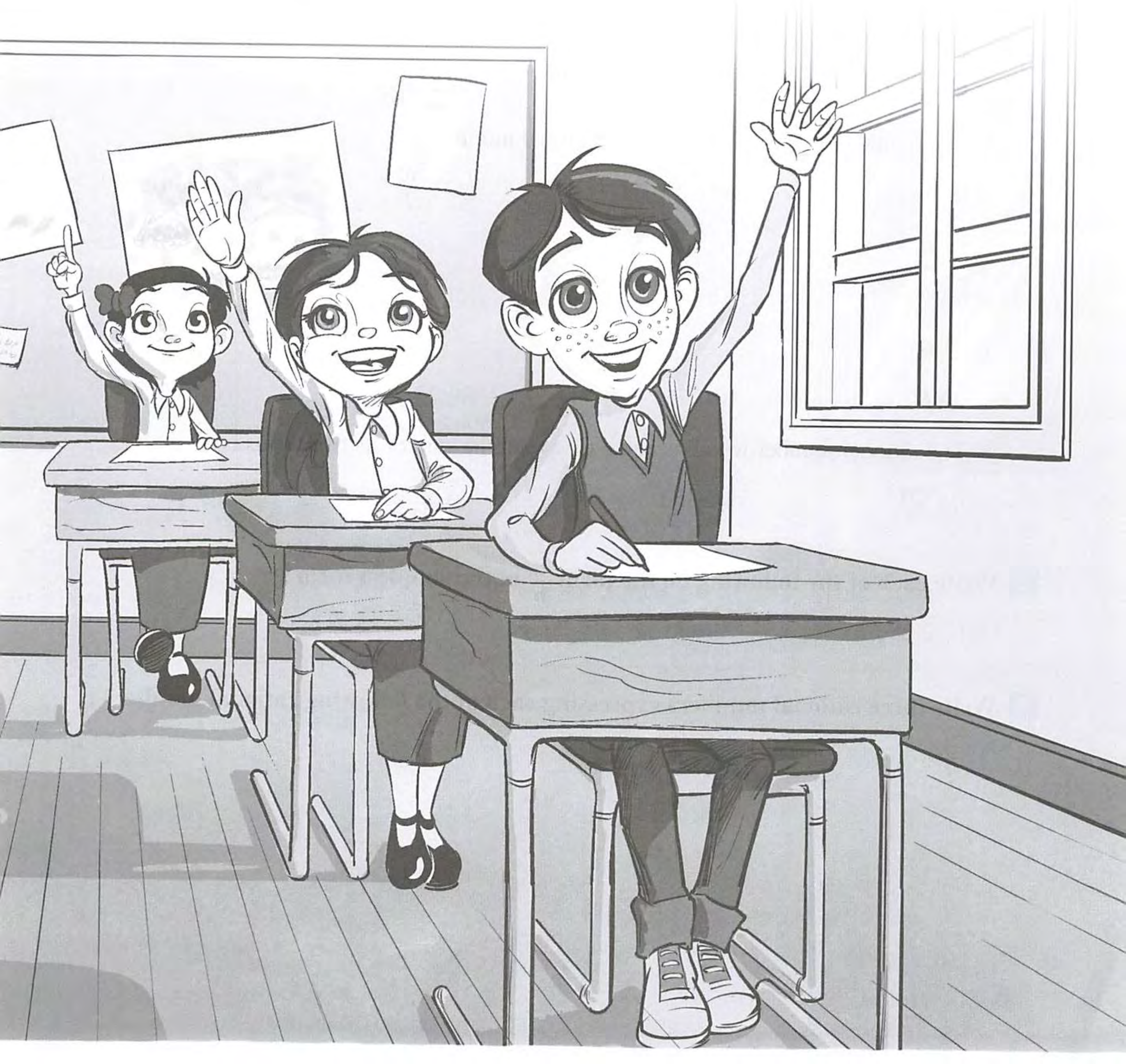
- 6 Accumulative tests
- Important questions
- Final revision
- Final examinations :
  - School book examinations (2 models + model for the merge students)
  - 15 schools examinations

First Algebra and Statistics



Accumulative Tests

on Algebra and Statistics



Accumulative test 1 on lesson 1 – unit 1

1 **Choose the correct answer from those given :**

1 The number $0.\dot{3}$ in the form of $\frac{a}{b}$ is ....

(a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{1}{3}$ (d) $\frac{3}{10}$

2 If $\frac{X}{X-2}$ is a rational number , then $X \neq$ ....

(a) zero (b) 1 (c) 3 (d) 2

3 The number $\frac{3-X}{7-X} = 0$ when $X =$ ....

(a) 7 (b) -7 (c) 3 (d) -3

4 The required condition for : $\frac{X+3}{2X-5}$ to be a rational number is ....

(a) $X \neq 0$ (b) $X \neq -5$ (c) $X \neq \frac{5}{2}$ (d) $X \neq -3$

5 The number .... is a positive rational number.

(a) $|-2|$ (b) -5 (c) $-\frac{3}{7}$ (d) zero

6 $\frac{2}{3} =$ ....

(a) $\frac{1}{3}$ (b) $\frac{4}{6}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

7 $\frac{3}{5} =$ .... %

(a) 60 (b) 75 (c) 25 (d) 100

8 The rational number which is equal to $\frac{2}{7}$ and the sum of its two terms is 27 is ....

(a) $\frac{20}{7}$ (b) $\frac{7}{20}$ (c) $\frac{6}{21}$ (d) $\frac{3}{24}$

---

2 **Write each of the following on the form $\frac{a}{b}$ in the simplest form :**

1 $|-2.25|$ 2 35 %

---

3 **Write three rational numbers expressing each of the following rational numbers :**

1 $\frac{5}{7}$ 2 $\frac{2}{9}$

Accumulative test 2 till lesson 2 – unit 1

1 Choose the correct answer from those given :

1. $\left|\frac{-3}{7}\right|$ .... zero

   (a) $>$ (b) $<$ (c) $=$ (d) $\leq$

2. $\frac{4}{7}$ .... $\frac{3}{5}$

   (a) $>$ (b) $<$ (c) $=$ (d) $\geq$

3. The number of integers lying between $\frac{7}{3}$, $\frac{11}{6}$ is ....

   (a) zero (b) 1 (c) 2 (d) infinite number

4. If $\frac{X-5}{X-7}=0$, then $X=$ ....

   (a) -7 (b) 5 (c) 7 (d) -5

5. If $\frac{X-3}{X+2}$ is a rational number , then $X \neq$ ....

   (a) 3 (b) -3 (c) 2 (d) -2

6. The number of rational numbers lying between $\frac{2}{5}$, $\frac{4}{5}$ is ....

   (a) 1 (b) 2 (c) 3 (d) infinite number

7. The rational number $\frac{X}{-3}$ is negative if X ....

   (a) > 0 (b) < 0 (c) ≤ 0 (d) $= 0$

8. The rational number that lies between $\frac{5}{7}$, $\frac{6}{7}$ is ....

   (a) $\frac{9}{14}$ (b) $\frac{10}{14}$ (c) $\frac{11}{14}$ (d) $\frac{12}{14}$

---

2 Find two rational numbers lying between :

$\frac{1}{5}$, 0.25

---

3 Find three rational numbers lying between :

$\frac{5}{4}$, $\frac{2}{3}$ such that one of them is an integer.

Accumulative test 3 till lesson 3 – unit 1

1 **Choose the correct answer from those given :**

1. If $\frac{x+4}{x-3}$ is not a rotional number , then $x - 2 =$ ....
   (a) 1 (b) -3 (c) 4 (d) $\frac{2}{3}$

2. $\frac{3}{4} + 50\ \% =$ ....
   (a) 75 % (b) 150 % (c) $\frac{5}{4}$ (d) $\frac{3}{2}$

3. The remainder of subtracting $\frac{4}{9}$ from $\frac{-5}{9}$ is ....
   (a) $\frac{-1}{9}$ (b) $\frac{1}{9}$ (c) 1 (d) -1

4. If $x < 0 < y$, $|x| > y$, then $x + y$ 0
   (a) $>$ (b) $\geq$ (c) $<$ (d) $=$

5. The rational number $\frac{x}{-4}$ is positive , if x .... zero.
   (a) $>$ (b) $<$ (c) $\geq$ (d) $=$

6. $0.75 - \frac{1}{4} =$ .... %
   (a) 75 (b) 50 (c) 25 (d) 20

7. $\frac{3}{5} +$ .... $= 0$
   (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $-\frac{5}{3}$ (d) $-\frac{3}{5}$

8. The additive inverse of $\left(\frac{2}{5}\right)^0$ is ....
   (a) -1 (b) $\frac{2}{5}$ (c) 1 (d) $\frac{-2}{5}$

---

2 If $\frac{x-2}{x+3} = 0$

Find three rational numbers lying between : $\frac{1}{x}$, $\frac{2}{1+x}$

---

3 If $x = \frac{3}{8}$, $y = \frac{1}{2}$, $z = -\frac{3}{4}$

Find the value of : $(x - y) + z$

Accumulative test 4 till lesson 4 – unit 1

1 Choose the correct answer from those given :

1 If $\frac{3}{4} \times y = 1$, then $y = \ldots\ldots\ldots\ldots$

(a) 1 (b) $\frac{4}{3}$ (c) $-\frac{3}{4}$ (d) $\frac{3}{4}$

2 The multiplicative inverse of $\left(\frac{1}{2}\right)^0$ is $\ldots\ldots\ldots\ldots$

(a) 2 (b) -2 (c) 1 (d) -1

3 If $\frac{|-5|}{X} = 1$, then $X = \ldots\ldots\ldots\ldots$

(a) -5 (b) 5 (c) 1 (d) -1

4 $\frac{a}{b} = \frac{2}{3}$, then $\frac{3a}{2b} = \ldots\ldots\ldots\ldots$

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$ (c) 1 (d) $\frac{3}{2}$

5 $\frac{1}{4}$ of $\frac{1}{2} = \ldots\ldots\ldots\ldots$

(a) $\frac{3}{10}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$

6 $\frac{2}{5}$ is more than $\frac{-2}{5}$ by $\ldots\ldots\ldots\ldots$

(a) zero (b) $\frac{4}{5}$ (c) $\frac{-4}{5}$ (d) 1

7 The rational number that has no multiplicative inverse is $\ldots\ldots\ldots\ldots$

(a) 1 (b) zero (c) -1 (d) 2

8 $\frac{X+3}{X+5}$ is a rational number if $X \neq \ldots\ldots\ldots\ldots$

(a) -3 (b) -5 (c) 5 (d) zero

---

2 If $a = \frac{7}{4}$, $b = -\frac{1}{2}$, **find the value of :** $\frac{a-b}{a+b}$

---

3 Using the distribution property , find the value of : $\frac{3}{7} \times 9 + \frac{3}{7} \times 6 - \frac{3}{7}$

Accumulative test 5 till lesson 5 – unit 1

1 **Choose the correct answer from those given :**

1 If $a \times \frac{b}{3} = \frac{a}{3}$, then b = ...

(a) 1 (b) 3 (c) 9 (d) 6

2 If 3 a = 12 and ab = 1 , then b = ...

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) 4 (d) 3

3 If $\frac{2}{5} X = 10$, then $\frac{4}{5} X =$ ....

(a) 25 (b) 15 (c) 20 (d) 5

4 The used property in : $\frac{6}{7} \times 1 = \frac{6}{7}$ is ....

(a) associative. (b) commutative.

(c) multiplicative identity. (d) additive inverse.

5 The multiplicative inverse of $1\frac{2}{3}$ is ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{3}{5}$

6 The rational number that lies half the way between $\frac{1}{2}$, $\frac{3}{4}$ is ....

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{5}{8}$ (d) $\frac{1}{6}$

7 If $\frac{X}{y} = 1$, then $5X - 5y =$ ....

(a) 10 (b) 5 (c) zero (d) 25

8 $\frac{X + 5}{|X| - 3}$ is a rational number when $X \neq$ ....

(a) 3 (b) – 3 (c) – 5 (d) ± 3

---

2 Find a rational number lying at third the way between $\frac{-1}{2}$, $\frac{-2}{5}$ from the side of the greater number.

---

3 If $X = \frac{1}{2}$, $y = \frac{2}{3}$ **Find in the simplest form :** $2X^2 + 3Xy$

Accumulative test 6 till lesson 1 – unit 2

1 **Choose the correct answer from those given :**

1 If the algebraic term : $4x^3 y^{n+1}$ is of the third degree , then n = ……………

(a) zero (b) 1 (c) – 1 (d) 2

2 The algebraic term : $2^3 x^4 y^2$ is of the …………… degree.

(a) ninth (b) sixth (c) seventh (d) eighth

3 If the algebraic term : $x^3 y^m$ is of the sixth degree , then m = ……………

(a) 2 (b) 1 (c) 6 (d) 3

4 The algebraic expression : 5 a + 5 a b is of the …………… degree.

(a) first (b) second (c) third (d) zero

5 $3\frac{1}{4} \times$ …………… = 1

(a) $\frac{4}{13}$ (b) $4\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{13}{4}$

6 If $\frac{5}{3x}$ is a rational number , then $x \neq$ ……………

(a) 3 (b) zero (c) – 3 (d) – 5

7 $\frac{3}{4} + \frac{3}{4} = \frac{\cdots\cdots}{8}$

(a) 3 (b) 6 (c) 12 (d) 24

8 If the algebraic expression : $a x^3 + 5x^2 + 7x - 9$ is of the second degree , then a = ……………

(a) 1 (b) 3 (c) – 2 (d) zero

---

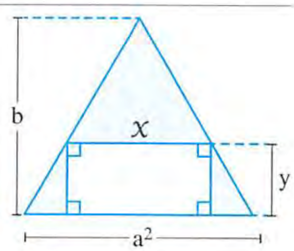
2 If $a = \frac{1}{2}$, $b = \frac{2}{5}$, $c = \frac{1}{5}$

Find the numerical value of the expression : (a + b) ÷ c

---

3 **In the opposite figure :**

Write the algebraic expression which expresses the area of the shaded part and state its degree.



Accumulative test 7 till lesson 2 – unit 2

1 Choose the correct answer from those given :

1 The remainder of subtracting $-2x$ from $7x$ is ....

(a) $5x$ (b) $9x$ (c) $-9x$ (d) $-5x$

2 The algebraic term : $6c^2b^3$ is of the .... degree.

(a) sixth (b) second (c) third (d) fifth

3 $\frac{x+b}{x+5}$ is a rational number , if $x \neq$ ....

(a) -7 (b) -5 (c) 7 (d) 5

4 The increase of $2x$ than $-3x$ is ....

(a) x (b) $-x$ (c) $5x$ (d) $-5x$

5 The remainder of substracting $2x$ from $-3x$ is ....

(a) $-5x$ (b) $2x$ (c) $5x$ (d) $6x^2$

6 $4a^3 + 3a^3 =$ ....

(a) $12a^6$ (b) $7a^7$ (c) $7a^3$ (d) $12a^3$

7 The algebraic term : xy^m is of the fifth degree , then $m =$ ....

(a) 4 (b) 5 (c) 2 (d) 3

8 $5z - 9z + 7z =$ ....

(a) $21z$ (b) $3z$ (c) $21z^3$ (d) $3z^3$

---

2 Reduce to the simplest form : $5x^2 - 2x + 8 - 7x - 3 + x^2$

---

3 Using the distribution property , find : $\frac{5}{19} \times 12 + \frac{5}{19} \times 5 + \frac{10}{19}$

Accumulative test 8 till lesson 3 – unit 2

1 Choose the correct answer from those given :

1. The algebraic term : 3^2 is of the .... degree.
   (a) first (b) third (c) zero (d) fourth
2. $\left|\frac{-3}{2}\right|$ is the additive inverse of ....
   (a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$
3. $2x + 3y$ is more than $3y - 2x$ by ....
   (a) $-6y$ (b) $-4y$ (c) $4x$ (d) $6y$
4. If $\frac{x}{y} = 30$, then $\frac{2x}{3y} =$ ....
   (a) 10 (b) 20 (c) 30 (d) 40
5. The remainder of subtracting $-7x$ from $-9x$ is ....
   (a) $2x$ (b) $-16x$ (c) $-2x$ (d) $16x$
6. What is the decrease of $3a - 2b$ than $2b + 4a$?
   (a) $4b$ (b) $7a$ (c) $4b + a$ (d) $4b - a$
7. The perimeter of the rectangle whose dimensions are $(2x + 1)$ cm. and $(3 - 2x)$ cm. is .... cm.
   (a) $2x$ (b) 4 (c) x (d) 8
8. The additive inverse of the number : $x + 2$ is .... cm.
   (a) $x - 2$ (b) $-x - 2$ (c) $2 - x$ (d) 2

---

2 What is the increase of : $5x + 5y - z$ than the sum of $7x - 6y - z$, $y - 3x - 5z$?

---

3 Subtract : $5x^2 - 3xy + y^2$ from $3y^2 + 6x^2 - 2xy$

Accumulative test 9 till lesson 4 – unit 2

1 Choose the correct answer from those given :

1. $3x \times 4x^2 =$ ....

(a) $7x^3$ (b) x^2 (c) $12x^3$ (d) $12x$

2. $(3x \div x) +$ .... $= 0$ where $x \neq 0$

(a) 2 (b) $-3x$ (c) -3 (d) -1

3. $2x^2y \times$ .... $= 12x^5y$

(a) $6xy$ (b) $6x^3$ (c) 6 (d) $6x^3y$

4. The degree of the algebraic expression : $3x^2y^2 + 5x^2y$ is ....

(a) second (b) third (c) fourth (d) fifth

5. The rational number which lies in the middle of the way between $\frac{1}{3}$, $\frac{5}{9}$ is ....

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{9}$ (d) $\frac{5}{27}$

6. $\frac{y^5}{y^3} + y^2 =$ .... , $y \neq 0$

(a) y^4 (b) $4y^4$ (c) $2y^2$ (d) $2y^4$

7. $-6x^3 \div 3x^2 =$ .... where $x \neq 0$

(a) $-18x^3$ (b) -2 (c) $2x$ (d) $-2x$

8. If the rational number $\frac{x-3}{5}$ has a multiplicative inverse , then $x \neq$ ....

(a) -5 (b) -7 (c) 3 (d) -3

---

2 Add $6x^2 + y^2 - 5xy$, $x^2 - y^2$, then find the numerical value of the result when $x = 1$, $y = -2$

---

3 Find three rational numbers lying between : $\frac{2}{3}$, $\frac{3}{4}$

Accumulative test 10 till lesson 5 – unit 2

1 **Choose the correct answer from those given :**

1. $2x(\cdots\cdots\cdots + 3) = 2x^2 + 6x$

   (a) x^3 (b) x^2 (c) x (d) 1

2. If $x + 2y = 5$, then $x + 2(3 + y) = \cdots\cdots\cdots$

   (a) 5 (b) 6 (c) 11 (d) 15

3. If $\frac{x}{y} = \frac{2}{3}$, then $\frac{3x}{2y} = \cdots\cdots\cdots$

   (a) $\frac{2}{3}$ (b) 1 (c) $\frac{3}{2}$ (d) -1

4. $3x(2x + 5y) = 6x^2 + \cdots\cdots\cdots$

   (a) $6x^2$ (b) $15x^2$ (c) $15y^2$ (d) $15xy$

5. $-3(y + 3) = \cdots\cdots\cdots$

   (a) $-3y + 6$ (b) $-3y - 9$ (c) $-3y - 6$ (d) $-3y$

6. If $\frac{x - 5}{x - 7} = 0$, then $x = \cdots\cdots\cdots$

   (a) -7 (b) 5 (c) 7 (d) -5

7. If the algebraic term : $9xy^n$ is of the third degree , then $n = \cdots\cdots\cdots$

   (a) 1 (b) 2 (c) 3 (d) 4

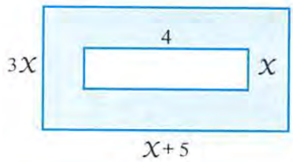
8. The remainder of subtracting $(-5x)$ from $3x$ is $\cdots\cdots\cdots$

   (a) $2x$ (b) $8x$ (c) $15x$ (d) $-2x$

---

2 **In the opposite figure :**

Find the algebraic expression which expresses the area of the colored part



---

3 If $x + 5y = 6$, $z = 2$, **find the numerical value of :** $x + 5(y + z)$

Accumulative test 11 till lesson 6 – unit 2

1 **Choose the correct answer from those given :**

1. If $(X - 3)(X + 3) = X^2 + k$, then $k =$ ....

(a) 3 (b) -3 (c) 9 (d) -9

2. The middle term of hte expansion of $(X + 3)^2$ is ....

(a) $3X$ (b) $6X$ (c) $5X$ (d) $9X$

3. If $(X + 2)(X - 2) = X^2 + kX - 4$, then $k =$ ....

(a) -4 (b) zero (c) 4 (d) 8

4. If $(X + 4)(X - 3) = X^2 + m - 12$, then $m =$ ....

(a) $-X$ (b) X (c) $-7X$ (d) $7X$

5. The square of the sum of a , b is ....

(a) $a^2 + b^2$ (b) $(a + b)^2$ (c) $2a + b$ (d) $a^2 b^2$

6. $\frac{X + 3}{X + 5}$ is a rational number if $X \neq$ ....

(a) -3 (b) -5 (c) 5 (d) zero

7. If $(X + y)^2 = 15$, $X^2 + y^2 = 9$, then $Xy =$ ....

(a) 5 (b) 2 (c) 3 (d) 4

8. If $\frac{14}{X} = \frac{7}{2}$, then $X =$ ....

(a) 49 (b) 4 (c) 7 (d) 2

---

2 **Reduce to the simplest form :** $(X + 3)(3X - 7) + (X + 5)(X - 5)$

---

3 If $X + y = \frac{7}{3}$, $y + z = \frac{2}{3}$

Find the value of : $X + 2y + z$

Accumulative test 12 till lesson 7 – unit 2

1 **Choose the correct answer from those given :**

1 $(15 x^4 + 5 x^3) \div 5 x^3 =$ ....

(a) $3 x^2 + x$ (b) $3 x^2 + 1$ (c) $3 x + 1$ (d) $4 x^4$

2 $(x^3 + x^2) \div x =$ .... where $x \neq 0$

(a) $x + 1$ (b) $x^2 + x$ (c) $x^4 + x^3$ (d) 1

3 If $x^2 = 1$, $y^2 = 9$, $x y = 3$, then $(x - y)^2 =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

4 $(27 a^3 - 9 a) \div (-3 a) =$ .... where $a \neq 0$

(a) $-9 a^2 - 3$ (b) $9 a^2 + 3 a$ (c) $9 a^2 + 3$ (d) $-9 a^2 + 3$

5 $(4 x^3 y^2 -$ ....$) \div 4 x y = x^2 y - 2$ where $x y \neq 0$

(a) $8 x^2 y$ (b) $8 x^2 y^2$ (c) $-8 x y$ (d) $8 x y$

6 $7 x$ increases than $2 x$ by ....

(a) -5 (b) 5 (c) $5 x$ (d) $-5 x$

7 $(12 a - 3) \div 3 =$ ....

(a) $4 a$ (b) $4 a - 3$ (c) $4 a - 1$ (d) $4 a + 3$

8 $\dfrac{4 a^2 b - 8 a b - 12 a}{\text{...............}} = 2 a b - 4 b - 6$ where $a \neq 0$

(a) $4 a b$ (b) $4 a$ (c) $2 a^2$ (d) $2 a$

---

2 **Reduce to the simplest form :** $(x + 3)^2 - (x - 3)(x + 3) - 6 x$

---

3 A rectangle of area $(24 x^3 + 18 x^2 + 42 x)$ cm^2. and its width is $6 x$ cm.

Find its length in terms of x

Accumulative test 13 till lesson 8 – unit 2

1 Choose the correct answer from those given :

1 If $x - y = 3$, $x + y = 5$, then $x^2 - y^2 =$ ....

(a) 3 (b) 5 (c) 10 (d) 15

2 The value of k that makes the expression $x^2 - 5x + k$ is divisible by $(x - 2)$ is ....

(a) 6 (b) 2 (c) 5 (d) 1

3 The quotient of : $x^2 - 12x + 20$ by $x - 2$ is .... (where $x \neq 2$)

(a) $x - 4$ (b) $x + 4$ (c) $x - 10$ (d) x

4 The multiplicative inverse of $0.\dot{4}$ in the simplest form is ....

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$

5 The area of a rectangle is $35x^2$ cm^2. and its length is 7 cm.

, then its width = ... cm.

(a) $5x$ (b) $35x$ (c) $4x$ (d) $12x$

6 If the algebraic term $3x^2y^n$ is of the fifth degree , then n = ...

(a) 3 (b) 4 (c) 5 (d) 6

7 If $\frac{x}{y} = 1$, then $5x - 5y =$ ....

(a) zero (b) 1 (c) 2 (d) 3

8 The coefficient of the algebraic term : $4xy^2$ is ....

(a) 3 (b) 4 (c) 2 (d) 1

2 Find the quotient of : $2x^3 - 5x^2 - 22x - 15$ by $2x + 3$ $\left(\text{where } x \neq \frac{-3}{2}\right)$

3 Find : $(x + 2)^2 - 4(x + 1)$, then find the numerical value of the result when $x = 2$

Accumulative test 14 till lesson 9 – unit 2

1 **Choose the correct answer from those given :**

1 $25x^3 + 15x^2 + 35x = \dots\dots\dots (5x^2 + 3x + 7)$

(a) $5x^3$ (b) $5x^2$ (c) $5x$ (d) 5

2 The H.C.F. of the expression : $3x^2y - 6x$ is ....

(a) $3x$ (b) $6x$ (c) $3xy$ (d) $3x^2$

3 The simplest form of the expression $(x-2)(x+2)+4$ is ....

(a) x^2+4 (b) x^2-4 (c) x^2 (d) 4

4 $3 \times \dots\dots\dots = -1$

(a) -3 (b) $\frac{1}{3}$ (c) 3 (d) $\frac{-1}{3}$

5 The H.C.F. of the expression : $3x^3y^2 + 15xy$ is ....

(a) $3xy$ (b) $6x$ (c) $5xy$ (d) xy

6 If $3x \times k = 12x^3$, then $k = \dots\dots\dots$

(a) $2x^4$ (b) $6x^2$ (c) $4x^2$ (d) $4x$

7 $8c^3 = 4c \times \dots\dots\dots$

(a) $2c^2$ (b) $2c$ (c) $4c^2$ (d) $4c$

8 $6a^2 + 12b^2 = 6(\dots\dots\dots)$

(a) a^2+b^2 (b) a^2+2b^2 (c) $6a^2+b^2$ (d) $a+2b$

2 **Factorize by identifying the highest common factor :**

$18x^2y^3 + 6x^3y^2 - 3x^2y^2$

3 If $a+b-c=5$, then what is the numerical value of :

$(a+b+c)(a+b-c) - 2c(a+b-c)$

Accumulative test 15 till lesson 1 – unit 3

1 Choose the correct answer from those given :

1 The arithmetic mean of the values : 3 , 5 , 2 , 6 is ....

(a) 5 (b) 7 (c) 3 (d) 4

2 The arithmetic mean of the values : 8 , 6 , – 5 , 7 is ....

(a) 3 (b) 4 (c) 10 (d) 16

3 If the arithmetic mean of the marks of 5 students is 20 , then the sum of their marks is ....

(a) 100 (b) 4 (c) 5 (d) 20

4 The arithmetic mean of the numbers : 6 , $3 - X$, $5 + X$, 6 is ....

(a) 3 (b) 5 (c) 6 (d) 16

5 The algebraic term : $2X^2y$ is of the .... degree.

(a) first (b) second (c) third (d) fourth

6 $(X - 2)(X + 5) = X^2 +$ .... $- 10$

(a) $-2X$ (b) $5X$ (c) $3X$ (d) $7X$

7 The arithmetic mean of two supplementary angles is ....°

(a) 70 (b) 90 (c) 180 (d) 40

8 If the arithmetic mean of the values : 2 , 4 , X , 5 is 4 , then $X =$ ....

(a) 4 (b) 3 (c) 5 (d) 6

---

2 If the arithmetic mean of the values : 8 , 7 , 5 , 9 , 4 , 3 , k + 4 is 6 , find the value of k

---

3 Reduce to the simplest form : $(X + 2)^2 - (X + 2)(X - 2)$

, then find the value of the result when $X = 1$

Accumulative test 16 till lesson 2 – unit 3

1 **Choose the correct answer from those given :**

1 If the order of the median of a set of values is the fourth , then the number of these values is ....

(a) 3 (b) 5 (c) 7 (d) 9

2 If the order of the median of a set of values is 6 , then the number of these values is ....

(a) 10 (b) 12 (c) 11 (d) 16

3 The median of the values : 8 , 5 , 12 , 2 , 3 is ....

(a) 2 (b) 5 (c) 8 (d) 12

4 If the order of the median of a set of values is the fourth and fifth , then the number of these values is ....

(a) 14 (b) 8 (c) 15 (d) 7

5 The middle term of the expansion of $(X + 3)^2$ is ....

(a) $3X$ (b) $6X$ (c) $5X$ (d) $9X$

6 The number that lies in the middle of the way between $\frac{1}{4}$, $\frac{1}{2}$ is ....

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{7}{8}$

7 The median of the values : 7 , 5 , 3 , 1 is ....

(a) 5 (b) 4 (c) 3 (d) 7

8 The simplest form of the expression : $(X - 2)(X + 2) + 4$ is ....

(a) $X^2 + 4$ (b) $X^2 - 4$ (c) X^2 (d) 4

---

2 If the sum of marks of Yossef in 3 consecutive months in maths is 276 , what is the mark of the fourth month , if the arithmetic mean of his marks is 93.5 marks ?

---

3 **If the median of the values :**

k + 9 , k + 3 , k + 8 , k + 12 , k + 7 is 7 **Find :** the value of k

Accumulative test 17 till lesson 3 – unit 3

1 Choose the correct answer from those given :

1 The mode of the values : 2 , 3 , 4 , 3 , 6 is ....

(a) 6 (b) 4 (c) 3 (d) 2

2 If the mode of the values : 5 , 7 , $X + 5$, 5 , 9 is 7 , then $X =$ ....

(a) 4 (b) 5 (c) 1 (d) 2

3 If the mode of the values : 7 , 5 , $2X + 3$, 5 , 7 is 5 , then $X =$ ....

(a) 2 (b) -1 (c) 1 (d) -2

4 The rational number $\frac{3 + X}{7 + X} = 0$ when $X =$ ....

(a) 7 (b) -7 (c) 3 (d) -3

5 If $(3X + 4)^2 = 9X^2 + kX + 16$, then $k =$ ....

(a) 12 (b) -12 (c) 7 (d) 24

6 If $\frac{1}{3}X = 5$, then $X =$ ....

(a) 10 (b) 15 (c) 20 (d) 25

7 The arithmetic mean of the values : $8 + X$, $7 - X$, 6 is ....

(a) $7X$ (b) $3X$ (c) 7 (d) 21

8 The number lies half the way between : $\frac{1}{2}$, $\frac{3}{4}$ is ....

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{5}{8}$ (d) $\frac{1}{6}$

---

2 If the arithmetic mean of the values : 2 k , 3 k , 10 , 4 is 6 , find the value of k

---

3 The following table shows the marks of 40 pupils in an exam :

Mark	5	6	7	8	9	10	Total
Frequency	4	7	2	8	13	6	40

Find the mode mark.

Important Questions

on Algebra and Statistics

Important questions on Unit One

First Multiple choice questions

1 If $\frac{3}{X-2}$ is a rational number , then $X \neq$ ....

(a) 3 (b) -2 (c) zero (d) 2

2 If $\frac{3}{3+X}$ is a rational number , then $X \neq$ ....

(a) 3 (b) -3 (c) zero (d) 7

3 If $\frac{X+4}{X-7} = 0$, then $X =$ ....

(a) 4 (b) 7 (c) -4 (d) -7

4 The number that lies in the middle of the way between $\frac{1}{8}$ and $\frac{7}{8}$ is ....

(a) $\frac{1}{4}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) 1

5 The property used in the operation : $\frac{6}{7} \times 1 = \frac{6}{7}$ is ....

(a) closure. (b) commutative.

(c) multiplicative identity. (d) multiplicative inverse.

6 The number $0.\dot{6}\dot{3}$ as a fraction is ....

(a) $\frac{63}{100}$ (b) $\frac{7}{11}$ (c) $\frac{7}{9}$ (d) $\frac{7}{111}$

7 The additive identity element in $\mathbb{Q}$ is ....

(a) 1 (b) -1 (c) zero (d) 2

8 The multiplicative identity element in $\mathbb{Q}$ is ....

(a) zero (b) 1 (c) -1 (d) $\varnothing$

9 The multiplicative inverse of the number $\left(\frac{-2}{3}\right)^2$ is ....

(a) $\frac{4}{9}$ (b) $-\frac{4}{9}$ (c) $\frac{9}{4}$ (d) $-\frac{9}{4}$

10 The multiplicative inverse of the number 0.04 is ....

(a) $\frac{1}{25}$ (b) $\frac{-1}{25}$ (c) 25 (d) -25

11 The multiplicative inverse of the number $1\frac{3}{5}$ is ....

(a) $3\frac{5}{3}$ (b) $\frac{8}{3}$ (c) $\frac{8}{5}$ (d) $\frac{5}{8}$

12 $0.7 + 0.\dot{3} =$ ....

(a) 7.1 (b) 3 (c) $0.3\dot{7}$ (d) $1\frac{1}{30}$

---

13 If $\frac{x}{y} = 1$, then $4x - 4y =$ ....

(a) 1 (b) zero (c) 4 (d) 5

---

14 If $\frac{2}{5}x = 10$, then $\frac{1}{5}x =$ ....

(a) 1 (b) zero (c) 4 (d) 5

---

15 The number that has no multiplicative inverse is ....

(a) zero (b) 1 (c) -1 (d) 2

---

16 Subtracting $\frac{1}{3}$ from $\frac{4}{3}$ gives ....

(a) 1 (b) -1 (c) zero (d) 3

---

17 If $5x = 20$, $xy = 1$, then $y =$ ....

(a) 4 (b) -4 (c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

---

18 If $\frac{x}{y} = \frac{4}{5}$, then $\frac{5x}{4y} =$ ....

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$ (c) $\frac{25}{16}$ (d) 1

---

19 The number of rational numbers lying between $\frac{2}{5}$ and $\frac{4}{5}$ is ....

(a) 1 (b) 2 (c) 3 (d) infinite number

---

20 If $\frac{4}{7} \times x = \frac{4}{7}$, then $x =$ ....

(a) 1 (b) zero (c) 4 (d) 7

---

21 75 % .... $0.\dot{3}$

(a) $>$ (b) $=$ (c) $<$ (d) $\leq$

---

22 The rational number $\frac{x}{-5}$ is negative if x ....

(a) > 0 (b) < 0 (c) ≤ 0 (d) $= 0$

---

23 If $\frac{1}{3} < \frac{x}{6} < \frac{2}{3}$, then $x =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

---

24 The number $\frac{x}{|x| - 2}$ does not represent a rational number if $x =$ ....

(a) zero (b) -1 (c) 5 (d) ± 2

Second Complete questions

1 The additive inverse of the number $(5)^0$ is ....

2 The additive inverse of the number $\left|\frac{-3}{4}\right|$ is ....

3 $\frac{2}{5} \times X = 1$, then $X =$ ....

4 $20\ \% - \left|\frac{-1}{5}\right| =$ ....

5 If $\frac{3}{7}\ X = 3$, then $\frac{5}{7}\ X =$ ....

6 $\frac{1}{3} =$ .... (as a recurring decimal)

7 The multiplicative inverse of the number (-1) is ....

8 $\frac{X-3}{X-4} = 0$ when $X =$ ....

9 If $\frac{X+3}{X-7} \in \mathbb{Q}$, then $X \neq$ ....

10 $\frac{1}{2} =$ .... %

11 The multiplicative inverse of the number $\left|\frac{-5}{7}\right|$ is ....

12 The multiplicative inverse of the number $(-7)^0$ is ....

13 If $X = \frac{10}{3}$, $y = \frac{5}{3}$, then $X \div y =$ ....

14 If $\frac{a}{b} = \frac{1}{2}$, then $\frac{2\,a}{b} =$ ....

15 $0.\dot{5} - \left|-\frac{1}{9}\right| =$ ....

16 The sum of any rational number and its additive inverse equals ....

17 If $X + y = y + X = y$, then X is called an additive ....

18 If $\frac{4}{5} + x = 0$, then $x =$ ....

19 The number which is equal to its additive inverse is ....

20 The rational number which lies third the way between 8 and 12 from the side of the smaller number is ....

Third Essay questions

1 Find three rational numbers lying between $\frac{1}{2}$ and $\frac{1}{3}$

2 Find four rational numbers lying between $\frac{2}{3}$ and $\frac{3}{7}$

3 Use the distribution property to find the value of : $\frac{3}{8} \times 3 + \frac{3}{8} \times 9 - \frac{3}{8} \times 4$

4 Use the distribution property to find the value of : $\frac{5}{7} \times 3 + \frac{5}{7} \times 5 - \frac{5}{7}$

5 Use the distribution property to find the value of : $\frac{4}{9} \times \frac{5}{6} + \frac{4}{9} \times \frac{7}{6} - \frac{4}{9}$

6 If $x = \frac{1}{4}$, $y = \frac{2}{5}$, $z = 4$ Find the numerical value of the expression : $x\,y\,z$

7 Find the value of : $\left(\frac{3}{5} \div \frac{2}{5}\right) \times \frac{2}{3}$

8 Find a rational number lying at on third of the way between $\frac{4}{7}$, $1\frac{3}{4}$ from the side of the smaller number.

9 If $x = \frac{1}{2}$, $y = 9$, $z = \frac{2}{3}$ Find the value of : $3\,x\,z + y$

10 If $x = \frac{1}{2}$, $y = \frac{3}{4}$, find : $\frac{x + y}{x - y}$

11 If $a = \frac{1}{2}$, $b = -\frac{3}{4}$, then find in the simplest form the value of : $a\,b - \frac{1}{3}$

12 Prove that the number $\frac{5}{12}$ lies between $\frac{1}{3}$ and $\frac{1}{2}$

13 If $x = \frac{3}{2}$, $y = \frac{-1}{4}$, $z = -2$ Find the numerical value of : $x - (z \div y)$

14 If $\frac{x - 2}{x + 3} = 0$ Find three rational numbers lying between : $\frac{1}{x}$ and $\frac{2}{1 + x}$

Important questions on Unit Two

First Multiple choice questions

1 The algebraic term : $6 x^3 y^2$ is of .... degree.

(a) third (b) fourth (c) fifth (d) sixth

2 The algebraic term : $6 x y$ is of .... degree.

(a) second (b) third (c) fourth (d) sixth

3 If the algebraic term : $9 x y^n$ is of third degree , then n = ...

(a) 1 (b) 2 (c) 3 (d) 4

4 The degree of the algebraic expression : $2 x^2 y - 6 x y^2 + 9 x^2 y^2$ is ....

(a) first. (b) second. (c) third. (d) fourth.

5 If $5 x^m + 2 x^n = 7 x^6$, then m + n = ...

(a) zero (b) 3 (c) 8 (d) 12

6 The perimeter of the rectangle whose dimensions are $8 x$, $5 x$ length unit is .... length unit.

(a) $40 x^2$ (b) $13 x$ (c) $40 x$ (d) $26 x$

7 $3 x \times 4 x =$ ....

(a) $7 x$ (b) $7 x^2$ (c) $12 x$ (d) $12 x^2$

8 The result of subtracting (– 3 a) from (2 a) is ....

(a) 5 a (b) – 5 a (c) a (d) – a

9 $4 x$ increases $(- 4 x)$ by ....

(a) zero (b) $8 x$ (c) $- 8 x$ (d) $16 x$

10 If $(x - 3)(x + 3) = x^2 - k$, then k = ...

(a) 3 (b) 9 (c) 6 (d) – 9

11 If $(x - 8)(x + 8) = x^2 + k$, then k = ...

(a) – 16 (b) 16 (c) 64 (d) – 64

12 $(x-3)^2 = x^2 - \text{...............} + 9$

(a) $6x$ (b) $-6x$ (c) $9x$ (d) $5x$

13 The middle term in the expansion of $(2x-5y)^2$ is ....

(a) $-10x^2y^2$ (b) $20xy$ (c) $10x^2y^2$ (d) $-20xy$

14 A cube of edge length 2 b cm. , then its volume is .... cm^3.

(a) $4b^2$ (b) $2b^3$ (c) $4b^3$ (d) $8b^3$

15 $2x^3 + 4x^3 = \text{...............}$

(a) $6x^3$ (b) $6x^6$ (c) $8x^3$ (d) $8x^6$

16 If $x + y = 5$, $x^2 + xy = 20$, then $x = \text{...............}$

(a) 4 (b) 5 (c) 15 (d) 20

17 The square of the sum of the two terms a , b is ....

(a) $a^2 + b^2$ (b) $2ab$ (c) $(a+b)^2$ (d) $2(a+b)$

18 If the price of 5 pens is x pounds , then the price of 50 pens is .... pounds.

(a) $\frac{x}{50}$ (b) $10x$ (c) $50x$ (d) $\frac{5x}{2}$

19 $6x^3 + 18x^2 = \text{...............}(x+3)$

(a) 6 (b) $6x$ (c) x^2 (d) $6x^2$

20 $(x^2 + x) \div x = \text{...............}$ where $x \neq 0$

(a) zero (b) x (c) $2x+1$ (d) $x+1$

Second Complete questions

1 $(3x-y)(2x+5y) = 6x^2 + 13xy$ ....

2 $2x^3y^2 \times 5xy = \text{...............}$

3 $3x^2y^3 \times \text{...............} = 15x^4y^4$

4 The degree of the algebraic term : $2xy$ is ....

5 If the algebraic term $2x^2y^m$ is of third degree , then m = ...

6 The H.C.F. of the expression : $8x^2y + 16xy^2$ is ....

7 $5x^2 + 15xy = 5x$ (.............. + ...)

8 $(x + 5)(2x - 3) = 2x^2 +$ .... $- 15$

9 3 a + 4 b decreases than 5 b + 3 a by ....

10 The result of subtracting $2x$ from $5x$ is ....

11 $x(x^2 + 3x) =$ .... $+ 3x^2$

12 $\frac{y^5}{y^3} + y^2 =$ .... , $y \neq 0$

13 The middle term of the expansion of $(2x + 3)^2$ is ....

14 If $(x - 5)(x + 5) = x^2 + 2kx - 25$, then k = ...

15 If a – b = 7 , then a (a – b) + b (b – a) = ...

16 If a + 3 b = 5 , c = 3 , then a + 3 (b + c) = ...

17 If $(x + y)^2 = 26$, $x^2 + y^2 = 14$, then xy = ...

18 If the algebraic term : $4xy^{k+1}$ is of fifth degree , then k = ...

Third Essay questions

1 Find in the simplest form : $(5x + 6)(2x - 3)$

2 Simplify : $(x + 2)^2 - (x - 8)(x + 2)$

3 Find the quotient of : $15x^2y + 9xy^2$ by $3xy$ where $xy \neq 0$

4 Divide : $20a^3b^2 + 15a^2b^3 + 10ab$ by $5ab$ where $ab \neq 0$

5 Simplify : $(3x + 5)(3x - 5) + 8$, then find the numerical value of the result when $x = -2$

6 Add : $4x + 3y - 5$ and $3x - 4y + 7$

7 Add : $x^2 + 5xy - 2y^2$ and $x^2 - 3xy - 7y^2$

8 What is the increase of $x^2 - 5x - 1$ than $3x^2 + 2x - 3$?

9 Subtract : $5x^2 + y^2 - 3xy$ from $x^2 - 2xy + 3y^2$

10 Factorize by identifying the H.C.F. : $5x^3 + 15x^2 + 10x$

11 Factorize by identifying the H.C.F. : $4x^3y^3 - 6x^2y^2 + 2xy$

12 Factorize by identifying the H.C.F. : $3a(4a + 5b) - 2b(4a + 5b)$

13 Find the quotient of : $x^2 - 5x + 6$ by $x - 2$ where $x \neq 2$

14 A peice of rectangular land of area $(x^2 + 5x + 6)$ square metres and its width is $(x + 2)$ metres find its length.

15 Add : $x^2 + 2x - 3$, $4x^2 - 5x + 6$, $1 + 3x - 3x^2$

16 If $x + y = \frac{5}{3}$, $y + z = \frac{1}{3}$ find the value of : $x + 2y + z$

17 Simplify : $(x + 2)^2 - 4(x + 1)$, then find the value of the result when $x = 3$

18 Simplify : $(x + 2)^2 - (x + 2)(x - 2)$

19 Simplify : $(x + 1)(x + 2) - x^2$, then find the numerical value of the result when $x = \frac{2}{3}$

20 Find the value of k which makes the expression : $x^2 - 9x + k$ is divisible by $x - 4$ without a remainder

21 A rectangle of length $(2x + 3)$ cm. , width $(x + 2)$ cm. Find the area of the rectangle in terms of x , then find the area when $x = 5$

Important questions on Unit Three

First Multiple choice questions

1 The arithmetic mean of the values 1 , 6 , 4 , 8 , 6 is ....

(a) 4 (b) 5 (c) 6 (d) 7

---

2 The median of the values : 8 , 4 , 3 , 5 , 7 is ....

(a) 3 (b) 4 (c) 5 (d) 7

---

3 The mode of the values : 11 , 1 , 2 , 2 , 11 , 2 is ....

(a) 1 (b) 2 (c) 11 (d) 14

---

4 If the arithmetic mean of the values : 9 , 4 , 5 , X is 5 , then X = ...

(a) 2 (b) 3 (c) 4 (d) 5

---

5 If the mode of the values 7 , 5 , $X + 4$, 7 is 7 , then X = ...

(a) 5 (b) 4 (c) 3 (d) 11

---

6 If the order of the median of a set of ordered values is the eleventh , then the number of values is ....

(a) 5 (b) 10 (c) 20 (d) 21

---

7 The arithmetic mean of the values 5 , $X + 1$, $3 - X$ is ....

(a) 1 (b) 2 (c) 3 (d) 5

---

8 If the sum of four values is 28 , then the arithmetic mean of these values is ....

(a) 4 (b) 5 (c) 6 (d) 7

---

9 If the arithmetic mean if six values is 4 , then the sum of these values is ....

(a) 4 (b) 6 (c) 1 (d) 24

---

10 If the median of the values : k + 3 , k + 2 , k + 4 is 8 where k is a positive integer , then k ....

(a) 2 (b) 3 (c) 4 (d) 5

11 The order of the median of the values : 6 , 2 , 9 , 3 , 10 is ....

(a) second (b) third (c) sixth (d) ninth

12 If the arithmetic mean of the values : $2X$, $X - 1$, 1 is 2 , then X = ...

(a) 1 (b) 2 (c) 3 (d) 4

Second Complete questions

1 If the median of the values 27 , 45 , 19 , 24 , 28 is X , then X = ...

2 If the order of the median of a set of values is seventh , then the number of values is ....

3 .... is the most common value.

4 The arithmetic mean of : $X + 6$ and $8 - X$ is ....

5 The mode of the values 3 , 2 , 12 , 2 , 1 , 2 is ....

6 If the arithmetic means of the marks of five students is 20 , then the sum of their marks is .... marks.

7 If the arithmetic mean of 3 , 4 , k is 5 , then k = ...

8 The arithmetic mean of five values of sum 20 is ....

Third Essay questions

1 If the mode of the values : a + 2 , a + 1 , a + 3 , a + 2 is 18 , find the value of a

2 **The following table shows the marks of a student in maths :**

Month	October	November	December	February	March	April
Mark	25	37	34	48	44	52

Find the arithmetic mean of these marks.

3 **The following table shows the marks of 24 students in one exam :**

Mark	7	8	9	10	Total
No. of students	6	5	7	6	24

Find the mode mark.

4 The following table shows the marks of three students :

Name \ Subject	Arabic	English	Social Studies	Maths	Science
Ahmed	23	33	24	27	23
Hassan	24	21	29	26	20
Mahmoud	25	27	30	18	10

First : Find the median of Mahmoud's marks.

Second : Find the arithmetic mean of Hassan's marks.

Third : Find the mode of Ahmed's marks.

---

5 Find the arithmetic mean , median and mode of the values : 42 , 30 , 51 , 35 , 42 , 40

---

6 If the arithmetic mean of the values : 8 , 7 , 5 , 9 , 4 , 3 , k + 4 is 6 Find the value of k

---

7 The following table shows the marks of the students of a class in maths , find the mode mark , then find the number of students who got less than 6 marks

Marks	2	4	6	8	10
Frequency	5	7	13	6	4

---

8 If the studying hours of Hanan during 6 days are as follows

Day	**Sat.**	**Sun.**	**Mon.**	**Tues.**	**Wedn.**	**Thur.**
Number of hours	$3\frac{1}{2}$	2	$2\frac{1}{2}$	3	4	3

Find the mean of daily studying hours.

---

9 If the arithmetic mean of the values : 9 , 2 k , 5 , 4 k is 8 Find the value of k

---

10 If the median of the values : $X + 5$, $X + 3$, $X + 8$ is 9 Find the value of X

Final Revision

of Algebra and Statistics

Revision for the important rules of Algebra and Statistics

Remember **The rational numbers**

The rational number is the number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Examples for rational numbers : $\frac{5}{7}$, $-\frac{3}{8}$, 7 , zero , $4\frac{3}{7}$, 0.35 , 43 % , $0.\dot{3}$

Each integer is a rational number with denominator = 1 *i.e.* $\mathbb{Z} \subset \mathbb{Q}$

For example : $7 = \frac{7}{1}$, $0 = \frac{0}{1}$

If $\frac{a}{b}$ is a rational number , then $b \neq 0$

For example : If $\frac{3}{x-5} \in \mathbb{Q}$, then $x - 5 \neq 0$ *i.e. :* $x \neq 5$

If the rational number $\frac{a}{b} = 0$, then $a = 0$

For example : If the rational number $\frac{3x}{x+3} = 0$, then $3x = 0$ *i.e. :* $x = 0$

Zero is the additive identity element (additive neutral element) in $\mathbb{Q}$

For example : $0 + \frac{1}{2} = \frac{1}{2} + 0 = \frac{1}{2}$

The number 1 is the multiplicative identity element (multiplicative neutral element) in $\mathbb{Q}$

For example : $1 \times \frac{-2}{3} = \frac{-2}{3} \times 1 = \frac{-2}{3}$

The additive inverse :

For every rational number $\frac{a}{b}$ there exist an additive inverse to it that is $(-\frac{a}{b})$ where $\frac{a}{b} + (-\frac{a}{b}) = 0$ (The additive identity element)

For example : The additive inverse of the number $\frac{3}{7}$ is $(-\frac{3}{7})$ where $\frac{3}{7} + (-\frac{3}{7}) = 0$ (The additive identity element)

Notice that : The additive inverse of the number zero is itself.

The multiplicative inverse :

For every rational number $\frac{a}{b}$ except zero there is a multiplicative inverse that is the rational number $\frac{b}{a}$ where $\frac{a}{b} \times \frac{b}{a} = 1$ (The multiplicative identity element)

For example :

The multiplicative inverse of the number $\frac{3}{7}$ is $\frac{7}{3}$

where $\frac{3}{7} \times \frac{7}{3} = 1$ (The multiplicative identity element)

Notice that

- There is no multiplicative inverse for the number 0 because $\frac{a}{0}$ is meaningless.
- The multiplicative inverse of the number 1 is itself.
- The multiplicative inverse of the number –1 is itself.

The number that lies at the middle of the way (halfway) between two numbers

= The smaller number $\oplus$ $\frac{1}{2}$ |The difference between the two numbers|

= The greater number $\ominus$ $\frac{1}{2}$ |The difference between the two numbers|

$= \frac{\text{First number + second number}}{2}$

For example :

The number that lies at the middle of the way between $\frac{3}{4}$ and $\frac{1}{8} = \frac{\frac{3}{4} + \frac{1}{8}}{2} = \frac{7}{16}$

The number that lies at one third of the way between two numbers :

1. From the side of the smaller number

   = the smaller number $\oplus$ $\frac{1}{3}$ |The difference between the two numbers|
2. From the side of the greater number

   = the greater number $\ominus$ $\frac{1}{3}$ |The difference between the two numbers|

For example :

The number that lies at one third of the way between $\frac{3}{8}$ and $-\frac{5}{6}$:

1. From the side of the smaller number $= -\frac{5}{6} + \frac{1}{3}\left|\frac{3}{8} - \left(-\frac{5}{6}\right)\right| = -\frac{31}{72}$
2. From the side of the greater number $= \frac{3}{8} - \frac{1}{3}\left|\frac{3}{8} - \left(-\frac{5}{6}\right)\right| = -\frac{1}{36}$

Remember | The algebraic term and the algebraic expression and their degree

The algebraic term is formed from the product of two or more factors.

For example : $5\,x$ is an algebraic term is formed from :

- 5 numerical factor (Coefficient of the term)
- x algebraic factor (Symbolic factor)

The degree of the algebraic term is the sum of the indices of the algebraic factors (symbolic factors) in this term.

For example :
- The term $5\,x$ is of the 1^{st} degree.
- The term $-9\,z^2$ is of the 2^{nd} degree.
- The term $7\,x^2\,y$ is of the 3^{rd} degree.
- The term 7 is of zero degree.

The algebraic expression is formed from an algebraic term or more.

For example :
- $7\,a + 5\,b$ is an algebraic expression consisting of two terms.
- $3\,x^2 + 4\,x^2 y - 7\,x$ is an algebraic expression consisting of three terms.

The degree of the algebraic expression is the highest degree of the terms forming it.

For example :
- $3\,y + 7$ is of the 1^{st} degree.
- $4\,x^2 + 5\,x + 3$ is of the 2^{nd} degree.

Remember | Like algebraic terms

The algebraic terms are said to be like if the algebraic symbols forming their factors are like and the indices of these symbols are equal.

For example :
- $3\,x$, $-7\,x$ and x are like terms.
- $3\,x\,y^2$ and $-5\,y^2\,x$ are like terms.

but :
- $6\,x^2$, $7\,x\,y$ and $-y^2$ are unlike terms because their symbols are different.
- $7\,x$, $-5\,x^2$ and $3\,x^3$ are unlike terms because their indices are different.

Remember **Adding and subtracting algebraic expressions**

We can add the two expressions : $3x - 5y + 7$ and $2y - 7x + 3$ by

The horizontal method

In this method, we use the commutative and associative properties.

$\therefore (3x - 5y + 7) + (2y - 7x + 3)$

$= (3x - 7x) + (-5y + 2y) + (7 + 3)$

$= -4x - 3y + 10$

The vertical method

In this method, we arrange the two expressions vertically such that the like terms lie under each other using the commutative property.

$$\begin{array}{l} \therefore\ 3x - 5y + 7 \\ \underline{-7x + 2y + 3} \\ -4x - 3y + 10 \end{array}$$

We can subtract $3x - 5y + 7$ from $2y - 7x + 3$ by

The horizontal method

In this method, we put the subtraction operation in the form :

The remainder

= The minuend – the subtrahend

$\therefore$ The remainder

$= (2y - 7x + 3) - (3x - 5y + 7)$

$= 2y - 7x + 3 - 3x + 5y - 7$

$= (-7x - 3x) + (2y + 5y) + (3 - 7)$

$= -10x + 7y - 4$

The vertical method

In this method, we arrange the terms of the subtrahend down its like terms of the minuend, then we add the minuend to the additive inverse of the subtrahend.

$$\begin{array}{ll} \text{The minuend :} & 2y - 7x + 3 \\ & \oplus \quad \ominus \quad \ominus \\ \text{The subtrahend :} & \underline{-5y + 3x + 7} \\ \text{The remainder} = & 7y - 10x - 4 \end{array}$$

Remember **Multiplying algebraic terms and expressions**

Multiplying a monomial by a monomial

When multiplying a monomial by another monomial :

1. Multiply the coefficients.
2. Multiply the symbols regarding adding the indices of the factors which have like bases.

For example :

- $5x \times (-3y) = -15xy$
- $-3x^2 \times 4xy = -12x^3y$

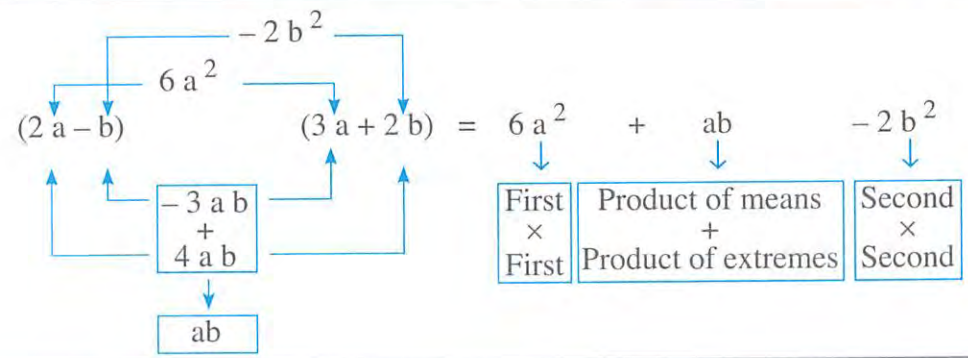
Multiplying a monomial by an algebraic expression

When we multiply a monomial by an algebraic expression , we multiply this monomial by each term of the algebraic expression.

For example : $3x(4y - x) = 12xy - 3x^2$

Multiplying two binomials

First Multiplying by inspection



Second The product of the sum of two terms and the difference between them :

$(a + b)(a - b) = a^2 - b^2$

i.e. Sum of two terms × difference between them = square of the first – square of the second

For example : $(a + 3b)(a - 3b) = a^2 - 9b^2$

Third The square of an expression containing two terms : $(a \pm b)^2 = a^2 \pm 2ab + b^2$

1. The square of an expression consisting of the sum of two terms =
The square of the first ⊕ 2 × the first × the second + the square of the second.

For example : $(2a + 3)^2 = (2a)^2 + (2 \times 2a \times 3) + 3^2 = 4a^2 + 12a + 9$

2. The square of an expression consisting of the difference between two terms =
The square of the first ⊖ 2 × the first × the second + the square of the second.

For example : $(2x - 5y)^2 = (2x)^2 - (2 \times 2x \times 5y) + (5y)^2 = 4x^2 - 20xy + 25y^2$

Remember | Dividing algebraic terms and expressions

Dividing a monomial by a monomial

When dividing a monomial by another monomial :

1. Divide the coefficients.
2. Divide the symbols regarding that the indices of the factors of like bases should be subtracted.

For example : • $-12\,a^3 \div 3\,a = -4\,a^2$

• $-28\,a^5 b^3 c^3 \div (-4\,a^3 b) = 7\,a^2 b^2 c^3$

Dividing an algebraic expression by a monomial

When we divide an algebraic expression by a monomial, we divide each term of the expression by this monomial.

For example : $(16\,x^3 y + 24\,x^2 y^4 - 8\,x^2 y) \div (-4\,x^2 y)$

$$= \frac{16\,x^3 y}{-4\,x^2 y} + \frac{24\,x^2 y^4}{-4\,x^2 y} - \frac{8\,x^2 y}{-4\,x^2 y} = -4\,x - 6\,y^3 + 2$$

Dividing an algebraic expression by another one

To perform the division operation of an algebraic expression by another one, we will remember the steps of division :

For example : To find the quotient of dividing $5\,a - 10\,a^2 + 6\,a^3 + 3$ **by** $3 + 2\,a^2 - 4\,a$

1. Arrange the terms of each of the dividend and the divisor descendingly according to the power of a before performing of the division operation.
2. Divide $6\,a^3$ **by** $2\,a^2$, then the result $3\,a$
3. Multiply $3\,a$ **by** $(2\,a^2 - 4\,a + 3)$
4. Subtract $(6\,a^3 - 12\,a^2 + 9\,a)$ from $(6\,a^3 - 10\,a^2 + 5\,a + 3)$
5. Repeat the previous steps in the same order till the difference will be equal to zero, then the operation of division will be finished and the quotient will be : $3\,a + 1$

$$\begin{array}{c|l}
2\,a^2 - 4\,a + 3 & 6\,a^3 - 10\,a^2 + 5\,a + 3 \\
3\,a + 1 & \ominus 6\,a^3 \overset{\oplus}{-} 12\,a^2 \overset{\ominus}{+} 9\,a \\
\hline
 & \qquad 2\,a^2 - 4\,a + 3 \\
 & \qquad \ominus 2\,a^2 \overset{\oplus}{-} 4\,a \overset{\ominus}{+} 3 \\
\hline
 & \qquad 0 \qquad 0 \qquad 0
\end{array}$$

Remember **The steps of factorization by identifying the highest common factor (H.C.F.)**

1. Find H.C.F. of the algebraic terms of the expression.
2. Put H.C.F. outside two brackets.
3. Divide each term of the algebraic expression by the H.C.F. and write the quotients inside the two brackets.

For example : $21\,a^3\,b^2 - 7\,a^2\,b^2 - 35\,a^2\,b^3 = 7\,a^2\,b^2\,(3\,a - 1 - 5\,b)$

Remember **Statistics**

The arithmetic mean of a set of values $= \dfrac{\text{Sum of these values}}{\text{Number of these values}}$

For example : The arithmetic mean of the values : 2 , 5 , 7 , 5 , 4 , 9 , 3

is $\dfrac{2+5+7+5+4+9+3}{7} = \dfrac{35}{7} = 5$

The median of a set of values is the value which divides this set such that the number of values which are greater than it is equal to the number of values which are smaller than it.

To get the median, we arrange the values ascendingly or descendingly

If the number of values is odd , then :

The median is the value which is in the middle exactly.

For example :

- **If the values are :**

42 , 23 , 17 , 30 , 20

Then its ascending order is :

17 , 20 , 23 , 30 , 42

$\therefore$ The median = 23

If the number of values is even , then :

The median $= \dfrac{\text{The sum of the two middle values}}{2}$

For example :

- **If the values are :**

27 , 13 , 23 , 24 , 13 , 21

Then its ascending order is :

13 , 13 , 21 , 23 , 24 , 27

$\therefore$ The median $= \dfrac{21+23}{2} = 22$

The mode of a set of values is the most common value.

For example : The mode of the values : 6 , 2 , 3 , 11 , 6 , 8 , 6 , 3 is 6

Final Examinations

on Algebra and Statistics

Model Examinations of the School Book on Algebra and Statistics

Model 1

Answer the following questions :

1 Complete each of the following :

1. $2\frac{1}{5} \times$ .... $= 1$
2. If the order of the median of a set of values is the fourteenth , then the number of these values equals ....
3. $0.18 - 30\ \% =$ ....
4. $7\,x^3\,y^2 \times$ .... $= 21\,x^3\,y^5$
5. $(2\,x - 3)\,(x + 5) = 2\,x^2 +$ .... $- 15$

---

2 Choose the correct answer from those given :

1. The rational number that lies one third of the way between 8 and 12 from the smaller is ....

   (a) $8\frac{1}{3}$ (b) 10 (c) $9\frac{1}{3}$ (d) $10\frac{2}{3}$

2. If the mode of the values 7 , 5 , $x + 4$, 5 , 7 is 5 , then $x =$ ....

   (a) 1 (b) 4 (c) 5 (d) 7

3. If $\Delta + \square = 20$, $\Delta + \Delta + \square = 35$, then $\Delta =$ ....

   (a) 15 (b) 20 (c) 5 (d) 10

4. The arithmetic mean of the values 1 , 6 , 4 , 8 , 6 is ....

   (a) 25 (b) 5 (c) 6 (d) 8

5. If $\frac{2}{5}\,x = 10$, then $\frac{3}{5}\,x =$ ....

   (a) 25 (b) 15 (c) 20 (d) 5

6. $0.7 + 0.\dot{3} =$ ....

   (a) 1 (b) 3.7 (c) $0.\dot{3}7$ (d) $1\frac{1}{30}$

---

3 **[a] Subtract :** $5\,x^2 + y^2 - 3\,x\,y + 1$ **from** $6\,x^2 - 2\,x\,y + 3\,y^2$

[b] Use the distribution property to find the value of :

$$\frac{27}{16} \times \frac{11}{7} + \frac{27}{16} \times \frac{11}{7} - \frac{27}{16} \times \frac{6}{7}$$

---

4 **[a] Simplify to the simplest form :** $(2\,x - 3)\,(2\,x + 3) + 7$, then calculate the numerical value of the result when $x = -1$

[b] Find three rational numbers that lie between : $\frac{1}{2}$ and $\frac{1}{3}$

5 **[a] Divide :** $2x^3+3x^2-4x-6$ **by** $2x+3$ $\left(\text{where } x \neq -\frac{3}{2}\right)$

[b] The following table shows Gehad's marks in mathematics exam in 6 months :

Month	October	November	December	February	March	April
Mark	30	35	42	37	44	50

Find the arithmetic mean of the marks.

Model 2

Answer the following questions :

1 **Complete each of the following :**

1. $24x^4y^6 = 6x^2y^3 \times$ ....
2. The remainder of subtracting $-3x$ from $2x$ is ....
3. 1 , 1 , 2 , 3 , 5 , 8 , ... (in the same pattern)
4. If the mode of the values 7 , 5 , a + 3 , 5 , 7 is 7 , then a = ...
5. $5x^2 + 15xy = 5x$ (.............. + ...)

---

2 **Choose the correct answer from those given :**

1. The algebraic term $6x^3y^2$ is of the .... degree.

   (a) third (b) fourth (c) fifth (d) sixth

2. The rational number that lies in half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....

   (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{9}$ (d) $\frac{5}{27}$

3. The multiplicative inverse of the number $\left(\frac{1}{2}\right)^0$ is ....

   (a) 2 (b) – 2 (c) 1 (d) –1

4. If $\frac{5}{x+2}$ is a rational number , then $x \neq$ ....

   (a) – 2 (b) 0 (c) 2 (d) 5

5. The median of the values 5 , 4 , 7 is ....

   (a) 4 (b) 5 (c) 7 (d) 16

6. If the arithmetic mean of the values 3 , 5 and $x+2$ is 4 , then the arithmetic mean of the two values $5-x$, $5+2x$ is ....

   (a) 6 (b) 4 (c) 3 (d) 2

3 [a] **Using the distribution property , find the value of :** $\frac{3}{7} \times 2 + \frac{3}{7} \times 6 - \frac{3}{7}$

[b] **Find three rational numbers that lie between :** $\frac{1}{2}$ and $\frac{1}{3}$

---

4 [a] **What is the increase of :** $7 X + 5 y + z$ **than** $2 X + 6 y + z$?

[b] **Divide :** $14 X^2 y - 35 X y^2 + 7 X y$ **by** $7 X y$ where $X \neq 0$ and $y \neq 0$

---

5 [a] **Simplify to the simplest form :** $(X - 3)(X + 3) + 9$, then

calculate the numerical value of the result when $X = 5$

[b] **If the arithmetic mean of the numbers :** 8 , 7 , 5 , 9 , 4 , 3 , k + 4 is 6

, then find the value of : k

Model examination for the merge students

Answer the following questions :

1 Complete each of the following :

1 The algebraic term $5 \mathcal{X} y$ is of the .... degree.

2 $(\mathcal{X} - 3)$ (............... + ...) $= \mathcal{X}^2 - 9$

3 The rational number which hasn't a multiplicative inverse is ....

4 The median of the values 3 , 4 , 5 is ....

5 The number $\frac{4}{\mathcal{X}}$ is a rational number if $\mathcal{X} \neq$ ....

---

2 Choose the correct answer from those given :

1 If $\frac{4}{7} \mathcal{X} = \frac{4}{7}$, then $\mathcal{X} =$ ....

(a) 1 (b) 0 (c) 4 (d) 7

2 The arithmetic mean of the values 2 , 3 , 8 , 2 , 5 equals ....

(a) 3 (b) 2 (c) 4 (d) 8

3 The additive inverse of the number – 3 is ....

(a) – 3 (b) 3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

4 The remainder of subtracting $7 \mathcal{X}$ from $9 \mathcal{X}$ equals ....

(a) $2 \mathcal{X}$ (b) $16 \mathcal{X}$ (c) $-2 \mathcal{X}$ (d) 0

5 The mode of the values 3 , 3 , 4 , 4 , 5 , 3 is ....

(a) 4 (b) 22 (c) 5 (d) 3

---

3 [a] Using the distribution property , complete to find :

$\frac{5}{7} \times 8 + \frac{5}{7} \times 5 + \frac{5}{7} = \frac{5}{7}$ (............... + ... + ...) $= \frac{5}{7}$ (...............) = ...

[b] If $a = \frac{1}{2}$, $b = -2$, **complete the following :**

$b \div a =$ (...............) ÷ (...............) = (...............) × (...............) = ...

---

4 Put true (✔) or false (✘) :

1 The quotient of $12 \mathcal{X}^4 + 6 \mathcal{X}$ by $6 \mathcal{X}$ is $2 \mathcal{X}^3 + 1$ ( )

2 The H.C.F. of : $15 \mathcal{X}^5 + 5 \mathcal{X}$ is $5 \mathcal{X}^5$ ( )

3 The rational number that lies between $\frac{1}{4}$ and $\frac{3}{4}$ is $\frac{1}{2}$ ( )

4 $5x + 3x = 8x$ ( )

5 If $(x + 4)^2 = x^2 + k + 16$, then $k = 4x$ ( )

5 Match from column (A) to column (B) :

Column (A)	Column (B)
1 If $\frac{x-7}{5} = 0$, then $x =$	3
2 $3x^2 + 15y =$ .... $(x^2 + 5y)$	7
3 $(3x + 5) + (4x - 5) =$	50
4 $\frac{1}{2} =$ .... %	1
5 If $\frac{a}{b} = \frac{1}{2}$, then $\frac{2a}{b} =$	$7x$

Some Schools Examinations on Algebra and Statistics

1 **Cairo Governorate** — El-Waily Educ. Admin. St.Joseph Maronite Language Schools

Answer the following questions :

1 **Choose the correct answer :**

1. The mode of the values : 6 , 8 , 6 , 1 , 1 , 9 , 8 , 2 , 8 is ....
   (a) 1 (b) 6 (c) 8 (d) 9
2. $a^3 b \times a b^2 =$ ....
   (a) $a^3 b^2$ (b) $3 a^3 b^4$ (c) $a^4 b^3$ (d) $a^3 b^3$
3. The multiplicative inverse of $\left|\frac{-7}{8}\right|$ is ....
   (a) $\frac{-7}{8}$ (b) $\frac{8}{7}$ (c) $\frac{7}{8}$ (d) $\frac{-8}{7}$
4. The degree of the expression : $x^3 + 2xy + 3y^2x^2$ is the .... degree.
   (a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 4^{th}
5. $(-5x) + (-3x) - x =$ ....
   (a) $-9x$ (b) $9x$ (c) $8x$ (d) $-8x$
6. $(3a + 2b)^2 = 9a^2 +$ .... $+ 4b^2$
   (a) 6 ab (b) 12 ab (c) 24 ab (d) 36 ab

---

2 **Complete :**

1. The arithmetic mean of the values : 22 , 18 , 15 , 25 and 30 is ....
2. $\frac{-1}{4} +$ .... $= 0$
3. $(x + 4)(x - 4) =$ ....
4. The median of the values : 23 , 16 , 12 , 28 , 21 , 32 , 9 is ....
5. $7x(x + 5y) = 7x^2 +$ ....

---

3 [a] **By using the distribution property find :** $\frac{5}{9} \times \frac{2}{7} + \frac{5}{9} \times \frac{1}{7} + \frac{5}{9} \times \frac{4}{7}$

[b] **Subtract :** $5x^2 + 2x - 1$ **from** $8x^2 - 3x + 7$

[c] **Add :** $6x^2 - 5x + 3$ **and** $4x^2 + 2x + 5$

4 [a] If $a = \frac{1}{2}$, $b = \frac{-2}{3}$, $c = 3$, **find the value of :** $c^2 - 6\,a\,b$

[b] **Divide :** $x^2 + 12\,x + 35$ **by** $x + 5$ (where $x \neq -5$)

---

5 [a] **Factorize by using identifying the H.C.F. :** $6\,x^4\,y^3 - 12\,x^3\,y^4 + 2\,x^3\,y^3$

[b] **Find three rational numbers between :** $\frac{1}{2}$ and $\frac{1}{3}$

[c] **The following table shows the marks of 50 students :**

Marks	4	6	9	12	15	18
Frequency	6	13	16	7	5	3

Find the mode of these marks.

2 Cairo Governorate

Zietoun Educational Administration
Gomhouria Language School

Answer the following questions :

1 Choose the correct answers :

1 The degree of the algebraic term $(-2)^3$ is ....

(a) zero. (b) first. (c) second. (d) third.

2 The remainder of subtracting $\frac{-3}{7}$ from $\frac{4}{7}$ is ....

(a) $\frac{-1}{7}$ (b) $\frac{1}{7}$ (c) 1 (d) -1

3 The multiplicative inverse of $\left(\frac{-2}{3}\right)^0$ is ....

(a) 1 (b) -1 (c) $\frac{-3}{2}$ (d) zero.

4 If the order of the median for a set of values is the nineth , then number of values = ...

(a) 17 (b) 14 (c) 15 (d) 9

5 The arithmetic mean of the values : $2\,x$, 6 , 3 , 2 , 4 , 1 is 5 , then $x =$ ....

(a) 7 (b) 30 (c) 15 (d) 6

6 If $\bigcirc + \triangle = 9$, $\bigcirc + \triangle + \triangle = 14$, then $\triangle =$ ....

(a) 7 (b) 4.5 (c) 5 (d) 9

---

2 Complete each of the following :

1 If $\frac{x + 4}{x - 5} = 0$, then $x =$ ....

2 $(x - 5)^2 = x^2$ .... $+ 25$

3 The rational number that lies at half way between $\frac{3}{5}$ and $\frac{2}{5}$ is ....

4 The mode of the values : 15 , 9 , $X + 1$, 9 , 15 is 9 , then $X =$ ....

5 Third of a number is 20 , then $\frac{3}{4}$ of this number = ...

3 [a] **Find the quotient of :** $3X^2 y - 6Xy^2 + 12Xy$ **by** $-3X$ where $X \neq 0$

[b] **Write three rational numbers between :** $\frac{5}{7}$, $\frac{1}{4}$

4 [a] **Use the distribution property to find the value of :** $\frac{5}{9} \times 2 + \frac{5}{9} \times 6 + \frac{5}{9}$

[b] **Simplify to the simplest form :**

$(X - 3)(X - 3) + 9$ and find the value of the result if $X = 2$

5 [a] **Factorize by identifying the H.C.F. :**

$X(a - b) + y(a - b)$ and find the value when : $a - b = 5$, $X + y = 2$

[b] **Find the mean and the mode of the values :** 15 , 7 , 4 , 5 , 8 , 7

3 Cairo Governorate

El-Sahel Educational Zone
Tarek Ibn Zaid Governmental Language School

Answer the following questions :

1 **Complete each of the following :**

1 The mean of the values : 10 , 21 , 13 , 16 is ....

2 If $|X| = 5$, then $X =$ .... or ....

3 The increase of : $-3X$ than $4X$ is ....

4 $2\frac{1}{5} \times$ .... $= 1$

5 The mode of the values : 7 , 12 , 7 , 12 , 14 , 7 is ....

2 **Choose the correct answer :**

1 The algebraic term $5X^3 y$ of the .... degree.

(a) first (b) second (c) third (d) fourth

2 2 hours = ... min.

(a) 20 (b) 60 (c) 100 (d) 120

3 25% of 400 = ...

(a) 10 (b) 100 (c) 200 (d) 2500

4 If $\frac{X + 2}{X - 3}$ is a rational number , then $X \neq$ ....

(a) 2 (b) -2 (c) 3 (d) -3

5 The median of the values : 5 , 9 , 12 , 15 , 8 is ....

(a) 8 (b) 12 (c) 7 (d) 9

6 If $5X = -15$, then $X =$ ....

(a) 3 (b) -3 (c) 45 (d) -45

---

3 [a] **Simplify :** $(2X - 3)(2X + 3) + 7$, then find the numerical value of the result when $X = 1$

[b] **Use the distributive property to find the value of :** $\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25}$

---

4 [a] **Add :** $2X + 3y - 7$ **and** $5X + 4 - 5y$

[b] **If :** $a = \frac{7}{4}$, $b = -\frac{1}{2}$, **find the value of expression :** $(a - b) \div (a + b)$

---

5 [a] **Divide :** $(18a^3b + 12ab^3 - 6ab)$ **by** $6ab$ (where $ab \neq 0$)

[b] **The following table shows the distribution of 45 students in an examination :**

Marks	10	12	14	16	18	20
No. of students	4	6	15	10	7	4

1 Find the mode of these marks.

2 Find the number of students who got less than 14 marks.

4 Giza Governorate

Dokki Educational Directorate
Modern Narmer Language School

Answer the following questions :

1 Choose the correct answer :

1 $\frac{2}{X - 3}$ is not a rational number if $X =$ ....

(a) 0 (b) 1 (c) 2 (d) 3

2 The additive identity rational number is ....

(a) 0 (b) 1 (c) -1 (d) 10

3 The term .... is of third degree.

(a) $3a$ (b) a^3b^3 (c) $2a^2b$ (d) 3

4 Subtract $(2X)$ from $(-X)$ equals ....

(a) X (b) $3X$ (c) $-3X$ (d) $-X$

5 $\frac{4}{7}$ …………… $\frac{3}{5}$

(a) $>$ (b) $=$ (c) $<$ (d) $\geq$

6 If the order of the median of a set of values is the fourth , then the number of values is ……………

(a) 7 (b) 8 (c) 5 (d) 6

---

2 Complete :

1 $8\, a^2 \div (-2\, a) =$ …………… (where $a \neq 0$)

2 $(x-2)(x+5) = x^2 +$ …………… $- 10$

3 The arithmetic mean of the values : 3 , 5 and 7 is ……………

4 The rational number $\frac{x-2}{x-3}$ equals zero at $x =$ ……………

5 The degree of the algebraic expression : $x^2 - 5\,x + 3$ is ……………

---

3 [a] **Find three rational numbers between :** $\frac{1}{3}$ and $\frac{1}{2}$

[b] **Find the sum of :** $(3\,x^2 - 4\,x - 2)$ **and** $(7 - 4\,x - x^2)$

[c] **Subtract :** $(-x^2 - 5\,x\,y + 4\,y^2)$ **from** $(3\,x^2 - 2\,x\,y - 2\,y^2)$

---

4 [a] **Multiply , then find the numerical value of the expression at $x = -2$:**

1 $(2\,x + 3)^2$ 2 $(2\,x - 1)(x + 3)$

[b] **Find the quotient of :** $x^2 + 5\,x + 6$ **by** $x + 2$ where $x \neq -2$

---

5 [a] **Factorize by identifying the H.C.F. :** $3\,x^2\,y^2 - 9\,x^3\,y^4 + 12\,x\,y$

[b] If the mode of the values : 5 , 4 , $x + 3$, 5 , 4 is 5 **find the value of :** x

[c] **Simplify :** $(x-2)(x+2) + 4$

5 Giza Governorate

6th October Directorate
Math Inspection

Answer the following questions :

1 Choose the correct answer :

1 If $\frac{x-3}{x-5}$ is a rational number , then $x \neq$ ……………

(a) -2 (b) zero (c) 2 (d) 5

2 The mode of the values : 4 , 5 , 4 , 3 , 7 , 5 , 4 is ……………

(a) 3 (b) 4 (c) 7 (d) 5

3 A rectangle of dimensions : $5\,x$ cm. and $3\,x$ cm. , then its perimeter = ... cm.

(a) $3\,x$ (b) $5\,x$ (c) $16\,x$ (d) $7\,x$

4 $(3\,x + 5)(x + 2) = 3\,x^2 +$ .... $+ 10$

(a) -7 (b) $11\,x$ (c) $5\,x$ (d) $4\,x$

5 The remainder of subtracting : $-2\,x$ from $2\,x$ equals ....

(a) $-4\,x$ (b) $4\,x$ (c) 0 (d) -4

6 $(x^2 + x) \div x =$ .... where $x \neq 0$

(a) zero (b) x (c) $2\,x + 1$ (d) $x + 1$

---

2 Complete the following :

1 If $\frac{x+6}{x-3} =$ zero , then $x =$ ....

2 $5\,x^2 + 15\,x\,y = 5\,x$ (.............. + ...)

3 The mode of the values : 5 , 7 , 4 , a + 1 , 6 , 10 is 4 , then a = ...

4 The number that lies half way between $\frac{1}{3}$ and $\frac{5}{9}$ is ....

5 The rational number that hasn't a multiplicative inverse is ....

---

3 [a] **Use the distribution property to find the value of :** $\frac{8}{13} \times 11 + \frac{8}{13} \times 9 - \frac{8}{13} \times 7$

[b] **Add :** $2\,a + 3\,b - c$ **and** $3\,a - 2\,b - 2\,c$

---

4 [a] **Simplify :** $(a - 4)^2 + 8\,a$

[b] **Divide :** $x^2 - 5\,x + 6$ **by** $x - 3$ (where $x - 3 \neq 0$)

---

5 [a] **Factorize by taking out the H.C.F. :** $12\,a^2\,b + 18\,a^3\,b^2$

[b] 1 **Find the mean of the values :** 2 , 5 , 3 , 6 , 9

2 **Find the median of the values :** 7 , 9 , 13 , 6 , 8

6 Alexandria Governorate

Eastern Educational Zone
Taymour English School

Answer the following questions :

1 Choose the correct answer :

1 The multiplicative inverse of the number $-\frac{2}{3}$ is ....

(a) $-\frac{3}{2}$ (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\left(\frac{-2}{3}\right)^2$

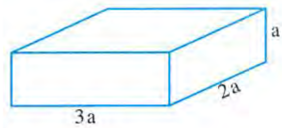
2 The rational number that lies at the half of the way between : $\frac{3}{5}$ and $\frac{4}{5}$ is ....

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{7}{10}$ (d) $\frac{5}{5}$

3 $36\,a^5\,b^8 = 12\,a^3\,b^2 \times$ ....

(a) $3\,a\,b$ (b) $3\,a^2\,b^4$ (c) $3\,a\,b^6$ (d) $3\,a^2\,b^6$

4 The volume of the opposite solid is ....



(a) $6\,a^2$ (b) $5\,a$

(c) $6\,a$ (d) $6\,a^3$

5 If $(2\,x + y)^2 = 4\,x^2 + k\,x\,y + y^2$, then k = ...

(a) 2 (b) 4 (c) 6 (d) 8

6 The mode for the set of values : 11 , 14 , 11 , 12 , 14 , 15 , 11 is ....

(a) 11 (b) 12 (c) 14 (d) 15

---

2 Complete the following :

1 The median of the numbers : 5 , 4 , 10 , 3 , 3 , 4 , 7 , 4 , 6 is ....

2 $\frac{(x^2 + x)}{x} =$ .... (where $x \neq 0$)

3 $5\,x^2 + 3$ is an algebraic expression of the .... degree.

4 $7\,x^2 + 14\,y^2 = 7$ (.............. + ...)

5 If $\frac{3}{x - 5}$ is a rational number , then $x \neq$ ....

---

3 [a] Find the sum of : $(3\,x - 2\,y + 5)$ **and** $(x + 2\,y - 2)$

[b] Subtract : $2\,x - 5\,y + z$ **from** $3\,x + y - 2\,z$

---

4 [a] Simplify : $5\,x^2 - 2\,x + 8 - 7\,x - 3 + x^2$

[b] If $a = \frac{3}{4}$, $b = -\frac{5}{2}$ **Find in the simplest form the numerical value of :** $\frac{a - b}{a + b}$

---

5 [a] Multiply : $(2\,y + 7)\,(3\,y + 4)$

[b] Divide : $2\,x^2 + 13\,x + 15$ **by** $x + 5$ where $x \neq -5$

[c] If the temperature for a full week in one of the cities : 25° , 27° , 31° , 23° , 22° , 22° , 18°

Calculate the arithmetic mean.

7 Alexandria Governorate

El-Montazah Educational Zone
Maths Supervision

Answer the following questions :

1 Choose the correct answer :

1. The smallest natural number is ....
   (a) -1 (b) 0 (c) 1 (d) 2

2. If $a \times \frac{b}{2} = \frac{a}{2}$, $a \neq 0$, then $b =$ ....
   (a) 1 (b) 0 (c) a (d) $\frac{a}{2}$

3. $(2\,x) \times (5\,x) =$ ....
   (a) $10\,x$ (b) $7\,x$ (c) $7\,x^2$ (d) $10\,x^2$

4. $\{3 , 5\} \cap \{5 , 4\} =$ ....
   (a) $\{3\}$ (b) $\{4\}$ (c) $\{5\}$ (d) $\{3 , 4 , 5\}$

5. If $\frac{5}{x+2}$ is a rational number , then $x \neq$ ....
   (a) 5 (b) 2 (c) -2 (d) 0

6. The median of the values : 4 , 8 , 3 , 5 , 7 is ....
   (a) 3 (b) 4 (c) 5 (d) 7

2 Complete each of the following :

1. The additive identity in $\mathbb{Q}$ is ....
2. The algebraic term : $5\,x^2\,y$ is of the .... degree.
3. If $\frac{2}{5}\,x = 10$, then $\frac{3}{5}\,x =$ ....
4. $(x^2 + x) \div x =$ .... (where $x \neq 0$)
5. The arithmetic mean of the set of values : 1 , 6 , 4 , 8 , 6 is ....

3 [a] **Use the distribution property to find the value of :** $5 \times \frac{3}{11} + 7 \times \frac{3}{11} - \frac{3}{11}$

[b] **Factorize by taking the H.C.F. :** $3\,x^2 + 15\,x\,y$

4 [a] **Add :** If $3\,a + 5\,b - 3\,c$ **and** $2\,a - 7\,b + 5\,c$

[b] **Find three rational numbers between :** $\frac{3}{5}$, $\frac{1}{4}$

5 **[a] Simplify :** $(3X - 5)(3X + 5)$

[b] The following table shows the distribution of marks 25 students in an exam :

Marks	6	9	12	16	19
No. of students	4	7	3	5	6

Find the mode of these marks.

8 El-Kalyoubia Governorate — Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1. The coefficient of the algebraic term $4X^2$ is ....
   (a) 1 (b) 2 (c) 3 (d) 4
2. If the median of a set of values : 27 , 45 , 19 , 24 , 28 is X , then X = ...
   (a) 24 (b) 27 (c) 28 (d) 45
3. $\mathbb{Z}_+ \cup \{0\}$ = ...
   (a) $\mathbb{Z}$ (b) $\varnothing$ (c) $\mathbb{Q}$ (d) $\mathbb{N}$
4. The number lies between 0.3 and $0.\dot{3}$ is ....
   (a) 0.33 (b) 0.34 (c) 0.35 (d) $\frac{3}{10}$
5. The remainder of subtracting : $-2a^2$ from $4a^2$ is ....
   (a) $-6a^2$ (b) $6a^2$ (c) $-2a^2$ (d) $6a$
6. The number which hasn't multiplicative inverse is ....
   (a) -2 (b) -1 (c) 0 (d) 1

2 **Complete each of the following :**

1. The mode is ....
2. One year ≃ .... week.
3. The number in fifth way from smaller between $\frac{-1}{5}$ and $\frac{9}{5}$ is ....
4. The arithmetic mean of the values : 2 , 6 , 5 , 8 , 4 is ....
5. If 20% of a number equals 100 , then the number is ....

3 **[a] Using the distributive property to find the result of :** $\frac{15}{17} \times \frac{23}{45} + \frac{19}{17} \times \frac{23}{45} - \frac{23}{45}$

[b] If $X = \frac{-2}{5}$, $y = \frac{2}{3}$ and $z = \frac{8}{15}$ **, find the numerical value of :** $(X + y) \div z$

[c] Find the rational number that lies in the half of the way between : $\frac{9}{4}$ and $\frac{17}{6}$

4 **[a] Factorize by taking H.C.F. :** $4x^4y - 6x^3y^2 + 2x^2y^3$

[b] Find the sum of : $3x + 4y - 5z$ **and** $2x - y + 4$

[c] Divide : $x^2 - 10x + 25$ **by** $x - 5$ where $x \neq 5$

---

5 **[a] Simplify :** $(x + 3)^2 - (x - 3)(x + 3)$

[b] The following table shows student's marks in 6 mathematics examinations :

Months	Oct.	Nov.	Dec.	Feb.	Mar.	Apr.
Marks	28	34	42	38	48	50

Find : 1 The arithmetic mean. 2 The median.

9 El-Sharkia Governorate

Minia El-Kameh Directorte
Minia El-Kameh Language School

Answer the following questions :

1 **Choose the correct answer :**

1 The mode of the values : 7 , 6 , 7 , 3 , 7 is ....

(a) 8 (b) 3 (c) 7 (d) 6

2 The additive identity element in $\mathbb{Q}$ is ....

(a) zero (b) 1 (c) -3 (d) 3

3 The mean of : 6 , 7 , 0 , 4 , 3 is ....

(a) 5 (b) 4 (c) 6 (d) 7

4 The rational number that lies in the half of the way between $\frac{1}{3}$ and $\frac{2}{3}$ is ....

(a) $\frac{2}{5}$ (b) $\frac{3}{7}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

5 The multiplicative inverse of $\frac{2}{3}$ is ....

(a) $\frac{-2}{3}$ (b) $\frac{5}{6}$ (c) $\frac{3}{2}$ (d) $\frac{7}{3}$

6 The rational number $\frac{x-3}{7} = 0$, when $x =$ ....

(a) 7 (b) zero (c) 6 (d) 3

---

2 **Complete the following :**

1 $5x^3y \times$ .... $= 15x^5y^2$

2 The algebraic term : $-3x^2y$ of .... degree.

3 The median of the values : 9 , 18 , 5 , 7 and 11 is ....

4 The number $\frac{x+3}{x-5} \in \mathbb{Q}$ if $x \neq$ ....

5 $(x-5)(x+5) =$ .... -25

3 [a] **Find three rational numbers between :** $\frac{1}{2}$ and $\frac{1}{3}$

[b] **Add :** $2a + 3b - c$ **and** $3a - 2b - 2c$

4 [a] **Factorize by taking the H.C.F. :** $12xy^2 + 18x^2y - 6x^2y^2$

[b] **Simplify :** $(x-6)(x+6) + 36$

5 [a] **Use the property of distribution to find the result of :** $\frac{13}{15} \times 17 + \frac{13}{15} \times 14 - \frac{13}{15}$

[b] **Find the quotient of :**

$6x^2 + 13x + 6$ **by** $2x + 3$ where $2x + 3 \neq 0$

10 El-Monofia Governorate

Quesna Education Administration
Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 If the order of the median for a set of values is the fourth , then the number of values equals ....

(a) 3 (b) 5 (c) 7 (d) 9

2 If the arithmetic mean of the numbers : 8 , 7 , 5 , 9 , 4 , 3 , k + 4 is 6 , then find the value of k = ...

(a) 1 (b) 2 (c) 4 (d) 5

3 If $\frac{x}{3} - 4 = 6$, then $\frac{x}{3} + \frac{2}{3} =$ ....

(a) 1 (b) x (c) $\frac{32}{3}$ (d) 10

4 If the algebraic term $x^k y^2$ of the third degree , then k = ...

(a) 1 (b) 2 (c) 3 (d) 4

5 The middle term in the expansion of $(2x - 5y)^2$ is ....

(a) $-10x^2y^2$ (b) $20xy$ (c) $10x^2y^2$ (d) $-20xy$

6 $(x^2 + x) \div x =$ .... (where $x \neq 0$)

(a) zero (b) x (c) $2x + 1$ (d) $x + 1$

2 **Complete each of the following :**

1. $24\,x^4\,y^6 = 6\,x^2\,y^3 \times$ ....
2. The remainder of subtracting $-3\,x$ from $2\,x$ is ....
3. $(x-3)$ (.............. + ...) $= x^2 - 9$
4. The rational number which hasn't multiplicative inverse is ....
5. $20\% - \left|\frac{-1}{5}\right| =$ ....

---

3 [a] **Use the distribution property to find the value of :** $\frac{5}{7} \times 8 + \frac{5}{7} \times 5 + \frac{5}{7}$

[b] If $\frac{x-2}{x+3} = 0$, find the three rational numbers lying between $\frac{1}{x}$ and $\frac{2}{1+x}$

---

4 [a] If $x = \frac{1}{2}$, $y = \frac{-2}{3}$, $z = 2$, **find the value of :** $\frac{y-z}{x}$

[b] **Factorize by identification the H.C.F. :**

$3\,a\,(a-2\,b) - 6\,b\,(a-2\,b)$, then find the value of the result when $(a-2\,b) = \left|-\frac{1}{3}\right|$

---

5 [a] **Find the quotient of the dividing :** $5\,a - 10\,a^2 + 6\,a^3 + 3$ **by** $2\,a^2 - 4\,a + 3$ where $2\,a^2 - 4\,a + 3 \neq 0$

[b] **Multiply :** $(6\,x - 2\,y)(x + 5\,y)$

11 El-Dakahlia Governorate — Maths Supervision

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $\frac{3}{4} =$ ....%

   (a) 0.75 (b) 75 (c) 7.5 (d) 0.0075

2. If the mode of : 7 , 5 , a + 3 , 5 , 7 is 7 , then a = ...

   (a) 2 (b) 4 (c) 7 (d) 10

3. The number that lies half way between $\frac{8}{13}$ and $\frac{12}{13}$ is ....

   (a) $\frac{9}{13}$ (b) $\frac{10}{13}$ (c) $\frac{20}{13}$ (d) $\frac{10}{26}$

4. If $(x+3)(x-3) = x^2 - k$, then k = ...

   (a) -3 (b) 3 (c) -9 (d) 9

5. The order of median for the values : 4 , 12 , 9 , 8 , 2 is ....

   (a) 8 (b) 9 (c) third (d) fourth

6. If $\triangle + \square = 40$, $\triangle + \triangle + \square = 55$, then $\triangle =$ ....

   (a) 5 (b) 15 (c) 20 (d) 25

2 **Complete each of the following :**

1 $3\frac{1}{2} \times$ .... $= 1$

2 The arithmetic mean of the values : 2 , 3 , 8 , 2 , 5 is ....

3 If $\frac{x+4}{x-2}$ is a rational number , then $x \neq$ ....

4 $7x^3 y^2 \times$ .... $= 35x^4 y^2$

5 The remainder of subtracting $(-3y)$ from $(4y)$ is ....

---

3 [a] **Use the distribution property to find the value of :** $\frac{27}{16} \times 11 + \frac{27}{16} \times 7 - \frac{27}{16} \times 2$

[b] **Divide :** $35x^3y^3 + 25x^2y^2 - 15xy$ **by** $5xy$ (when $x \neq 0$, $y \neq 0$)

---

4 [a] **Find three rational numbers lie between :** $\frac{1}{2}$ and $\frac{1}{3}$

[b] **Add :** $2x^2 - 5x + 3$ **and** $4x - x^2 - 2$

---

5 [a] 1 **Factorize by identifying the H.C.F. :** $6x^2 + 15xy - 3x$

2 **Subtract :** $5a + 2b - 1$ **from** $2a - 5b + 3$

[b] **From the following values :** 2 , 5 , 9 , 6 , 3

1 Find the arithmetic mean.

2 Find the median.

12 Ismailia Governorate

Directorate of Education
Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The mean of : 17 , 15 , 13 , 25 , 10 is ....

(a) 15 (b) 16 (c) 17 (d) 18

2 If $\frac{x-2}{x-5} = 0$, then $x =$ ....

(a) 2 (b) 5 (c) 7 (d) 10

3 The number that lies half way between $\frac{2}{3}$ and $\frac{4}{3}$ is ....

(a) $\frac{3}{4}$ (b) 2 (c) 1 (d) $\frac{5}{3}$

4 The median of : 12 , 7 , 6 , 14 , 9 is ....

(a) 6 (b) 9 (c) 7 (d) 14

5 $(2x-3)(2x+3) = \cdots\cdots - 9$

(a) x^2 (b) $2x$ (c) $4x$ (d) $4x^2$

6 The algebraic term $4^2 a b^2$ of the .... degree.

(a) second (b) fifth (c) third (d) fourth

2 Complete :

1 The greatest negative integer is ....

2 If a = 3 , b = 6 , then 2 a b = ...

3 The mode of : 8 , 7 , 6 , 4 , 7 , 9 is ....

4 $\{4, 5, 6\} \cap \{5, 6, 7\} = \cdots\cdots$

5 $\frac{2}{5} \div \cdots\cdots = \frac{2}{5} \times \frac{3}{7}$

3 [a] **Divide :** $18 a^4 b^3 + 14 a^3 b^3 - 6 a^2 b^3$ **by** $2 a^2 b^2$ where $a \neq 0$, $b \neq 0$

[b] **Simplify :** $(x+3)(x+4) - 12$ **, then find the numerical value of the result when :** $x = -1$

4 [a] **Find using the distributive property :** $\frac{4}{9} \times 8 + \frac{4}{9} \times 11 - \frac{4}{9}$

[b] **Subtract :** $2a + 3b - 1$ **from** $5a - 6b + 4$

5 [a] Write 3 rational numbers between $\frac{2}{7}$ and $\frac{3}{4}$

[b] **In the following table :**

Height in cm.	100	110	120	130	140
Number of children	6	9	4	5	6

Find the mode height.

[c] **In the following table :**

Name	Ali	Hany	Sara	Mona	Nour
Marks	25	30	35	45	40

Find the mean of marks.

13 Damietta Governorate — Maths Supervision

Answer the following questions :

1 Choose the correct answer from those given :

1 If $\frac{5}{x+2}$ is a rational number , then $x \neq \cdots\cdots$

(a) -2 (b) 0 (c) 2 (d) 5

2 The algebraic term : $2 x^3$ is of the .... degree.

(a) second (b) third (c) fourth (d) fifth

3 The median of the values : 5 , 7 , 11 , 3 , 9 is ....

(a) 5 (b) 7 (c) 9 (d) 11

4 The value of number 5 in the number 0.2457 is ....

(a) $\frac{5}{10}$ (b) $\frac{5}{100}$ (c) $\frac{5}{1000}$ (d) $\frac{5}{10000}$

5 If $(x - 7)(x + 7) = x^2 + m$, then m = ...

(a) 0 (b) 14 (c) 49 (d) – 49

6 The number of rational numbers lying between $\frac{1}{5}$, $\frac{3}{5}$ is ....

(a) 1 (b) 2 (c) 3 (d) infinite number.

2 Complete the following :

1 If $\frac{3}{4} + x = 0$, then $x =$ ....

2 7.1 kilogram = ... gram.

3 The remainder of subtracting – 3 A from 2 A is ....

4 If the mode of the values : 8 , 9 , k + 2 , 6 is 8 , then k = ...

5 $(10 y^5 - 2 y^2) \div 2 y^2 =$ .... – 1 (where $y \neq 0$)

3

[a] **Add :** $2 x + 3 y - 3$ **and** $5 x - 2 y + 1$

[b] **Using the distribution property , find :** $\frac{5}{12} \times 7 + \frac{5}{12} \times 6 - \frac{5}{12}$

4

[a] **Reduce to the simplest form :** $(x + 2)^2 - 4 x$, then find the numerical value of the result when $x = 3$

[b] **Factorize by identifying the highest common factor :** $5 y^3 + 35 x y$

5

[a] **Find the quotient of :** $x^2 + 7 x + 10$ **by** $x + 5$ (where $x \neq -5$)

[b] **The following table shows the marks of a student in maths :**

Month	Oct.	Nov.	Dec.	Feb.	Mar.	Apr.
Mark	25	25	29	25	28	30

Find : 1 The mode mark. 2 The arithmetic mean of these marks.

14 Beni Suef Governorate — Maths Supervision

Answer the following questions :

1 Choose the correct answer from those given :

1 The number $\frac{3}{X-5}$ is a rational number if $X \neq$ ....

(a) -5 (b) -3 (c) 3 (d) 5

2 $\frac{1}{2}$ .... $\left|-\frac{3}{2}\right|$

(a) $>$ (b) $<$ (c) $=$ (d) $\geq$

3 $1\frac{3}{4} =$ .... %

(a) 25 (b) 50 (c) 75 (d) 175

4 If $(X-3)(X+3) = X^2 + k$, then $k =$ ....

(a) -9 (b) -6 (c) 6 (d) 9

5 $\frac{24}{5} =$ ....

(a) $2\frac{4}{5}$ (b) $3\frac{2}{5}$ (c) $4\frac{1}{5}$ (d) $4\frac{4}{5}$

6 The mode of the values : 7 , 10 , 4 , 7 , 3 , 4 , 3 , 10 , 3 is ....

(a) 3 (b) 4 (c) 7 (d) 10

2 Complete :

1 The rational number that lies at one fifth of the way between $\frac{5}{6}$ and $\frac{2}{3}$ from the smaller is .... in the simplest form.

2 If the mode of the values : 6 , 3 , k + 2 , 3 , 6 is 6 , then k = ...

3 $5X^2 + 3$ is an algebraic expression of the .... degree.

4 $(-5) \times [7 + (-3)] =$ .... in the simplest form.

5 If the arithmetic mean of the numbers : 9 , 4 , 5 , X is 5 , then $X =$ ....

3 [a] 1 **Find the value of :** $\left[\frac{3}{2} + \left(-\frac{1}{4}\right)\right] \div \left[-\frac{1}{4} + 2\right]$ in the simplest form.

2 **Find a rational number between** $-\frac{1}{5}$ and $-\frac{1}{9}$

[b] **Use the distribution property to find the value of :**

$\frac{5}{9} \times 4 + \frac{5}{9} \times 6 - \frac{5}{9}$ in the simplest form.

4 **[a] Subtract :** $3y^2 - 2xy + x^2$ **from** $y^2 - 5xy + 3x^2$

[b] Factorize by identifying the H.C.F. : $10a^5x^2 + 15a^4x^4 - 30a^3x^3$

5 **[a] Find the quotient of :** $x^2 + 5x + 6$ **by** $x + 3$ (where $x \neq -3$)

[b] The following table shows the marks of Gehad in maths tests in 6 months :

The month	October	November	December	February	March	April
The mark	40	35	47	37	44	48

Find : The median of the previous marks.

15 Aswan Governorate

Kom Ombo Educational Directorate
Al-Qahmury English The formal Language School

Answer the following questions :

1 **Choose the correct answer :**

1 If $\frac{5}{x+2}$ is a rational number , then $x \neq$ ....

(a) -2 (b) 0 (c) 2 (d) 5

2 If $(x-3)(x+3) = x^2 + k$, then $k =$ ....

(a) 9 (b) 6 (c) -9 (d) 3

3 The mode of the values : 7 , 5 , k + 4 , 5 , 7 is 5 , then k = ...

(a) 1 (b) 4 (c) 5 (d) 7

4 The coefficient of the algebraic term : $2x^3$ is ....

(a) 2 (b) 3 (c) 4 (d) 5

5 If $\frac{a}{b} = 1$, then $2a - 2b =$ ....

(a) 3 (b) 2 (c) 1 (d) 0

6 If $\frac{4}{7}y = \frac{4}{7}$, then $y =$ ....

(a) 1 (b) 0 (c) 4 (d) 7

2 **Complete the following :**

1 The multiplicative inverse of the number $\frac{-2}{3}$ is ....

2 The arithmetic mean of the set of values : 1 , 6 , 4 , 8 , 6 is ....

3 $0.18 - 30\% =$ ....

4 The median of the set of values : 4 , 8 , 3 , 5 , 7 is ....

5 1 , 1 , 2 , 3 , 5 , 8 , ... "in the same pattern"

---

3 **[a] Use the distribution property to find :** $\frac{27}{16} \times \frac{11}{7} + \frac{27}{16} \times \frac{11}{7} - \frac{27}{16} \times \frac{6}{7}$

[b] Factorize by identifying the H.C.F. : $5\ a^3\ b^2 + 35\ a^2\ b^3$

---

4 **[a]** Write three rational numbers between $\frac{4}{5}$ and $\frac{2}{3}$

[b] What is the increase of : $3\ x - 5\ y + 1$ **than** $x + 2\ y - 3$?

---

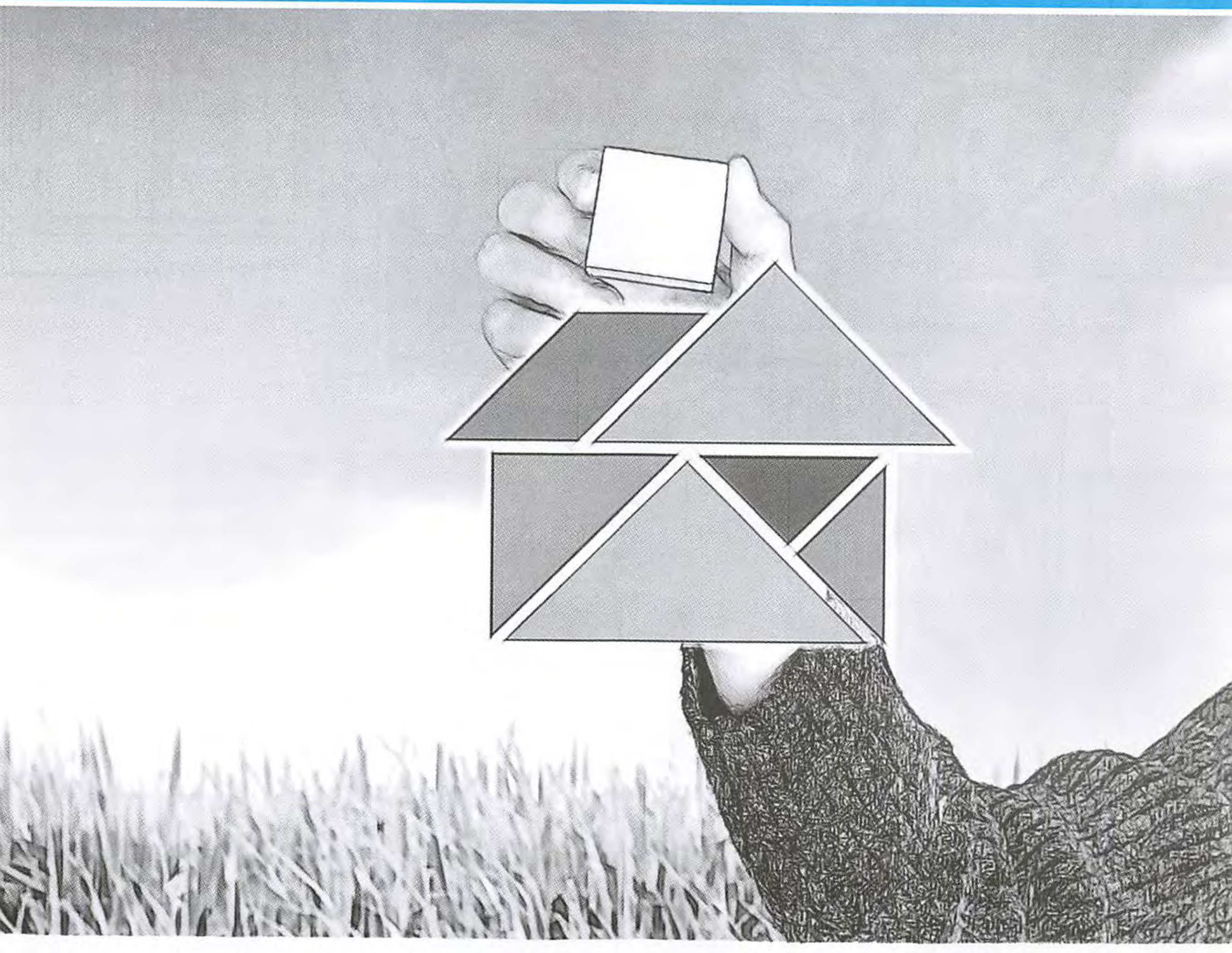
5 **[a] Divide :** $10\ x^2\ y - 4\ x\ y^2 + 2\ x\ y$ **by** $2\ x\ y$ where $x\ y \neq 0$

[b] The following table shows distribution of marks of 30 students in a test :

The marks	4	6	9	12	15	18	**Total**
Number of students	6	13	16	7	5	3	**50**

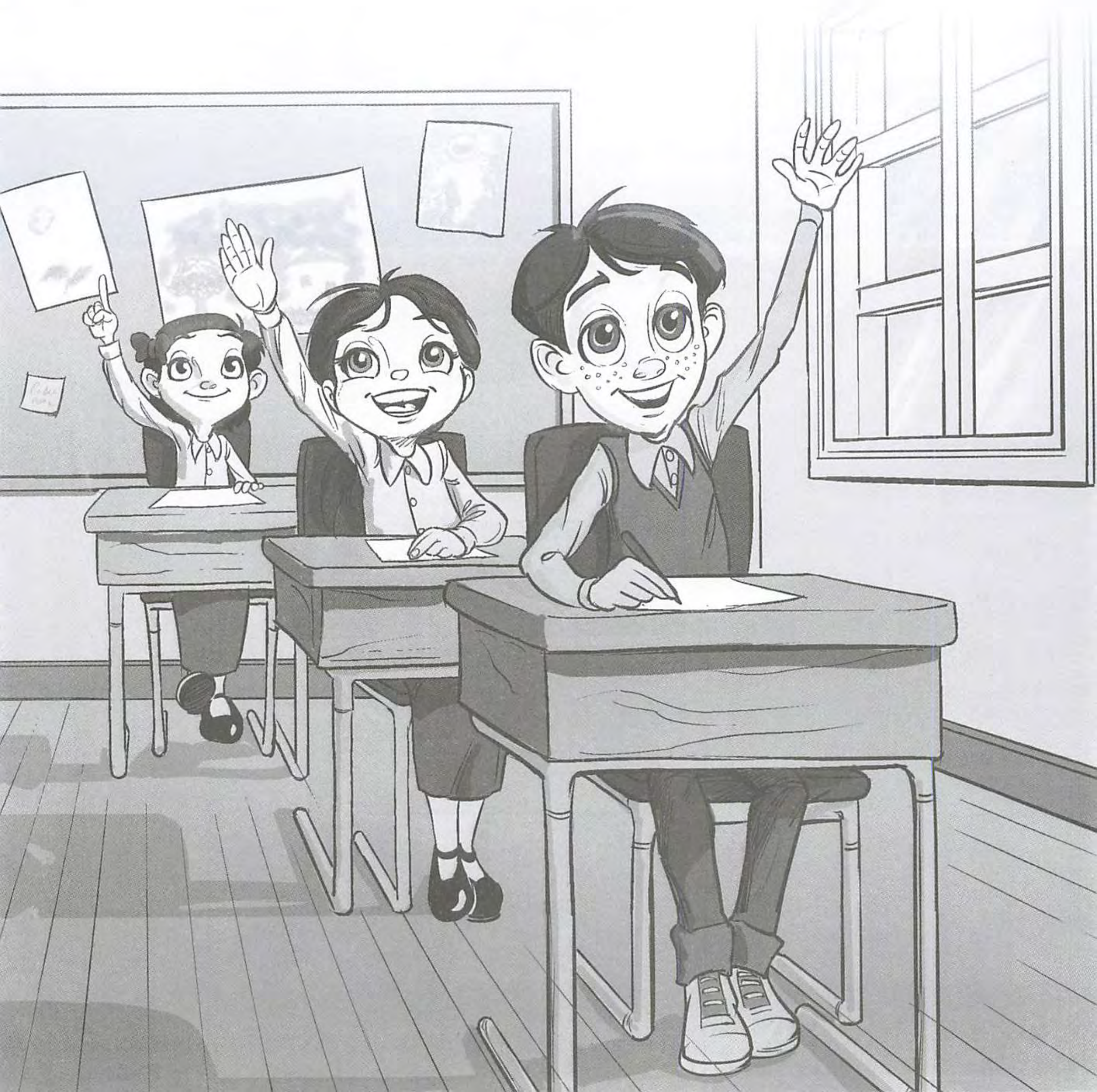
Find the mode mark.

Second Geometry



Accumulative Tests

on Geometry



Accumulative test 1 on lesson 1 – unit 4

1 **Choose the correct answer from those given :**

1 The angle whose measure is 30° supplements an angle of measure ....

(a) 40° (b) 50° (c) 120° (d) 150°

2 The two complementary equal angles , the measure of each angle is ....

(a) 180° (b) 45° (c) 360° (d) 90°

3 If m (∠ A) = 80° , then m (reflex ∠ A) = ...

(a) 10° (b) 100° (c) 180° (d) 280°

4 If a line segment is extended from only one terminal without limit , then it is ....

(a) a line segment. (b) a ray. (c) a straight line. (d) an angle.

5 The two adjacent complementary angles , their outer sides are ....

(a) perpendicular. (b) parallel. (c) collinear. (d) coincided.

6 The type of the angle of measure $89° \, 53'$ is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

7 If the ratio between the measures of two supplementary angles is 4 : 5 , then the measure of the greater angle is ....

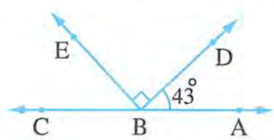
(a) 80° (b) 90° (c) 100° (d) 120°

8 The acute angle supplements the .... angle.

(a) right (b) acute (c) obtuse (d) straight

---

2 **In the opposite figure :**



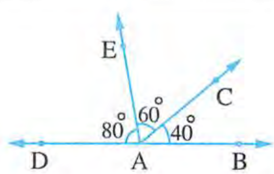
m (∠ DBE) = 90°

, m (∠ ABD) = 43°

Find : m (∠ EBC) , m (∠ DBC)

---

3 **In the opposite figure :**



m (∠ BAC) = 40° , m (∠ CAE) = 60°

, m (∠ EAD) = 80°

Are $\overrightarrow{AD}$, $\overrightarrow{AB}$ on the same straight line ?

Give reason.

Accumulative test 2 till lesson 2 – unit 4

1 Choose the correct answer from those given :

1. If the two vertically opposite angles are complementary , then the measure of each angle equals ....
   (a) 90° (b) 180° (c) 45° (d) 360°

2. The two bisectors of the two supplementary adjacent angles are ....
   (a) parallel. (b) perpendicular.
   (c) coincided. (d) including an acute angle.

3. If m (∠ A) = 2 m (∠ B) , ∠ A supplements ∠ B , then m (∠ B) = ...
   (a) 30° (b) 60° (c) 90° (d) 120°

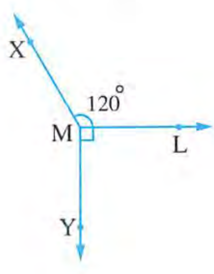
4. If ∠ A complements ∠ B , ∠ A complements ∠ C , then m (∠ B) .... m (∠ C)
   (a) > (b) < (c) = (d) ≥

5. The sum of measures of the accumulative angles at a point is ....
   (a) 90° (b) 4 rights (c) 180° (d) 270°

6. If $\overrightarrow{BD}$ bisects ∠ ABC , then m (∠ ABC) .... m (∠ ABD)
   (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) 3

7. The right angle supplements .... angle.
   (a) a zero (b) an acute (c) a right (d) an obtuse

8. **In the opposite figure :**
   m (∠ XMY) = ...
   (a) 100°
   (b) 150°
   (c) 160°
   (d) 140°

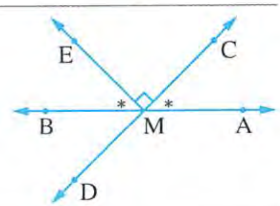


2 In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, m (∠ CME) = 90°

, m (∠ AMC) = m (∠ EMB)

Find : m (∠ AMC) , m (∠ AMD)

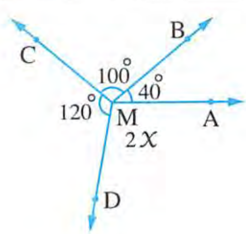


3 In the opposite figure :

m (∠ AMB) = 40° , m (∠ BMC) = 100°

, m (∠ CMD) = 120° , m (∠ AMD) = 2 X

Find : the value of X



Accumulative test 3 till lesson 3 – unit 4

1 **Choose the correct answer from those given :**

1 If $\angle$ X complements $\angle$ Y , $\angle X \equiv \angle Y$, then m ($\angle$ Y) = ...

(a) 45° (b) 90° (c) 180° (d) 30°

2 The two vertically opposite angles are ....

(a) complementary. (b) supplementary

(c) adjacent. (d) congruent.

3 If $\angle$ A complements $\angle$ B , $\angle$ B supplements $\angle$ C , m ($\angle$ A) = 35°
, then m ($\angle$ C) = ...

(a) 55° (b) 145° (c) 125° (d) 130°

4 If AB = CD , then $\overline{AB}$ .... $\overline{CD}$

(a) $\equiv$ (b) = (c) $\perp$ (d) bisects

5 If $\overline{AB} \equiv \overline{EF}$, then AB + EF = ...

(a) 1 (b) zero (c) 2 AB (d) AB

6 If the polygon ABCD $\equiv$ the polygon XYZL , then BC = ...

(a) XY (b) YZ (c) ZL (d) XL

7 If $\overline{AB} \equiv \overline{XY}$, then $\frac{AB}{XY}$ = ...

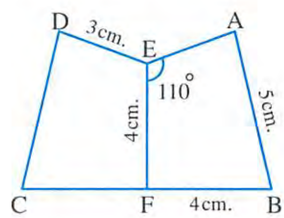
(a) zero (b) 5 (c) 4 (d) 1

8 If $\overrightarrow{XY}$ bisects $\angle$ LXN , m ($\angle$ LXY) = 60° , m ($\angle$ LXN) = ...

(a) 30° (b) 60° (c) 120° (d) 360°

---

2 **In the opposite figure :**



If F $\in \overline{BC}$, the figure ABFE $\equiv$ the figure DCFE

Complete the following :

1 The axis of symmetry of the figure is ....

2 AE = ... cm.

3 $\angle D \equiv \angle$ ....

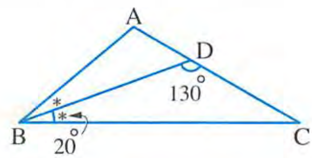
4 m ($\angle$ FED) = ...

5 m ($\angle$ EFB) = ...

6 The perimeter of the figure ABCDE = ... cm.

---

3 **In the opposite figure :**



$\overrightarrow{BD}$ bisects $\angle$ ABC , m ($\angle$ DBC) = 20°
, m ($\angle$ CDB) = 130°

Find : m ($\angle$ A)

Accumulative test 4 till lesson 4 – unit 4

1 **Choose the correct answer from those given :**

1 If $\Delta ABC \equiv \Delta XYZ$, $m(\angle X) + m(\angle Y) = 140°$, then $m(\angle C) =$ ....

(a) $180°$ (b) $140°$ (c) $90°$ (d) $40°$

2 If $\Delta ABC \equiv \Delta XYZ$, $m(\angle B) = 50°$, $m(\angle X) = 70°$, then $m(\angle C) =$ ....

(a) $50°$ (b) $60°$ (c) $70°$ (d) $120°$

3 If $\Delta ABC \equiv \Delta XYZ$, then ....

(a) $AB = YZ$ (b) $BC = XZ$ (c) $YX = CA$ (d) $ZY = CB$

4 The sum of measures of four accumulative angles at a point .... the sum of measures of five accumulative angles at a point.

(a) $<$ (b) $>$ (c) $=$ (d) $\neq$

5 The two triangles are congruent if .... in one triangle is congruent to its corresponding in the other triangle.

(a) each angle (b) each side (c) a side (d) an angle

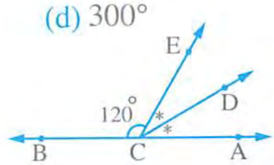
6 If $\Delta ABC \equiv \Delta DEF$, the perimeter of $\Delta ABC = 18$ cm. , $BC = 6$ cm. , then $DE + DF =$ ....

(a) 6 cm. (b) 12 cm. (c) 3 cm. (d) 24 cm.

7 If $\angle X$ supplements $\angle Y$, $m(\angle X) = 60°$, then $m(\text{reflex } \angle Y) =$ ....

(a) $120°$ (b) $180°$ (c) $240°$ (d) $300°$

8 **In the opposite figure :**



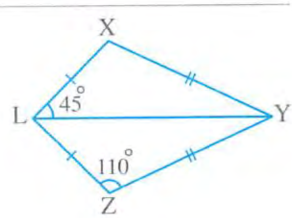
If $m(\angle ECB) = 120°$, $\overrightarrow{CD}$ bisects $\angle ACE$

, then $m(\angle DCA) =$ ....

(a) $60°$ (b) $30°$ (c) $240°$ (d) $180°$

---

2 **In the opposite figure :**



$XY = ZY$, $XL = ZL$

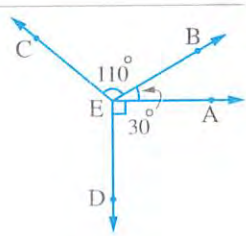
, $m(\angle Z) = 110°$, $m(\angle XLY) = 45°$

1 Mention the conditions of the congruence of ΔXYL , ΔZYL

2 Find : $m(\angle X)$, $m(\angle XYZ)$

---

3 **In the opposite figure :**



If $m(\angle AEB) = 30°$, $m(\angle BEC) = 110°$

, $m(\angle AED) = 90°$

Find : $m(\angle CED)$

Accumulative test 5 till lesson 5 – unit 4

1 **Choose the correct answer from those given :**

1. If a straight line intersects two parallel straight lines , then each two .... angles are equal in measure.

   (a) interior (b) complementary (c) acute (d) corresponding

2. L , M , N are three straight lines , $L \perp M$, $N \perp M$, then ....

   (a) $L \perp N$ (b) $M // L$ (c) $M // N$ (d) $L // N$

3. If $\overleftrightarrow{AB} // \overleftrightarrow{XY}$, then $\overleftrightarrow{AB} \cap \overleftrightarrow{XY} =$ ....

   (a) X (b) $\{0\}$ (c) B (d) $\varnothing$

4. If two straight lines are parallel to a third straight line , then these straight lines are ....

   (a) perpendicular. (b) parallel. (c) intersecting. (d) coincided.

5. If ABCD is a rectangle , then $\overline{BC} \equiv$ ....

   (a) $\overline{AC}$ (b) $\overline{BD}$ (c) $\overline{AD}$ (d) $\overline{DC}$

6. If m $(\angle A) = 200$, then the type of $\angle A$ is ....

   (a) straight. (b) right. (c) reflex. (d) obtuse.

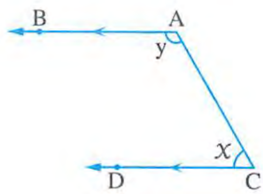
7. **In the opposite figure :**

   $\overrightarrow{AB} // \overrightarrow{CD}$, if $\frac{x}{y} = \frac{7}{11}$

   , then $x =$ ....

   (a) 60° (b) 70°

   (c) 100° (d) 110°

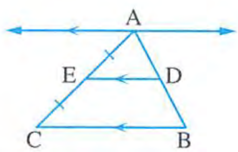


8. **In the opposite figure :**

   AD : AB = ...

   (a) 1 : 1 (b) 1 : 2

   (c) 1 : 3 (d) 1 : 4



---

2 **In the opposite figure :**

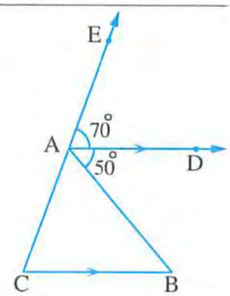
$\overrightarrow{AD} // \overrightarrow{BC}$

, $E \in \overrightarrow{CA}$

, m $(\angle DAE) = 70°$

, m $(\angle DAB) = 50°$

Find : the measures of the angles of Δ ABC

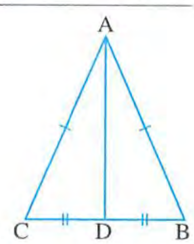


---

3 **In the opposite figure :**

AB = AC , BD = CD

Prove that : $\overrightarrow{AD}$ bisects $\angle A$



Accumulative test 6 till lesson 6 – unit 4

1 **Choose the correct answer from those given :**

1. The axis of symmetry of a line segment is ....
   (a) parallel to it. (b) equal to it.
   (c) perpendicular to it from its midpoint. (d) congruent to it.

2. If m (∠ A) = 2 m (∠ B) , ∠ A complements ∠ B , then m (∠ A) = ...
   (a) 15° (b) 30° (c) 45° (d) 60°

3. If the two adjacent angles are supplementary , then their outer sides are ....
   (a) perpendicular. (b) coincided.
   (c) parallel. (d) on the same straight line.

4. If two coplaner straight lines are perpendicular to a third one , then the two straight lines are ....
   (a) intersecting. (b) perpendicular. (c) parallel. (d) coincided.

---

2 Draw Δ ABC where AB = AC = 5 cm. , BC = 6 cm. , then draw $\overline{AD} \perp \overline{BC}$ where $\overline{AD} \cap \overline{BC} = \{D\}$

Find : the length $\overline{AD}$ by measuring. (Don't remove the arcs)

---

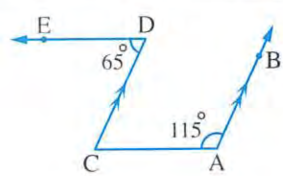
3 Using the geometric tools , draw $\overline{AB}$ of length 7 cm. , then draw its axis of symmetry. (Don't remove the arcs)

---

4 Using the geometric tools , draw ∠ ABC of measure 110° , then draw $\overrightarrow{BF}$ to bisect it. (Don't remove the arcs)

---

5 **In the opposite figure :**



$\overrightarrow{AB} // \overrightarrow{CD}$, m (∠ A) = 115°

, m (∠ D) = 65°

Find : m (∠ C) , then prove that : $\overline{AC} // \overrightarrow{DE}$

Important Questions

on Geometry

Important questions on Unit Four

First Multiple choice questions

1 If $\angle X$ supplements $\angle Y$, m ($\angle X$) = 60° , then m ($\angle Y$) = ……………

(a) 60° (b) 120° (c) 30° (d) 90°

---

2 The two complementary equal angles in measure , the measure of each angle is ……………

(a) 180° (b) 45° (c) 360° (d) 90°

---

3 The angle of measure $x°$ complements the angle of measure ……………

(a) $180° - x°$ (b) $90° - x°$ (c) $360° - x°$ (d) $90° + x°$

---

4 The angle of measure 70° is vertically opposite to an angle of measure ……………

(a) 20° (b) 110° (c) 70° (d) 360°

---

5 If m ($\angle A$) = 80° , then m (reflex $\angle A$) = ……………

(a) 360° (b) 180° (c) 100° (d) 280°

---

6 The measure of the straight angle = ……………

(a) 360° (b) 180° (c) 170° (d) 90°

---

7 The outer sides of the two adjacent complementary angles are ……………

(a) perpendicular. (b) coincided. (c) parallel. (d) collinear.

---

8 If $\angle X$ complements $\angle Y$, m ($\angle X$) = m ($\angle Y$) , then m ($\angle X$) = ……………

(a) 45° (b) 90° (c) 180° (d) 135°

---

9 The sum of measures of the accumulative angles at a point equals ……………

(a) 90° (b) 180° (c) 270° (d) 360°

---

10 m ($\angle A$) + m (reflex $\angle A$) = ……………

(a) 360° (b) 180° (c) 90° (d) 120°

---

11 The two supplementary equal angles , the measure of each angle is ……………

(a) 90° (b) 180° (c) 45° (d) 360°

12 The two bisectors of the two adjacent supplementary angles are ....

(a) parallel. (b) perpendicular.

(c) two axes of symmetry. (d) coincided.

---

13 The two straight lines are parallel to a third straight line , then they are ....

(a) perpendicular. (b) coincided. (c) parallel. (d) intersecting.

---

14 If Δ ABC $\equiv$ Δ XYZ , then AB = ...

(a) XY (b) YZ (c) XZ (d) BC

---

15 The two straight lines are perpendicular to a third straight line , then they are ....

(a) coincided. (b) perpendicular. (c) parallel. (d) intersecting.

---

16 If Δ ABC $\equiv$ Δ XYZ , m ($\angle$ A) + m ($\angle$ B) = 140° , then m ($\angle$ Z) = ...

(a) 100° (b) 40° (c) 80° (d) 140°

---

17 If $\overline{BC} \equiv \overline{XY}$, then BC ÷ XY = ...

(a) 2 (b) zero (c) 1 (d) XY

---

18 If the two vertically opposite angles are supplementary , then the measure of each angle is ....

(a) 45° (b) 90° (c) 180° (d) 60°

---

19 If the two vertically opposite angles are complementary , then the measure of each angle is ....

(a) 90° (b) 45° (c) 180° (d) 50°

---

20 If Δ ABC $\equiv$ Δ XYZ , m ($\angle$ A) = 60° , then m ($\angle$ X) = ...

(a) 20° (b) 120° (c) 60° (d) 30°

---

21 If Δ ABC $\equiv$ Δ LMN , then m ($\angle$ ABC) = m ($\angle$ ....)

(a) LMN (b) MNL (c) LNM (d) NLM

---

22 If L_1 , L_2 are two straight lines where $L_1 \cap L_2 = \varnothing$, then the two straight lines are ....

(a) perpendicular. (b) parallel. (c) intersecting. (d) coincided.

23 The type of the angle of measure $89° \, 60'$ is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

---

24 The right angle supplements .... angle.

(a) an acute (b) a right. (c) an obtuse. (d) a reflex.

---

25 If the ratio between the measures of two supplementary angles is 7 : 11 , then the measure of the smaller angle = ...

(a) 55° (b) 110° (c) 135° (d) 70°

---

26 If $\angle$ A supplements $\angle$ B , m ($\angle$ A) = 2 m ($\angle$ B) , then m ($\angle$ B) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

---

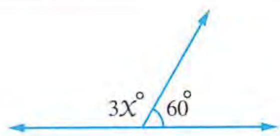
27 The straight line which is perpendicular to one of two parallel straight lines is .... to the other straight lines is the plane.

(a) perpendicular (b) parallel (c) coincided (d) otherwise

---

28 **In the opposite figure :**

x = ...



(a) 60° (b) 20°

(c) 40° (d) 80°

---

29 The axis of symmetry of a line segment is ....

(a) the perpendicular bisector to it. (b) parallel to it.

(c) equal to it. (d) congruent to it.

---

30 The sum of measures of the accumulative angles at a point equals the sum of measures of .... angles.

(a) 2 right (b) 3 right (c) 4 right (d) 5 right

---

31 If L_1 , L_2 , L_3 are straight lines in the same plane , $L_1 \perp L_3$, $L_2 \perp L_3$, then ....

(a) $L_1 // L_2$ (b) $L_1 \perp L_2$ (c) $L_1 // L_3$ (d) $L_2 // L_3$

---

32 If m (reflex $\angle$ A) = 240° , then the measure of the supplementary of $\angle$ A = ...

(a) 120° (b) 90° (c) 60° (d) 30°

---

33 The acute angle supplements .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

34 The angle whose measure is $95^\circ\ 60'$ supplements an angle of measure ……………

(a) 75° (b) 84° (c) 90° (d) 100°

---

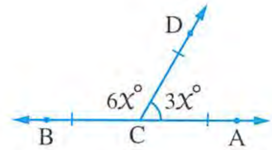
35 If $\angle X$ complements $\angle Y$, $3\ m(\angle X) + m(\angle Y) = 180^\circ$, then $m(\angle X) =$ ……………

(a) 60° (b) 120° (c) 45° (d) 90°

---

36 In the opposite figure :

If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{C\}$, then $x =$ ……………



(a) 30° (b) 20°

(c) 90° (d) 60°

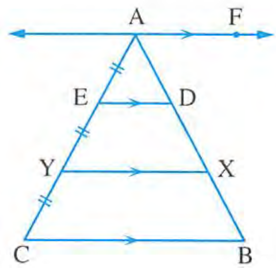
---

37 In the opposite figure :

$\overleftrightarrow{AF} \parallel \overline{ED} \parallel \overline{YX} \parallel \overline{CB}$

, $AE = EY = YC$

, then $AD : AB =$ ……………



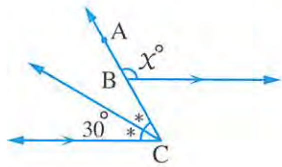
(a) $2 : 1$ (b) $1 : 2$

(c) $1 : 3$ (d) $2 : 3$

---

38 In the opposite figure :

$B \in \overline{AC}$, then $x =$ ……………



(a) 30° (b) 60°

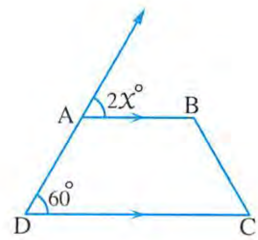
(c) 120° (d) 180°

---

39 In the opposite figure :

$\overline{AB} \parallel \overline{CD}$, $m(\angle D) = 60^\circ$

, then $x =$ ……………



(a) 30° (b) 60°

(c) 120° (d) 80°

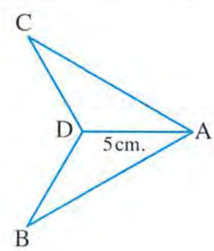
---

40 In the opposite figure :

If $\Delta ABD \equiv \Delta ACD$, $AD = 5$ cm.

, the perimeter of the figure ABDC = 30 cm.

, then the perimeter of $\Delta ABD =$ …………… cm.



(a) 35 (b) 20

(c) 30 (d) 15

Second Complete questions

1. The two outer sides of two adjacent complementary angles are ....

2. The two outer sides of two adjacent supplementary angles are ....

3. If m (∠ A) = 120° , then m (reflex ∠ A) = ...°

4. The perpendicular straight line to a line segment from its midpoint is called ....

5. The two right-angled triangles are congruent if ....

6. The two angles are congruent if they are ....

7. If the ratio between the measures of two complementary angles is 2 : 7 , then the measure of the greater angle = ...°

8. If a line segment is extended from both sides to infinity , then it is called ....

9. The obtuse angle supplements .... angle.

10. The two adjacent angles formed by a straight line and a ray with starting point on the straight line are ....

11. If C is the midpoint of $\overline{AB}$, then $\overline{AC} \equiv$ ....

12. The two triangles are congruent if .... and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

13. The two triangles are congruent if two angles and .... of one triangle are congruent to the corresponding parts of the other triangle.

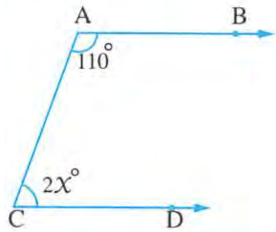
14. If a straight line intersects two parallel straight lines , then .... are supplementary.

15. If a straight line intersects two straight lines and there are two corresponding angles having the same measure , then the two straight lines are ....

16. If a straight line cuts two parallel straight lines , then each two altrenate angles are ....

17 **In the opposite figure :**

$x =$ ....



18 If the outer sides of two adjacent angles are on the same straight line , then the two angles are ....

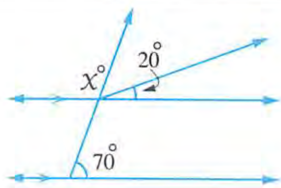
19 The perpendicular to one of two coplaner parallel straight line is .... to the other.

20 The axis of symmetry of a line segment is ....

21 If $\overline{AB} \equiv \overline{XY}$, then 3 AB – 3 XY = ...

22 **In the opposite figure :**

$x =$ ....



23 If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \varnothing$ and in the same plane , then ....

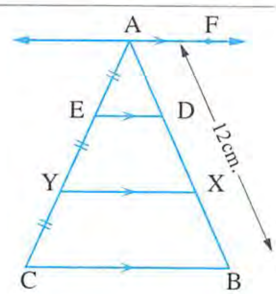
24 **In the opposite figure :**

$\overleftrightarrow{AF} \,//\, \overline{ED} \,//\, \overline{XY} \,//\, \overline{BC}$

, AE = EY = YC

, if AB = 12 cm.

, then AX = ... cm.



25 If an angle of measure 57° complements an angle of measure 3 a , then a = ...°

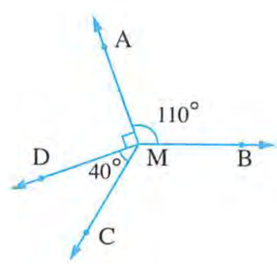
Third Essay questions

1 **In the opposite figure :**

m (∠ AMB) = 110° , m (∠ AMD) = 90°

, m (∠ DMC) = 40°

Find : m (∠ BMC)



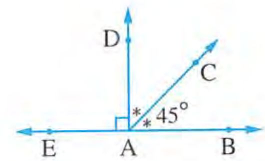
2 **In the opposite figure :**

$\overrightarrow{AC}$ bisects $\angle$ BAD

, m ($\angle$ DAE) = 90°

, m ($\angle$ BAC) = 40°

Show if the point B , A , E are on the same straight line

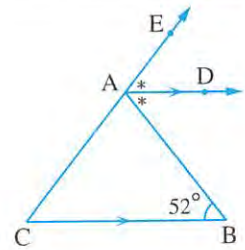


3 **In the opposite figure :**

$\overrightarrow{AD}$ // $\overline{CB}$

, $\overrightarrow{AD}$ bisects $\angle$ BAE , m ($\angle$ B) = 52°

Find : m ($\angle$ BAD) , m ($\angle$ C)



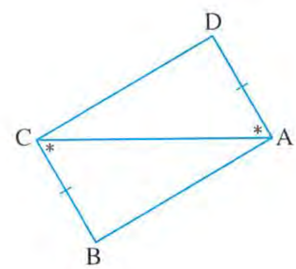
4 **In the opposite figure :**

AD = CB

, m ($\angle$ DAC) = m ($\angle$ BCA)

Prove that : 1 Δ ABC $\equiv$ Δ CDA

2 $\overline{AB}$ // $\overline{CD}$

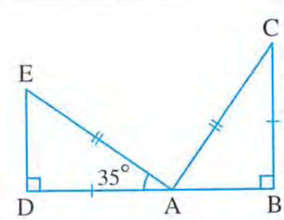


5 **In the opposite figure :**

m ($\angle$ DAE) = 35° , m ($\angle$ B) = m ($\angle$ D) = 90°

1 Prove that : Δ ABC $\equiv$ Δ EDA

2 Find with proof : m ($\angle$ C)

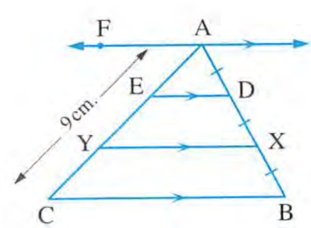


6 **In the opposite figure :**

$\overleftrightarrow{AF}$ // $\overline{DE}$ // $\overline{XY}$ // $\overline{BC}$

, AD = DX = XB , AC = 9 cm.

Find : the length of $\overline{AY}$

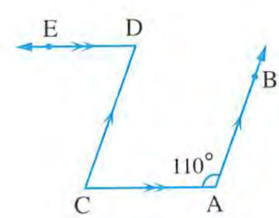


7 **In the opposite figure :**

$\overrightarrow{DE}$ // $\overline{AC}$, $\overrightarrow{AB}$ // $\overline{CD}$

, m ($\angle$ A) = 110°

Find : m ($\angle$ C) , m ($\angle$ D)

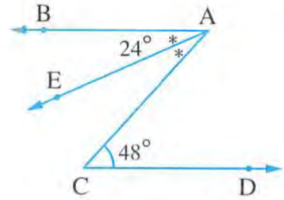


8 **In the opposite figure :**

$\overrightarrow{AE}$ bisects $\angle$ BAC , m ($\angle$ BAE) = 24°

, m ($\angle$ ACD) = 48°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{CD}$

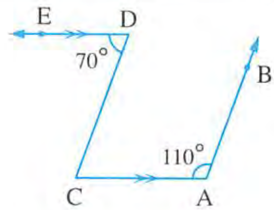


9 **In the opposite figure :**

$\overrightarrow{DE} \parallel \overline{AC}$, m ($\angle$ A) = 110°

, m ($\angle$ D) = 70°

Find : m ($\angle$ C) and is $\overrightarrow{AB} \parallel \overline{CD}$? State the reason.

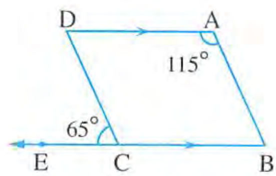


10 **In the opposite figure :**

$\overline{AD} \parallel \overrightarrow{BC}$, m ($\angle$ A) = 115°

, m ($\angle$ DCE) = 65°

Is $\overline{AB} \parallel \overline{DC}$? State the reason.

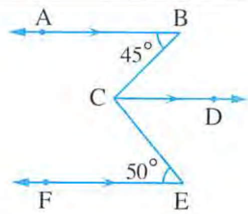


11 **In the opposite figure :**

$\overrightarrow{BA} \parallel \overrightarrow{CD} \parallel \overrightarrow{EF}$, m ($\angle$ B) = 45°

, m ($\angle$ E) = 50°

Find : m ($\angle$ BCE) giving the reason.



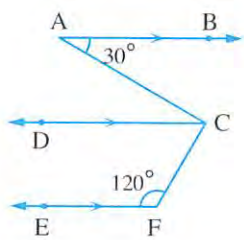
12 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD} \parallel \overrightarrow{FE}$

, m ($\angle$ A) = 30°

, m ($\angle$ F) = 120°

Find with reason : m ($\angle$ ACF)



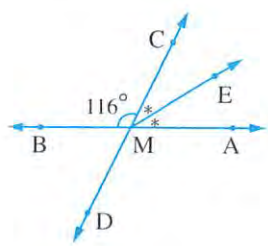
13 **In the opposite figure :**

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

, m ($\angle$ CMB) = 116°

, $\overrightarrow{ME}$ bisects $\angle$ AMC

Find : m ($\angle$ AMD) , m ($\angle$ AMC) , m ($\angle$ AME)

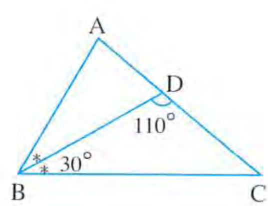


14 **In the opposite figure :**

m ($\angle$ CDB) = 110° , $\overrightarrow{BD}$ bisects $\angle$ CBA

, m ($\angle$ CBD) = 30°

Find : m ($\angle$ A)



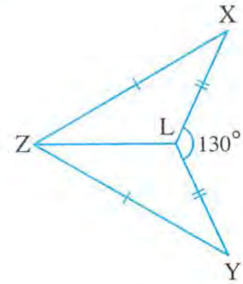
15 **In the opposite figure :**

$YZ = XZ$, $XL = YL$

, $m(\angle XLY) = 130°$

Prove that : $\Delta XLZ \equiv \Delta YLZ$

, then find : $m(\angle XLZ)$



16 Mention two cases of congruency of two triangles.

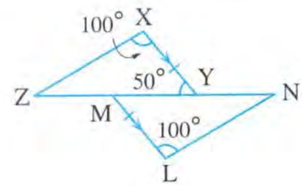
17 **In the opposite figure :**

$XY = LM$, $\overline{XY} // \overline{LM}$, $m(\angle L) = m(\angle X) = 100°$

, $m(\angle XYZ) = 50°$

1 Write the conditions of congruency of $\Delta\Delta$ XYZ , LMN

2 Find : $m(\angle N)$



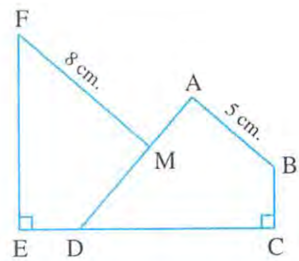
18 **In the opposite figure :**

$\overline{BC} \perp \overline{CD}$, $D \in \overline{CE}$

, the figure ABCD ≡ the figure MDEF

Find : 1 The length $\overline{AM}$

2 $m(\angle B) + m(\angle F)$



19 Using the geometric tools , draw $\angle ABC$ of measure 100° , then bisect it by the bisector $\overrightarrow{BD}$ (Don't remove the arcs)

20 Draw $\angle A$ of measure 120° , then divide it into 4 equal parts using the ruler and compasses.

21 Using the geometric tools , draw $\overline{XY}$ of length 8 cm. , then draw $\overleftrightarrow{DE}$ the axis of $\overline{XY}$ (Don't remove the arcs)

22 Draw ΔABC in which : $AB = AC = 5$ cm. , $BC = 6$ cm. , then draw $\overline{AD} \perp \overline{BC}$ using the ruler and compasses where $\overline{AD} \cap \overline{BC} = \{D\}$, then find : the length of $\overline{AD}$ (Don't remove the arcs)

23 Draw the equilateral triangle ABC of side length 4 cm. using the ruler and compasses draw the bisectors of $\angle ABC$, $\angle ACB$ to intersect at M , find : $m(\angle BMC)$ (Don't remove the arcs)

24 Draw an obtuse-angled triangle , then bisect each angle of its angles. (Don't remove the arcs)

Final Revision

of Geometry

The types of angles

Zero angle
Its measure is 0°

Obtuse angle
Its measure is more than 90° and less than 180°

Acute angle
Its measure is more than 0° and less than 90°

Straight angle
Its measure is 180°

Right angle
Its measure is 90°

Reflex angle
Its measure is more than 180° and less than 360°

The relation between the angles

The two vertically opposite angles are equal in measure.

Sum of measures of the accumulative angles at a point is 360°

Sum of measures of the two complementary angles is 90°

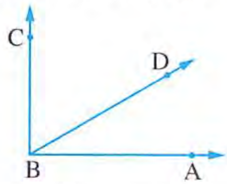
→ The complements of the same angle are equal in measure.

Sum of measures of the two supplementary angles is 180°

→ The supplements of the same angle are equal in measure.

The two adjacent angles

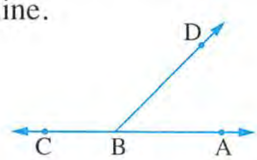
The two adjacent complementary angles :
Their outer sides are perpendicular.



If m (∠ ABD) + m (∠ DBC) = 90°

, then $\overline{AB} \perp \overline{BC}$

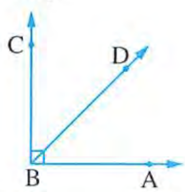
The two adjacent supplementary angles :
Their outer sides are on the same straight line.



If m (∠ ABD) + m (∠ DBC) = 180°

, then $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are on the same straight line.

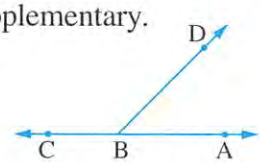
The two adjacent angles whose outer sides are perpendicular , are complementary



If $\overrightarrow{BA} \perp \overrightarrow{BC}$

, then m (∠ ABD) + m (∠ DBC) = 90°

The two adjacent angles formed by a straight line and a ray with a starting point on this straight line , are supplementary.



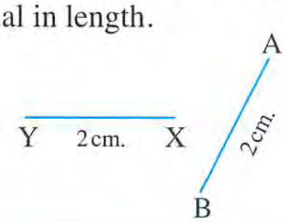
If B ∈ $\overleftrightarrow{AC}$

, then m (∠ ABD) + m (∠ DBC) = 180°

Congruence

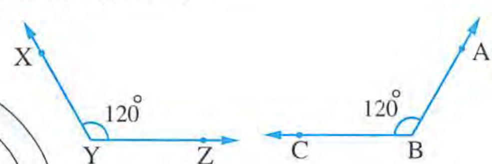
Congruence of two line segments

Two line segments are congruent if they are equal in length.



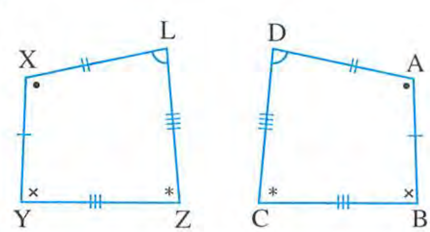
Congruence of two angles

Two angles are congruent if they are equal in measure.



Congruence of two polygons

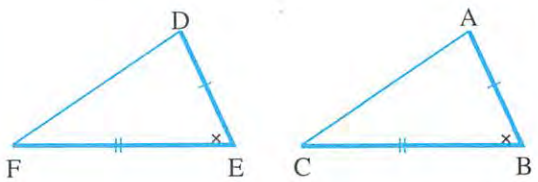
Two polygons are congruent if there is correspondence between their vertices such that each side and each angle in the first polygon is congruent to its corresponding element in the other polygon.



First case :

Two sides and the included angle (S.A.S.)

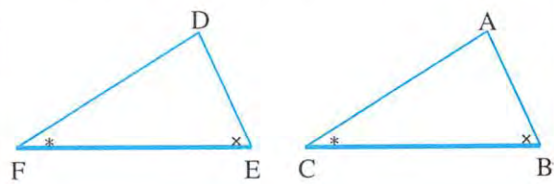
Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle.



Second case :

Two angles and one side (A.S.A.)

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle.

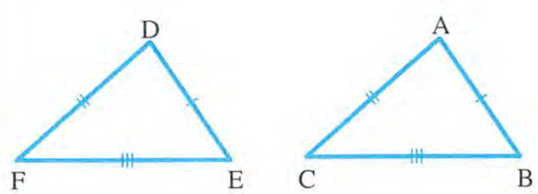


The cases of congruence of two triangles

Third case :

Three sides (S.S.S.)

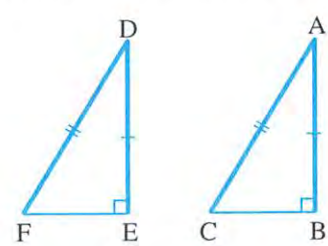
Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.



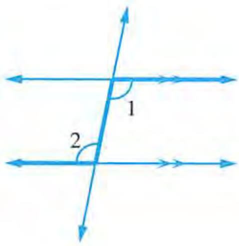
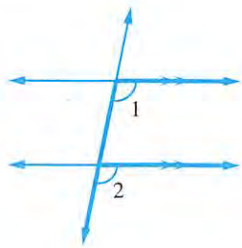
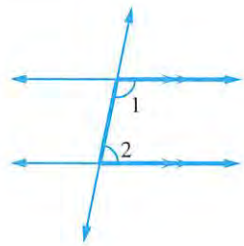
Fourth case :

Hypotenuse and one side in the right-angled triangle (R.H.S.)

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.



If a straight line intersects two parallel straight lines , then

Each two alternate angles are equal in measure.	**Each two corresponding angles are equal in measure.**	**Each two interior angles in the same side of the transversal are supplementary.**
		
For example : $m(\angle 1) = m(\angle 2)$ (alternate angles)	***For example :*** $m(\angle 1) = m(\angle 2)$ (corresponding angles)	***For example :*** $m(\angle 1) + m(\angle 2) = 180^\circ$

Remember **How to prove the parallelism of two straight lines**

The two straight lines are parallel if a third straight line intersects them and **one** of the following cases is satisfied :

1. Two alternate angles have the same measure.

or

2. Two corresponding angles have the same measure.

or

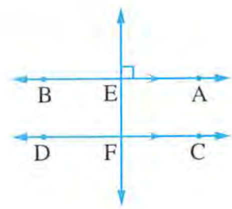
3. Two interior angles in the same side of the transversal are supplementary.

Remember that

The perpendicular to one of two coplaner parallel straight lines is perpendicular to the other.

If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{EF} \perp \overleftrightarrow{AB}$

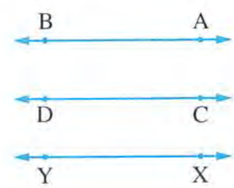
, then $\overleftrightarrow{EF} \perp \overleftrightarrow{CD}$



If two straight lines are parallel to a third straight line , then these two straight lines are parallel.

If $\overleftrightarrow{AB} \parallel \overleftrightarrow{XY}$, $\overleftrightarrow{CD} \parallel \overleftrightarrow{XY}$

, then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



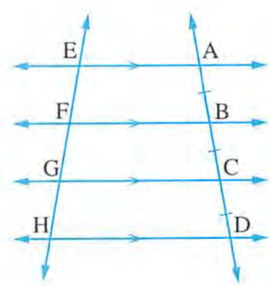
If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of equal lengths.

If $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{CG} \parallel \overleftrightarrow{DH}$

, $\overleftrightarrow{AD}$ and $\overleftrightarrow{EH}$ are transversals to them

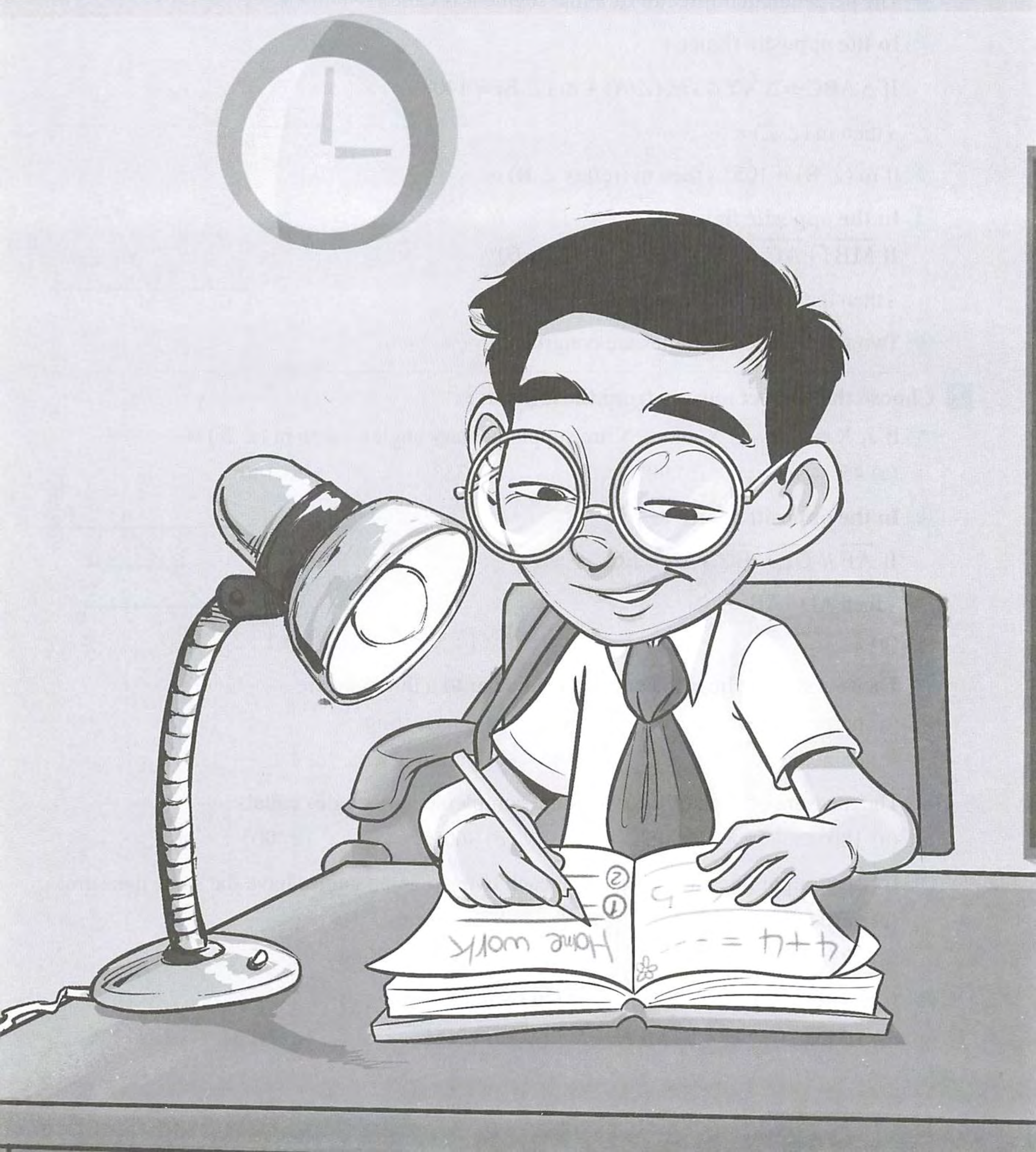
and AB = BC = CD

, then EF = FG = GH



Final Examinations

on Geometry



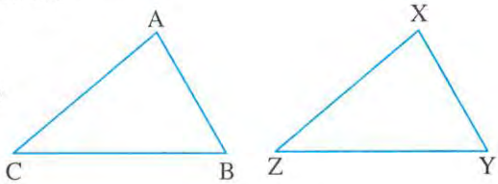
Model 1

Answer the following questions :

1 Complete each of the following :

1 The perpendicular bisector of a line segment is called ....

2 **In the opposite figure :**

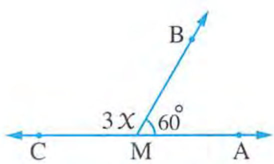


If $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) + m(\angle B) = 140°$

, then $m(\angle Z) =$ ....°

3 If $m(\angle B) = 105°$, then $m(\text{reflex } \angle B) =$ ....°

4 **In the opposite figure :**



If $\overrightarrow{MB} \cap \overleftrightarrow{AC} = \{M\}$, $m(\angle AMB) = 60°$

, then the value of X equals ....°

5 Two right-angled triangles are congruent if ....

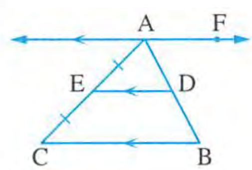
---

2 Choose the correct answer from those given :

1 If $\angle X \equiv \angle Y$, $\angle X$ and $\angle Y$ are supplementary angles , then $m(\angle X) =$ ....

(a) 45° (b) 90° (c) 135° (d) 180°

2 **In the opposite figure :**



If $\overleftrightarrow{AF} \,//\, \overline{DE} \,//\, \overline{BC}$, $AE = EC$

, then $AD : AB =$ ....

(a) 2 : 1 (b) 3 : 2 (c) 1 : 3 (d) 1 : 2

3 The two straight lines that are perpendicular to a third one are ....

(a) perpendicular. (b) intersecting.

(c) coincident. (d) parallel.

4 The measure of each of the two equal complementary angles equals ....

(a) 180° (b) 45° (c) 360° (d) 90°

5 If two straight lines intersect , then each two .... angles have the same measure.

(a) vertically opposite (b) adjacent

(c) alternate (d) corresponding

6 If $\Delta ABC \equiv \Delta LMN$, then $m(\angle ACB) = m(\angle$ ....$)$

(a) LMN (b) MLN (c) LNM (d) NLM

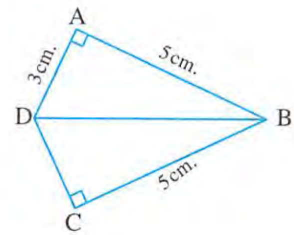
3 **[a] In the opposite figure :**

m (∠ BAD) = m (∠ BCD) = 90°

, AB = CB = 5 cm. , AD = 3 cm.

Mention the conditions for Δ ABD , Δ CBD to be congruent

, then find : The length of $\overline{CD}$

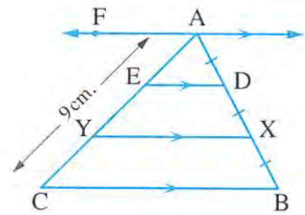


[b] In the opposite figure :

$\overleftrightarrow{AF}$ // $\overline{DE}$ // $\overline{XY}$ // $\overline{BC}$

, AD = DX = XB , AC = 9 cm.

Find : The length of $\overline{AY}$ (Give the reason)



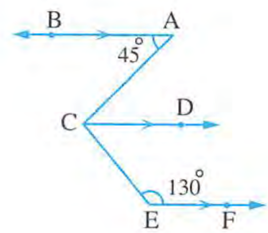
4 **[a] In the opposite figure :**

$\overrightarrow{AB}$ // $\overrightarrow{CD}$ // $\overrightarrow{EF}$

, m (∠ A) = 45°

, m (∠ E) = 130°

Find : m (∠ ACE)



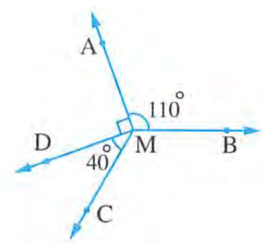
[b] In the opposite figure :

m (∠ AMB) = 110°

, m (∠ AMD) = 90°

, m (∠ DMC) = 40°

Find with steps : m (∠ BMC)



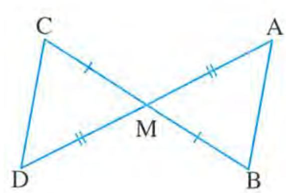
5 **[a] In the opposite figure :**

$\overline{AD} \cap \overline{BC} = \{M\}$

, BM = MC

, AM = MD

Write the conditions for Δ AMB , Δ DMC to be congruent.



[b] Using your geometric instruments , draw ∠ ABC of measure 110° , then draw $\overrightarrow{BF}$ to bisect the angle.

Model 2

Answer the following questions :

1 Complete each of the following :

1. The sum of the measures of the accumulative angles at a point equals ....°

2. If a straight line intersects two parallel straight lines , then each two corresponding angles are ....

3. If m (∠ A) = 110° , then m (reflex ∠ A) = ...°

4. Two right-angled triangles are congruent if ....

5. The two adjacent angles formed by the intersection of a straight line and a ray with a starting point on this straight line are ....

---

2 Choose the correct answer from those given :

1. If ∠ X complements ∠ Y and ∠ X ≡ ∠ Y , then m (∠ X) = ...

   (a) 45° (b) 90° (c) 180° (d) 360°

2. The number of triangles in the figure  equals ....

   (a) 4 (b) 6 (c) 7 (d) 8

3. If the ratio between the measures of two supplementary angles is 5 : 13 , then the measure of the smaller angle is ....

   (a) 50° (b) 130° (c) 150° (d) 180°

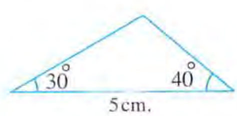
4. If Δ ABC ≡ Δ XYZ , m (∠ A) + m (∠ B) = 100° , then m (∠ Z) = ...

   (a) 50° (b) 80° (c) 90° (d) 100°

5. The two straight lines that are perpendicular to a third one are ....

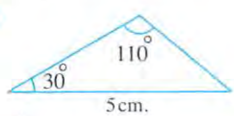
   (a) perpendicular. (b) parallel. (c) coincident. (d) intersecting.

6. The figure .... is not congruent to the opposite figure.

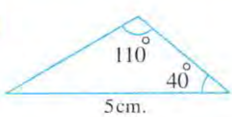


(a)

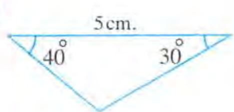
(b)



(c)



(d)



3 [a] Mention two cases of congruency of two triangles.

[b] **In the opposite figure :**

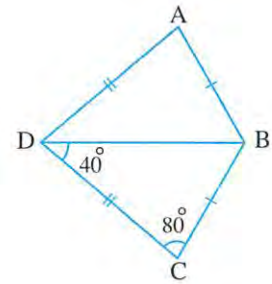
AB = BC , AD = DC

, m (∠ C) = 80°

, m (∠ BDC) = 40°

Prove that : Δ CBD ≡ Δ ABD

, then find : m (∠ ABD)



---

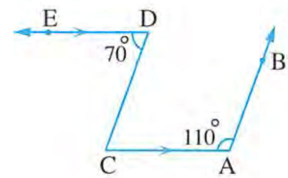
4 [a] **In the opposite figure :**

$\overrightarrow{DE}$ // $\overline{AC}$, m (∠ A) = 110°

, m (∠ D) = 70°

Find : m (∠ C)

Is $\overrightarrow{AB}$ // $\overline{CD}$? (Give the reason)



[b] Using the geometric instruments , draw ∠ ABC where m (∠ B) = 80° , then draw $\overrightarrow{BD}$ to bisect it. (Don't remove the arcs).

---

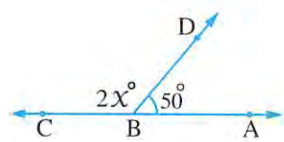
5 [a] **In the opposite figure :**

$\overleftrightarrow{AC} \cap \overrightarrow{BD} = \{B\}$

, m (∠ ABD) = 50°

, m (∠ DBC) = 2 x°

Find in degrees the value of x



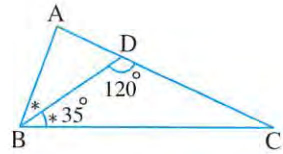
[b] **In the opposite figure :**

$\overrightarrow{BD}$ bisects ∠ ABC

, m (∠ DBC) = 35°

, m (∠ BDC) = 120°

Find : m (∠ A) in degrees.



Model examination for the merge students

Answer the following questions :

1 Complete each of the following :

1 If m (∠ A) = 100° , then m (reflex ∠ A) = ...°

2 The angle whose measure is 50° complements an angle of measure ....°

3 The two straight lines parallel to a third are ....

4 Two triangles are congruent if two sides and ....

5 If Δ ABC ≡ Δ XYZ , then m (∠ Z) = m (∠ ....)

---

2 Choose the correct answer from those given :

1 The sum of the measures of the accumulative angles at a point equals ....

(a) 630° (b) 180° (c) 90° (d) 360°

2 The axis of symmetry of a line segment is ....

(a) perpendicular to it from its midpoint. (b) parallel to it.

(c) equal to it in length. (d) congruent to it.

3 The supplement of the angle whose measure is 30° is an angle of measure ....

(a) 60° (b) 180° (c) 150° (d) 90°

4 The angle whose measure is more than 90° and less than 180° is .... angle.

(a) an obtuse (b) an acute (c) a right (d) a straight

5 If Δ ABC ≡ Δ XYZ , then AB = ...

(a) XY (b) XZ (c) YZ (d) BC

---

3 Put (✔) for the correct statement and (✘) for the incorrect statement :

1 The right-angled triangle is congruent to the equilateral triangle. ( )

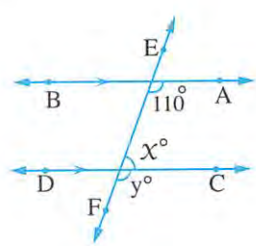
2 The two angles whose measures are 100° and 80° are supplementary. ( )

3 **From the opposite figure :**

(a) $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$ ( )

(b) $x = 70°$ ( )

(c) y = 180° ( )



4 **[a] In the opposite figure :**

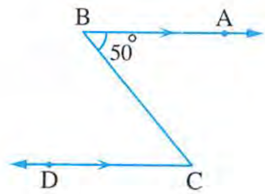
m (∠ ABC) = 50° , $\overrightarrow{BA}$ // $\overrightarrow{CD}$

Complete to find : m (∠ BCD)

$\overrightarrow{BA}$ // ....

, then m (∠ ABC) = m (∠ ....) (............... angles)

, m (∠ BCD) = ...°

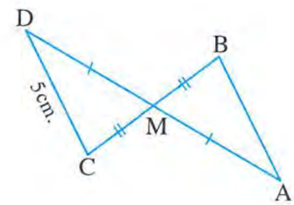


[b] From the opposite figure , complete :

1 Δ ABM ≡ Δ ....

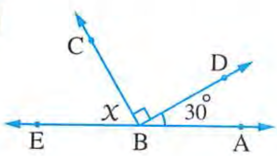
2 AB = ... cm.

3 m (∠ B) = m (∠ ....)



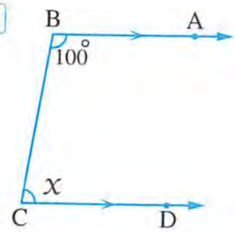
5 **[a] In each of the following figures , find the value of x :**

1



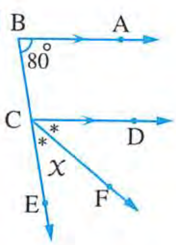
x = ...°

2



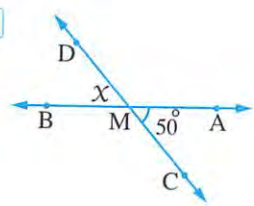
x = ...°

3



x = ...°

4



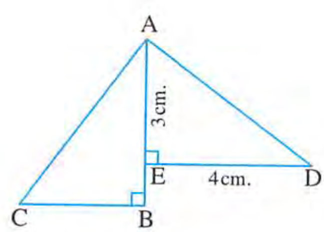
x = ...°

[b] In the opposite figure :

If Δ ABC ≡ Δ DEA ,

AE = 3 cm. and DE = 4 cm.

, **complete :** BE = ... cm.



1 Cairo Governorate

Nasr City Educational Zone
Thebes Language School

Answer the following questions :

1 Complete :

1 The angle whose measure is 50° complements an angle of measure ....

2 If Δ ABC $\equiv \Delta$ XYZ , then m ($\angle$ Z) = m ($\angle$ ....)

3 If a straight line intersects two parallel straight lines , then each two corresponding angles are ....

4 The sum of measures of the accumulative angles at a point is ....

5 If two adjacent angles are complementary , then their outer sides are ....

2 Choose the correct answer :

1 The angle whose measure is 179° is .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

2 The perpendicular bisector of a line segment is ....

(a) axis of symmetry. (b) parallel line.

(c) median. (d) coincident.

3 If $\overline{AB} \equiv \overline{CD}$, then AB – CD = ...

(a) 1 (b) 2 (c) zero (d) 5

4 If m ($\angle$ A) = 80° , then m ($\angle$ reflex A) = ...°

(a) 360 (b) 180 (c) 100 (d) 280

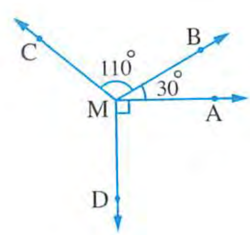
5 If $\angle$ X supplements $\angle$ Y and $\angle$ X $\equiv$ $\angle$ Y , then m ($\angle$ X) = ...°

(a) 45 (b) 90 (c) 135 (d) 180

6 The two straight lines parallel to a third straight line are ....

(a) intersecting. (b) parallel. (c) coincident. (d) perpendicular.

3 [a] In the opposite figure :

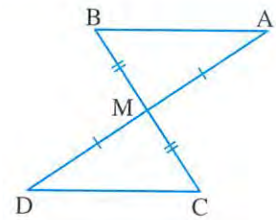


m ($\angle$ AMB) = 30° , m ($\angle$ BMC) = 110°

and m ($\angle$ AMD) = 90°

Find : m ($\angle$ CMD)

[b] In the opposite figure :



$\overline{AD} \cap \overline{BC} = \{M\}$

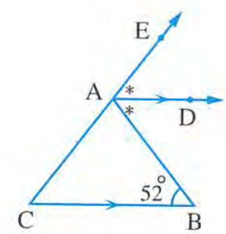
, BM = MC , AM = MD

Prove that : Δ AMB $\equiv$ Δ DMC

---

4 **[a]** Using the geometric tools , draw $\overline{XY}$ of length 8 cm. , then draw $\overleftrightarrow{DE}$ the axis of symmetry. (Don't remove the arcs)

[b] In the opposite figure :



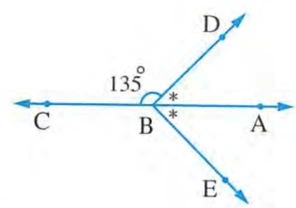
$\overrightarrow{AD}$ // $\overline{CB}$, $\overrightarrow{AD}$ bisects $\angle$ BAE

, m ($\angle$ B) = 52°

Find : m ($\angle$ BAD) , m ($\angle$ C)

---

5 **[a] In the opposite figure :**



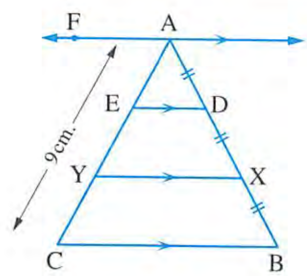
If B $\in \overleftrightarrow{AC}$, m ($\angle$ DBC) = 135°

and $\overrightarrow{BA}$ bisects $\angle$ DBE

Find : 1 m ($\angle$ ABD) 2 m ($\angle$ DBE)

3 m ($\angle$ CBE)

[b] In the opposite figure :



$\overleftrightarrow{AF}$ // $\overline{DE}$ // $\overline{XY}$ // $\overline{BC}$

, AD = DX = XB

, AC = 9 cm.

Find : The length of $\overline{AY}$

2 Cairo Governorate

El-Sayeda Zeinab Directorate
Mathematics Department

Answer the following questions :

1 **Choose the correct answer :**

1 The angle of measure 179° is .... angle.

(a) acute (b) obtuse (c) right (d) straight

2 If m ($\angle$ A) = 90° , then m (reflex $\angle$ A) = ...

(a) 0° (b) 90° (c) 180° (d) 270°

3 If Δ ABC $\equiv$ Δ XYZ , m ($\angle$ A) = 40° , m ($\angle$ Y) = 60° , then m ($\angle$ C) = ...

(a) 40° (b) 60° (c) 80° (d) 100°

4 The sum of measures of the accumulative angles at point equal .... right angles.

(a) 3 (b) 4 (c) 5 (d) 6

5 If Δ ABC $\equiv$ Δ XYZ , then XY = ...

(a) BC (b) AC (c) AB (d) YZ

2 Complete the following :

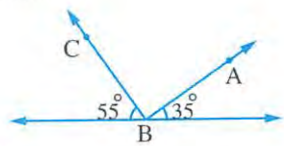
1 If two straight lines intersect , then each two vertically opposite angles are ....

2 If two straight lines are parallel to a third , then the two straight lines are ....

3 If two adjacent angles are supplementary , then their outer sides are ....

4 The angles of measure 35° complements an angles of measure ....

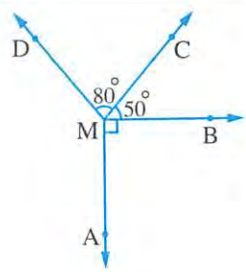
5 **In the opposite figure :**



m ($\angle$ ABC) = ...°

6 The sum of measures of the two supplementary angles is ....

3 [a] In the opposite figure :



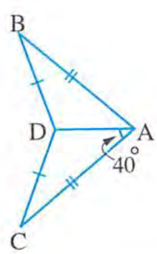
m ($\angle$ BMC) = 50°

, m ($\angle$ CMD) = 80°

, m ($\angle$ AMB) = 90°

Find : m ($\angle$ DMA)

[b] In the opposite figure :



AB = AC , BD = DC

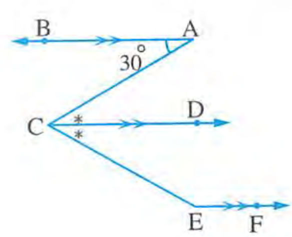
, m ($\angle$ CAD) = 40°

Prove that : Δ ABD $\equiv$ Δ ACD

Find : m ($\angle$ CAB)

4 [a] Draw an angles of measure 80° and bisect it (Don't remove the arcs).

[b] In the opposite figure :



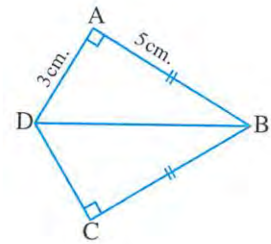
$\overrightarrow{AB}$ // $\overrightarrow{CD}$ // $\overrightarrow{EF}$

, m ($\angle$ A) = 30°

, $\overrightarrow{CD}$ bisects $\angle$ ACE

Find : m ($\angle$ ACD) , m ($\angle$ DCE) , m ($\angle$ CEF)

5 **[a] In the opposite figure :**



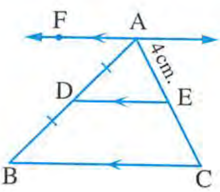
m (∠ A) = m (∠ C) = 90°

, AB = BC = 5 cm. , AD = 3 cm.

Prove that : Δ ABD ≡ Δ CBD

Find : The length of $\overline{CD}$

[b] In the opposite figure :



$\overleftrightarrow{AF}$ // $\overline{DE}$ // $\overline{BC}$

, AD = BD , AE = 4 cm.

Find : The length of $\overline{AC}$

3 Cairo Governorate

Good Shepherd Arabic Language School
Al-Khalifa and Al-Mokattam Directorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The sum of measures of the accumulative 4 angles at a point .... the sum of the measures of the accumulative 8 angles at a point.

(a) < (b) > (c) = (d) ≥

2 If m (∠ A) = 150° , then m (reflex ∠ A) = ...

(a) 210° (b) 15° (c) 30° (d) 360°

3 If Δ ABC ≡ Δ XYZ , and m (∠ X) + m (∠ Z) = 140 , then m (∠ B) = ...

(a) 70° (b) 220° (c) 40° (d) 140°

4 If L_1 , L_2 and L_3 are three coplanar straight lines , L_1 // L_2 , $L_3 \perp L_1$, then ....

(a) L_3 // L_2 (b) $L_3 \perp L_2$ (c) L_1 // L_3 (d) otherwise

5 The angle whose measure is 89° is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

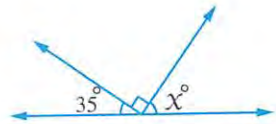
6 The acute angle supplements .... angle.

(a) an acute (b) a right (c) an obtuse (d) a zero

2 Complete each of the following :

1. If $\overline{AB} \equiv \overline{CD}$, AB = 5 cm. , then 2 AB – CD = ...

2. **In the opposite figure :**



The value of X = ...°

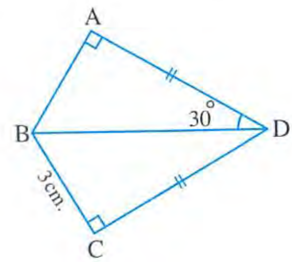
3. The angle of measure 70° complements an angle of measure .... and supplements an angle of measure ....

4. The sum of measures of the interior angles of triangle equals ....

5. The two adjacent angles whose outer sides are on the same straight line are ....

---

3 [a] **In the opposite figure :**



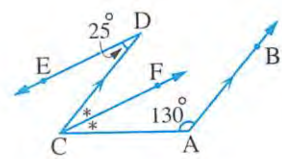
AD = DC , m (∠ ADB) = 30°

, m (∠ A) = m (∠ C) = 90° , BC = 3 cm.

Find with reasons :

m (∠ ABC) and the length of $\overline{AB}$

[b] **In the opposite figure :**



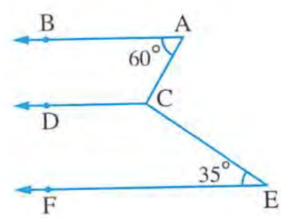
$\overrightarrow{AB}$ // $\overrightarrow{CD}$, $\overrightarrow{CF}$ bisects ∠ DCA

, m (∠ BAC) = 130° , m (∠ CDE) = 25°

Is $\overrightarrow{DE}$ // $\overrightarrow{CF}$? (Give reasons).

---

4 [a] **In the opposite figure :**



$\overrightarrow{AB}$ // $\overrightarrow{CD}$, $\overrightarrow{AB}$ // $\overrightarrow{EF}$

, m (∠ A) = 60° , m (∠ E) = 35°

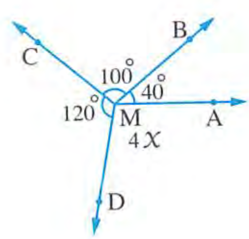
, **find :** m (∠ ACE)

[b] **Draw :** 1 ∠ ABC of measure 120° and bisect it.

2 $\overline{AB}$ of length 6 cm. and bisect it. "Don't remove arcs"

---

5 [a] **In the opposite figure :**



m (∠ AMB) = 40°

, m (∠ BMC) = 100°

, m (∠ CMD) = 120°

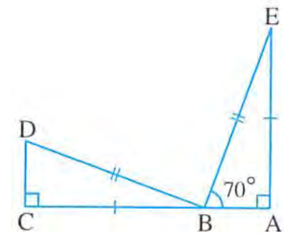
, m (∠ AMD) = 4 X

Find : The value of X

[b] In the opposite figure :

1 Is Δ BAE $\equiv \Delta$ DCB ? (Give reasons).

2 **Find :** m ($\angle$ EBD)



4 Giza Governorate — Math Inspection

Answer the following questions :

1 Complete :

1 If a straight line intersects two parallel straight lines , then each two alternate angles are ....

2 If Δ ABC $\equiv \Delta$ XYZ , m ($\angle$ X) = 60° , m ($\angle$ Z) = 30° , then m ($\angle$ B) = ...°

3 The outer sides of two adjacent complementary angles are ....

4 If m ($\angle$ B) = 110° , then m (reflex $\angle$ B) = ...°

5 The perpendicular bisector of a line segment is called ....

2 Choose the correct answer :

1 The measure of the straight angle = ...

(a) 45° (b) 90° (c) 180° (d) 360°

2 The sum of measures of the accumulative angles at a point = ... angles.

(a) 4 acute (b) 3 right (c) 4 right (d) 2 right

3 The two straight lines that are perpendicular to the third are ....

(a) perpendicular. (b) parallel. (c) intersecting. (d) coincident.

4 If $\overline{AB} \equiv \overline{XY}$, then $\frac{AB}{XY}$ = ...

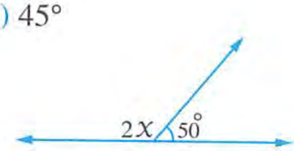
(a) 1 (b) 0 (c) 2 (d) 5

5 If $\angle$ A supplements $\angle$ B , and $\angle$ A $\equiv \angle$ B , then m ($\angle$ B) = ...

(a) 360° (b) 180° (c) 90° (d) 45°

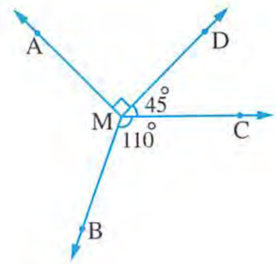
6 **In the opposite figure :**

The value of X = ...°



(a) 130 (b) 50 (c) 65 (d) 25

3 **[a] In the opposite figure :**



m (∠ DMC) = 45°

, m (∠ CMB) = 110°

, m (∠ AMD) = 90°

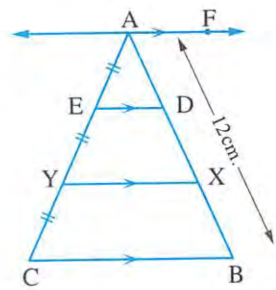
Find : m (∠ AMB) with steps.

[b] Using the geometric tools , draw ∠ XYZ of measure 80° , then draw $\overrightarrow{YE}$ to bisect it. (Don't remove the arcs)

---

4 **[a]** Mention two cases of congruency of two triangles.

[b] In the opposite figure :



If $\overleftrightarrow{AF}$ // $\overline{ED}$ // $\overline{XY}$ // $\overline{BC}$

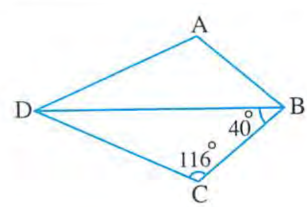
, AE = EY = YC

, and AB = 12 cm.

Find : The length of $\overline{AX}$

---

5 **In the opposite figure :**



AB = BC , AD = DC

1 **Prove that :** Δ CBD ≡ Δ ABD

2 **Find :** m (∠ ABD)

5 Giza Governorate — Awseem Educational Directorate

Answer the following questions :

1 **Complete the following :**

1 The perpendicular straight line to a line segment from its midpoint , is called ....

2 If a straight line cuts two parallel straight lines , then each two alternate angles are ....

3 If m (∠ B) = 115° , then m (reflex ∠ B) = ...°

4 The two adjacent angles formed by a straight line and a ray with starting point on the straight line are ....

5 If the triangle ABC ≡ the triangle XYZ , then m (∠ C) = m (∠ ....)

2 **Choose the correct answer :**

1 If m (∠ A) = 65° , then m (reflex ∠ A) = ……………

(a) 305° (b) 295° (c) 25° (d) 115°

2 The acute angle complements …………… angle.

(a) a right (b) an obtuse (c) an acute (d) a straight

3 ABCD is a rectangle , then $\overline{AC} \equiv$ ……………

(a) $\overline{BD}$ (b) $\overline{AD}$ (c) $\overline{DC}$ (d) $\overline{BC}$

4 The sum of measures of the accumulative angles at one point equals ……………

(a) 90° (b) 180° (c) 270° (d) 360°

5 If ∠ X supplements ∠ Y and m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ……………

(a) 30° (b) 45° (c) 60° (d) 120°

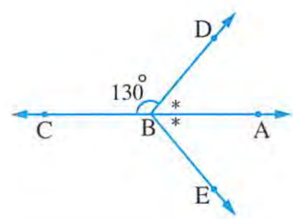
6 The two straight lines parallel to a third straight line are ……………

(a) intersecting. (b) parallel. (c) coincident. (d) perpendicular.

---

3 **[a]** Using the geometric instruments , draw ∠ ABC of measure 120° , then draw $\overrightarrow{BF}$ to bisect the angle. (Don't remove the arcs).

[b] In the opposite figure :



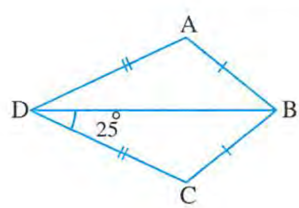
If B ∈ $\overleftrightarrow{AC}$, m (∠ DBC) = 130°

and $\overrightarrow{BA}$ bisects ∠ DBE

Find : m (∠ ABD) and m (∠ DBE) (Give reason).

---

4 **[a] In the opposite figure :**



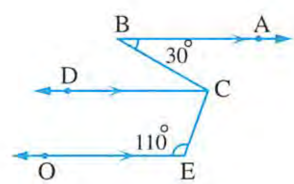
AB = CB , AD = CD

, m (∠ CDB) = 25°

Is Δ ABD ≡ Δ CBD ? Why ?

Find : m (∠ ADC)

[b] In the opposite figure :



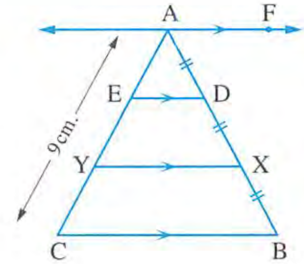
$\overrightarrow{BA}$ // $\overrightarrow{CD}$ // $\overrightarrow{EO}$

, m (∠ ABC) = 30°

, m (∠ CEO) = 110°

Find : m (∠ BCE)

5 **[a] In the opposite figure :**



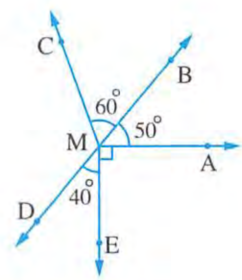
$\overleftrightarrow{AF} \parallel \overline{ED} \parallel \overline{YX} \parallel \overline{CB}$

, AD = DX = XB

, AC = 9 cm.

Find : The length of $\overline{AY}$

[b] In the opposite figure :



m (∠ AME) = 90°

, m (∠ AMB) = 50°

, m (∠ BMC) = 60°

, m (∠ DME) = 40°

Find : m (∠ DMC)

6 Alexandria Governorate

Amria Educational Zone
Mathematics Supervision

Answer the following questions :

1 **Choose the correct answer from those between brackets :**

1. The complement of an angle of measure 37° equals ....°

   (a) 53 (b) 143 (c) 180 (d) 90

2. If two lines are on the same plane and don't intersect , then they are ....

   (a) skew. (b) perpendicular. (c) parallel. (d) coincident.

3. If L_1 , L_2 and L_3 are three coplanar straight lines , $L_1 \parallel L_2$ and $L_1 \perp L_3$, then ....

   (a) $L_1 \perp L_2$ (b) $L_1 \parallel L_3$ (c) $L_2 \parallel L_3$ (d) $L_2 \perp L_3$

4. If Δ ABC ≡ Δ XYZ and m (∠ A) + m (∠ B) = 130° , then m (∠ Z) = ...

   (a) 50° (b) 65° (c) 130° (d) 180°

5. If the two adjacent angles are complementary , then their outer sides are ....

   (a) perpendicular. (b) parallel.

   (c) coincident. (d) on the same straight line.

6. The supplement of an angle of measure 70° is ....°

   (a) 10 (b) 20 (c) 110 (d) 180

2 **Complete each of the following :**

1. The sum of the measures of the accumulative angles at a point is ....°

2. If m (∠ B) = 120° , then m (reflex ∠ B) = ...°

3. If ∠ A supplements ∠ B and ∠ A ≡ ∠ B , then m (∠ B) = ...°

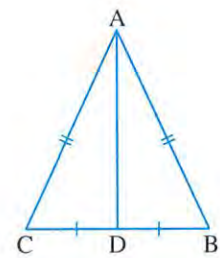
4. $\overline{AB}$ .... $\overrightarrow{AB}$

5. If two straight lines intersect , then each two vertically opposite angles are ....

---

3 [a] Mention two cases of congruency of two triangles.

[b] **In the opposite figure :**



AB = AC , BD = DC

Prove that :

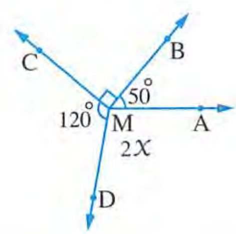
Δ ABD ≡ Δ ACD

---

4 [a] Draw ∠ ABC of measure 100° , using the ruler and compasses to bisects it by $\overrightarrow{BD}$?

(Don't remove the arcs)

[b] **In the opposite figure :**



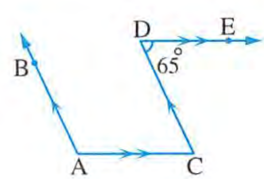
m (∠ AMB) = 50° , m (∠ BMC) = 90°

, m (∠ DMC) = 120° , m (∠ AMD) = 2 X

Find : The value of X ?

---

5 [a] **In the opposite figure :**

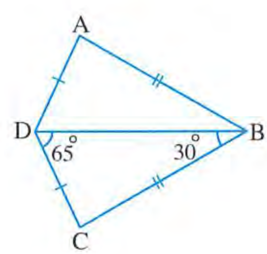


$\overrightarrow{AB}$ // $\overline{CD}$, $\overrightarrow{DE}$ // $\overline{AC}$

, m (∠ D) = 65°

Find : m (∠ ACD) and m (∠ A) ?

[b] **In the opposite figure :**



AB = CB , AD = CD

m (∠ CDB) = 65° , m (∠ CBD) = 30°

1. **Show that :** Δ ABD ≡ Δ CBD   2. **Find :** m (∠ A)

7 Alexandria Governorate

Borj Al-Arab Educational Administration
Math Inspection

Answer the following questions :

1 Choose the correct answer :

1 If two straight lines intersect , then each two vertically opposite angles are ....

(a) supplementary. (b) complementary.

(c) equal in measure. (d) adjacent

2 The sum of measures of two supplementary angles = ...

(a) 90° (b) 180° (c) 270° (d) 360°

3 The area of a square whose side length is 4 cm. equals .... cm^2.

(a) zero (b) 8 (c) 16 (d) 4

4 The complement of the angle whose measure is 30° is an angle of measure ....

(a) 60° (b) 30° (c) 120° (d) 150°

5 The two straight lines that are perpendicular to a third one are ....

(a) parallel. (b) intersecting. (c) equal in length. (d) perpendicular.

6 The sum of measures of the interior angles in a triangle is ....

(a) 90° (b) 180° (c) 270° (d) 360°

---

2 Complete :

1 The sum of the measures of the accumulative angles at a point equals ....

2 The perimeter of triangle whose side lengths are 3 cm. , 4 cm. , 5 cm. = ... cm.

3 If m (∠ B) = 160° , then m (reflex ∠ B) = ...

4 Two triangles are congruent if two sides ....

5 If Δ ABC ≡ Δ XYZ , then $\overline{AC}$ ≡ ....

---

3 [a] Using your geometric instruments , draw $\overline{AB}$ where AB = 8 cm. , then draw line of symmetry of it. (Don't remove the arcs)

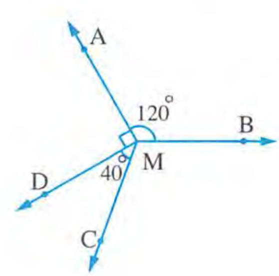
[b] **In the opposite figure :**

If m (∠ AMB) = 120°

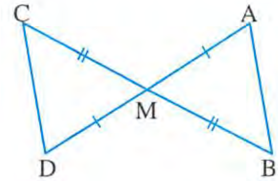
, m (∠ AMD) = 90°

, m (∠ DMC) = 40°

Find : m (∠ BMC)

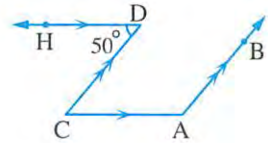


4 **[a] In the opposite figure :**



Is Δ AMB $\equiv$ Δ DMC ? Why ?

[b] In the opposite figure :



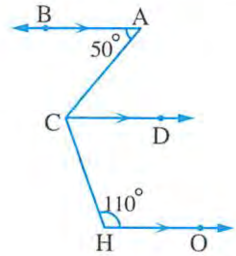
$\overrightarrow{DH} \,//\, \overline{AC}$, $\overrightarrow{AB} \,//\, \overline{DC}$

, m ($\angle$ D) = 50°

Find : m ($\angle$ C) and m ($\angle$ A)

---

5 **[a] In the opposite figure :**



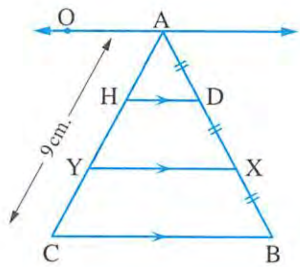
$\overrightarrow{AB} \,//\, \overrightarrow{CD} \,//\, \overrightarrow{HO}$

, m ($\angle$ A) = 50°

, m ($\angle$ H) = 110°

Find : m ($\angle$ ACH)

[b] In the opposite figure :



$\overleftrightarrow{AO} \,//\, \overline{HD} \,//\, \overline{XY} \,//\, \overline{BC}$

, AD = DX = XB

, AC = 9 cm.

Find : The length of $\overline{AY}$

8 El-Kalyoubia Governorate

Directorate of Education
Math Supervision

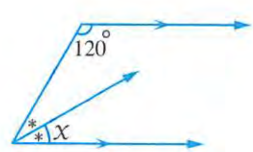
Answer the following questions :

1 **Choose the correct answer :**

1. The measure of the supplement of the angle whose measure is 40° equals ....
   (a) 50° (b) 140° (c) 90° (d) 180°

2. If two straight lines parallel to a third , then the two straight lines are ....
   (a) parallel. (b) perpendicular. (c) coincident. (d) intersecting.

3. **In the opposite figure :**



   x = ...°
   (a) 60 (b) 30
   (c) 90 (d) 45

4 If $\overline{AB} \equiv \overline{CD}$, AB = 5 cm. , then AB – CD = ...

(a) 0 (b) 1 (c) 5 (d) 10

5 Δ ABC is right-angled triangle at B , AB = 3 cm. and BC = 4 cm.

, then its area = ... cm^2.

(a) 12 (b) 6 (c) 24 (d) 14

6 The perimeter of the rhombus where its side length is 10 cm. = ... cm.

(a) 100 (b) 50 (c) 20 (d) 40

2 Complete each of the following :

1 The sum of measures of the accumulative angles at a point is ....°

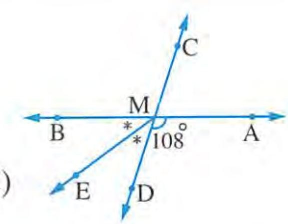
2 Two triangles are congruent if each .... is congruent with corresponding part of the other.

3 The complement of an angle of measure 25° is an angle of measure ....°

4 If Δ ABC $\equiv$ Δ XYZ , then m ($\angle$ CBA) = ...°

5 ABCD is parallelogram , m ($\angle$ A) = 55° , then m ($\angle$ B) = ...°

3 [a] In the opposite figure :

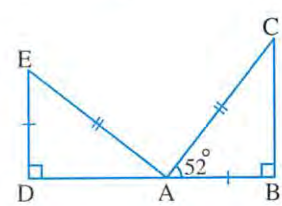


$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

, $\overrightarrow{ME}$ bisects $\angle$ BMD and m ($\angle$ AMD) = 108°

Find : m ($\angle$ BMD) , m ($\angle$ EMD) , m ($\angle$ AMC) and m ($\angle$ BMC)

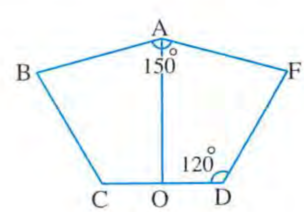
[b] In the opposite figure :



Mention the case and the conditions of congruence between Δ ABC and Δ EDA

, then find : m ($\angle$ EAD) and m ($\angle$ EAC)

4 [a] In the opposite figure :



The figure ABCO $\equiv$ the figure AFDO

, O $\in \overline{CD}$ and CD = 8 cm.

, m ($\angle$ FAB) = 150° and m ($\angle$ FDC) = 120°

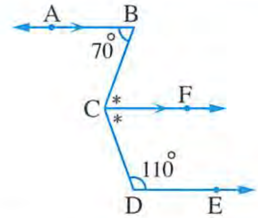
Find : m ($\angle$ BCO) , m ($\angle$ FAO) , m ($\angle$ AOD) and the length of $\overline{CO}$

[b] In the opposite figure :

If $\overrightarrow{BA} \parallel \overrightarrow{CF}$, m ($\angle$ ABC) = 70°

, m ($\angle$ CDE) = 110°

Find : m ($\angle$ BCF) and why $\overrightarrow{CF} \parallel \overrightarrow{DE}$?



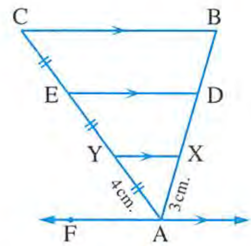
5 **[a] In the opposite figure :**

$\overleftrightarrow{AF} \parallel \overleftrightarrow{XY} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

, AY = YE = EC , AY = 4 cm.

, AX = 3 cm. and the perimeter of Δ ABC = 30 cm.

Find : The length of $\overline{BC}$



[b] Draw Δ ABC in which AB = 3 cm. , BC = 5 cm. , and AC = 7 cm. , then draw $\overrightarrow{BD}$ bisects $\angle$ ABC to intersect $\overline{AC}$ at D , find by measuring $\angle$ ABD (Don't remove the arcs).

9 El-Sharkia Governorate

East Zagazig Educational Administration
Omar Al-Farouk Governmental Language School

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If Δ ABC $\equiv$ Δ XYZ , m ($\angle$ X) + m ($\angle$ Y) = 150° , then m ($\angle$ C) = ...

(a) 75° (b) 30° (c) 100° (d) 50°

2 If Δ ABC $\equiv$ Δ XYZ , then $\overline{BC}$ $\equiv$ ....

(a) $\overline{XY}$ (b) $\overline{YZ}$ (c) $\overline{XZ}$ (d) $\overline{AB}$

3 The obtuse angle supplements .... angle.

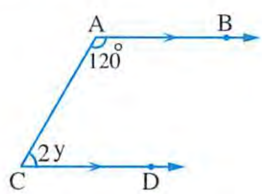
(a) acute (b) right (c) obtuse (d) reflex

4 **In the opposite figure :**

If $\overrightarrow{AB} \parallel \overrightarrow{CD}$, then y = ...°

(a) 30 (b) 60

(c) 100 (d) 120



5 If m ($\angle$ A) = 80° , then m (reflex $\angle$ A) = ...

(a) 360° (b) 180° (c) 90° (d) 280°

6 The angle whose measure is 60° complements an angle of measure ....

(a) 60° (b) 30° (c) 120° (d) 300°

2 **Complete the following :**

1 The sum of the measures of the accumulative angles at a point is ....°

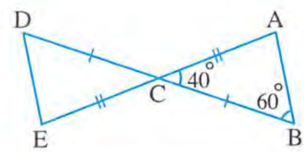
2 The two vertically opposite angles are .... in measure.

3 If two angles are supplementary and congruent , then the measure of each is ....°

4 If a straight line intersects two parallel straight lines , then each two alternate angles are ....

5 **From the opposite figure :**

m (∠ E) = ...°



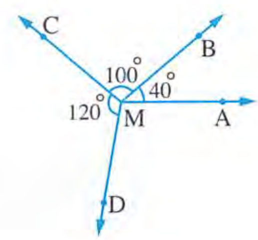
3 **[a] In the opposite figure :**

m (∠ AMB) = 40°

, m (∠ BMC) = 100°

, m (∠ CMD) = 120°

Find : m (∠ AMD)



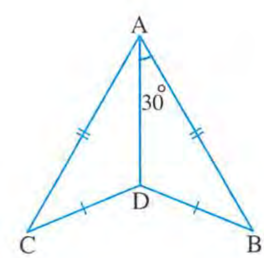
[b] In the opposite figure :

AB = AC , BD = CD

, m (∠ BAD) = 30°

1 Does Δ ADC ≡ Δ ADB ? Why ?

2 **Find :** m (∠ BAC)

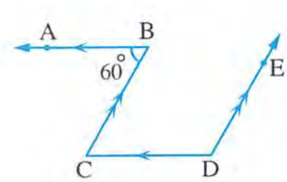


4 **[a] In the opposite figure :**

$\overrightarrow{BA} \parallel \overrightarrow{CD}$, $\overrightarrow{CB} \parallel \overrightarrow{DE}$

and m (∠ B) = 60°

Find : m (∠ C) and m (∠ D)



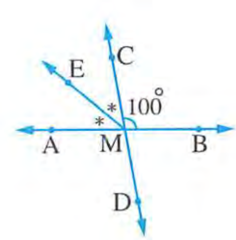
[b] In the opposite figure :

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

, $\overrightarrow{ME}$ bisects ∠ AMC

and m (∠ CMB) = 100°

Find : 1 m (∠ AMC) 2 m (∠ EMD)



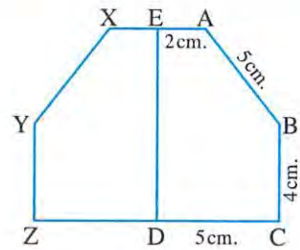
5 **[a] In the opposite figure :**

The figure ABCDE ≡ The figure XYZDE

Find :

1 m (∠ EDC)

2 The length of $\overline{ZC}$



[b] Draw the line segment $\overline{AB}$ of length 8 cm. , then draw its line of symmetry.

(Don't remove arcs)

10 El-Gharbia Governorate

East Tanta Educational Zone
Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The supplement of the angle whose measure is 75° is an angle whose measure is ....

(a) 60° (b) 180° (c) 105° (d) 90°

2 If Δ ABC ≡ Δ XYZ and m (∠ A) + m (∠ Y) = 120° , then m (∠ Z) = ...

(a) 50° (b) 60° (c) 70° (d) 80°

3 The sum of measures of the accumulative angles at a point equals ....

(a) 180° (b) 360° (c) 630° (d) 603°

4 If the two straight lines are perpendicular to a third , then the two straight lines are ....

(a) perpendicular. (b) coincident. (c) intersecting. (d) parallel.

5 If the perimeter of a square is 20 cm. , then its area is .... cm^2.

(a) 4 (b) 5 (c) 25 (d) 400

6 The triangle whose perimeter is 12 cm. and the lengths of its two sides are 2 cm. , 5 cm. , is called ....

(a) isosceles. (b) equilateral. (c) right. (d) scalene.

2 **Complete the following :**

1 The two diagonals are perpendicular in .... and ....

2 The perpendicular bisector of a line segment is called ....

3 The two right-angled triangles are congruent if ....

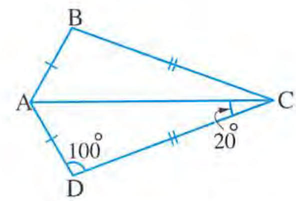
4 If $\angle A$ supplements $\angle B$ and $\angle A \equiv \angle B$, then m ($\angle B$) = ...

5 If $\overline{AB} \equiv \overline{XY}$, then AB – XY = ...

3 [a] State any two cases of congruency of two triangles.

[b] Draw a line segment $\overline{AB}$ of length 6 cm. , then draw its symmetry axis.

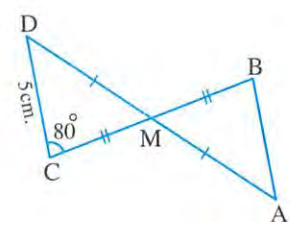
[c] **In the opposite figure :**



1 **Prove that :** $\Delta ABC \equiv \Delta ADC$

2 **Find :** m ($\angle BAC$)

4 [a] **In the opposite figure :**



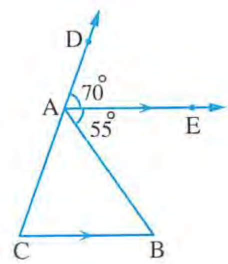
m ($\angle C$) = 80° , $\overline{CB} \cap \overline{AD} = \{M\}$

, MB = MC , MD = MA , CD = 5 cm.

Mention the conditions for ΔABM

, ΔDCM to be congruent and find m ($\angle B$)

[b] **In the opposite figure :**



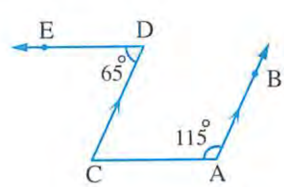
ABC is a triangle where the point $A \in \overrightarrow{CD}$

, $\overrightarrow{AE} \mathbin{//} \overline{CB}$, m ($\angle DAE$) = 70°

and m ($\angle EAB$) = 55° , calculate measure

of each angle in the triangle ABC

5 [a] **In the opposite figure :**

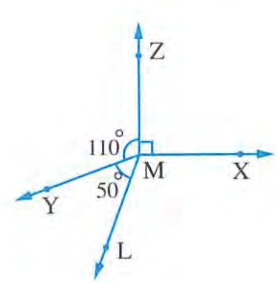


If $\overrightarrow{AB} \mathbin{//} \overrightarrow{CD}$, m ($\angle D$) = 65° , m ($\angle A$) = 115°

Find : m ($\angle C$)

Is $\overline{AC} \mathbin{//} \overrightarrow{DE}$? (Give the reason)

[b] **In the opposite figure :**



m ($\angle ZMY$) = 110°

, m ($\angle XMZ$) = 90°

, m ($\angle YML$) = 50°

Find by steps : m ($\angle XML$)

11 El-Dakahlia Governorate — Maths Supervision

Answer the following questions :

1 Complete each of the following :

1. If $\Delta ABC \equiv \Delta LMN$, then BC = ...
2. The number of axes of symmetry of a sqaure is ....
3. The sum of measures of the accumulative angles at point is ....°
4. If the ratio between two supplementary angles is 4 : 5 , then the measure of the greatest angle is ....
5. If a straight line intersects two parallel lines , then each two corresponding angles are ....

---

2 Choose the correct answer from those given :

1. If $\overline{AB} \equiv \overline{CD}$, then 3 AB – 3 CD = ...
   (a) 3 (b) 6 AB (c) 6 AC (d) zero
2. If m ($\angle$ A) = 120° , then m (reflex of $\angle$ A) = ...°
   (a) 60 (b) 120 (c) 240 (d) 360
3. The two straight lines that are perpendicular to third one are ....
   (a) perpendicular. (b) coincident. (c) intersecting. (d) parallel.
4. The sum of measures of the interior angles of any triangle equals ....°
   (a) 108 (b) 180 (c) 306 (d) 360
5. If $\angle X \equiv \angle Y$, m ($\angle$ X) + m ($\angle$ Y) = 140° , then m ($\angle$ X) = ...°
   (a) 40 (b) 70 (c) 140 (d) 220
6. The number of rectangles in the opposite figure is ....
   
   (a) 3 (b) 4 (c) 5 (d) 6

---

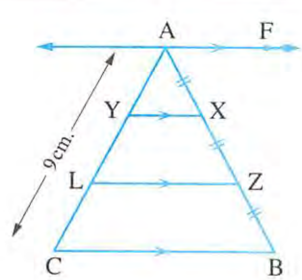
3 [a] In the opposite figure :

$\overleftrightarrow{AF} \mathbin{//} \overline{XY} \mathbin{//} \overline{ZL} \mathbin{//} \overline{BC}$

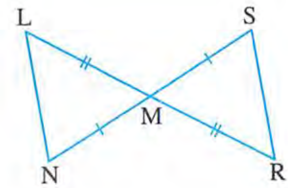
, AX = XZ = ZB

and AC = 9 cm.

Find : the length of $\overline{AL}$ (Give the reason)



[b] In the opposite figure :



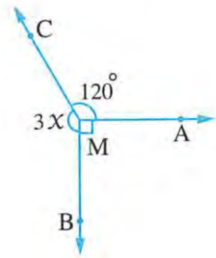
$\overline{SN} \cap \overline{LR} = \{M\}$

, MR = ML , MS = MN

Is Δ MSR ≡ Δ MNL ? Why ?

---

4 **[a] In the opposite figure :**



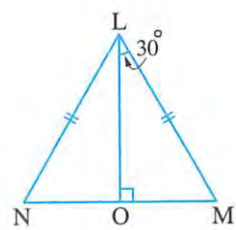
m (∠ AMC) = 120°

, m (∠ AMB) = 90°

, m (∠ CMB) = $(3\,x)°$

Find : The value of x ? (With steps)

[b] In the opposite figure :



$\overline{LM} \equiv \overline{LN}$, m (∠ MLO) = 30°

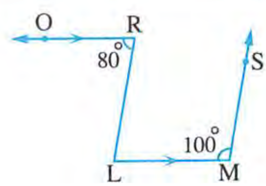
, m (∠ LOM) = m (∠ LON) = 90°

Mention the conditions for Δ LOM , Δ LON

to be congruent , then find m (∠ MLN)

---

5 **[a] In the opposite figure :**



$\overrightarrow{RO}$ // $\overline{ML}$, m (∠ ORL) = 80°

, m (∠ M) = 100°

1 **Find :** m (∠ L)   2 Is $\overrightarrow{MS}$ // $\overline{RL}$ (Give reason)

[b] Draw ∠ ABC where m (∠ B) = 70° , using the ruler and the compasses to bisect ∠ B by $\overrightarrow{BD}$ (Don't remove the arcs)

12 Suez Governorate — Math Inspectorate

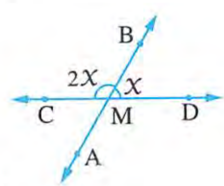
Answer the following questions :

1 **Choose the correct answer :**

1 If ∠ X ≡ ∠ Y , ∠ X complements ∠ Y , then m (∠ X) = ...°

(a) 90   (b) 45   (c) 180   (d) 360

2 **In the opposite figure :**



If $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

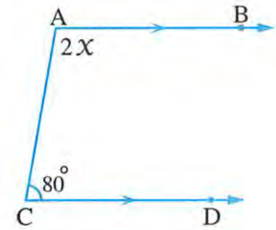
, then m (∠ AMC) = ...°

(a) 30   (b) 60   (c) 20   (d) 120

3 **In the opposite figure :**

If $\overrightarrow{AB} \parallel \overrightarrow{CD}$, m ($\angle$ C) = 80°

, then x = ...°



(a) 50 (b) 100

(c) 80 (d) 180

4 If $\angle$ X supplements $\angle$ Y , m ($\angle$ X) = 60° , then m ($\angle$ Y) = ...

(a) 60° (b) 30° (c) 90° (d) 120°

5 If two adjacent angles are complement , then their outer sides are ....

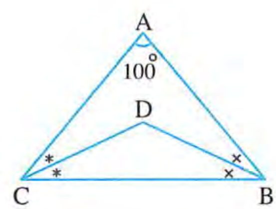
(a) perpendicular. (b) parallel

(c) coincident (d) on the same straight line.

6 **In the opposite figure :**

If m ($\angle$ A) = 100° , $\overrightarrow{BD}$ bisects $\angle$ B

, $\overline{CD}$ bisects $\angle$ C , then m ($\angle$ D) = ...°



(a) 100 (b) 80

(c) 140 (d) 90

---

2 **Complete :**

1 If m ($\angle$ X) = 160° , then m (reflex $\angle$ X) is ....°

2 If Δ ABC $\equiv$ Δ XYZ , m ($\angle$ X) + m ($\angle$ Y) = 100° , then m ($\angle$ C) = ...°

3 If two straight lines are parallel to a third line , then they are ....

4 If $\overline{XY} \equiv \overline{AB}$, XY = 5 cm. , then 2 AB + XY = ...

5 If a straight line intersects two parallel straight lines , then each two alternate angles are ....

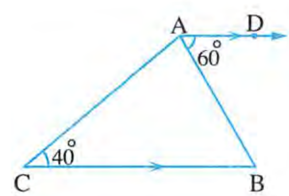
---

3 [a] **In the opposite figure :**

If $\overrightarrow{AD} \parallel \overline{BC}$, m ($\angle$ BAD) = 60°

, m ($\angle$ C) = 40°

Find : m ($\angle$ B) , m ($\angle$ BAC)

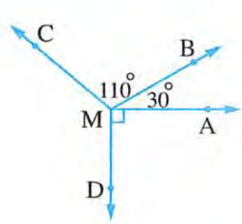


[b] **In the opposite figure :**

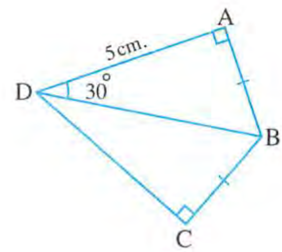
If m ($\angle$ AMB) = 30° , m ($\angle$ BMC) = 110°

, m ($\angle$ AMD) = 90°

Find : m ($\angle$ CMD)



4 [a] **In the opposite figure :**



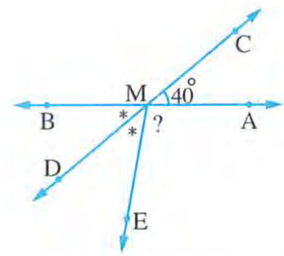
m ($\angle$ A) = m ($\angle$ C) = 90° , AB = CB

1 **Prove that :** Δ ABD $\equiv$ Δ CBD

2 **Find :** The length of $\overline{CD}$

3 **Find :** m ($\angle$ CBD)

[b] **In the opposite figure :**



$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$

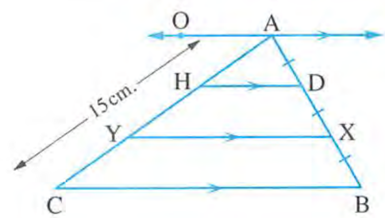
, m ($\angle$ AMC) = 40°

, $\overrightarrow{MD}$ bisect $\angle$ BME

Find : m ($\angle$ AME)

---

5 [a] **In the opposite figure :**



If $\overleftrightarrow{OA}$ // $\overline{HD}$ // $\overline{XY}$ // $\overline{BC}$

, AD = DX = XB

, AC = 15 cm.

Find : The length of $\overline{HC}$

[b] Draw line segment $\overline{AB}$ of length 6 cm. , then draw its axis of symmetry. (Don't remove arcs)

13 El-Beheira Governorate — Maths Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 The angle whose measure more than 180° and less than 360° is called ....

(a) acute. (b) obtuse. (c) straight. (d) reflex.

2 The sum of the measures of the accumulative angles at a point equals ....

(a) 90° (b) 180° (c) 270° (d) 360°

3 If the ratio between two supplementary angles is 2 : 7 , then the measure of smaller angle is ....

(a) 20° (b) 40° (c) 70° (d) 140°

4 If L_1 , L_2 and L_3 are three coplanar straight lines , L_1 // L_2 and $L_1 \perp L_3$, then L_2 .... L_3

(a) // (b) $\perp$ (c) coincides (d) =

5 The acute angle supplements .... angle.

(a) acute (b) right (c) obtuse (d) straight

6 If $\overline{AB} \equiv \overline{CD}$, then AB – CD = ...

(a) 0 (b) 1 (c) 2 (d) 5

---

2 Complete :

1 If m (∠ A) = 30° , then m (reflex ∠ A) = ...°

2 If two adjacent angles are supplementary , then their outer sides are ....

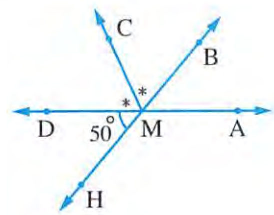
3 The perpendicular bisector of a line segment is called ....

4 If a straight line intersects two parallel straight lines , then every two interior angles in the same side of the transversal are ....

5 If Δ ABC ≡ Δ XYZ , m (∠ A) + m (∠ B) = 140° , then m (∠ Z) = ...°

---

3 [a] In the opposite figure :

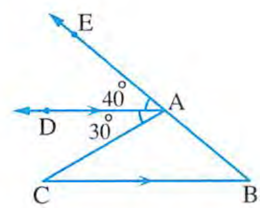


$\overleftrightarrow{AD} \cap \overleftrightarrow{BH} = \{M\}$, m (∠ HMD) = 50°

, $\overrightarrow{MC}$ bisects ∠ BMD

Find : m (∠ AMC)

[b] In the opposite figure :



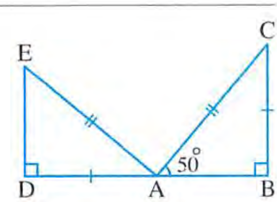
$\overrightarrow{AD}$ // $\overline{BC}$, m (∠ EAD) = 40°

, m (∠ CAD) = 30°

Find : m (∠ C) , m (∠ B) write the steps.

---

4 [a] In the opposite figure :

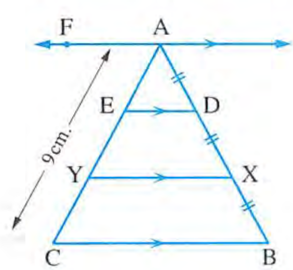


BC = AD , AC = AE

, m (∠ CAB) = 50°

Find : m (∠ EAD) , write the steps.

[b] In the opposite figure :



$\overleftrightarrow{AF}$ // $\overline{DE}$ // $\overline{XY}$ // $\overline{BC}$

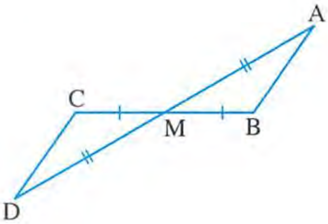
, AD = DX = XB , AC = 9 cm.

Find : length of $\overline{AY}$ (Give the reason)

5 **[a] In the opposite figure :**

$\overline{AD} \cap \overline{BC} = \{M\}$

, BM = MC , AM = MD



1 **Prove that :** Δ AMB $\equiv$ Δ DMC

2 Is $\overline{AB}$ // $\overline{CD}$? why ?

[b] By using geometrical tools , draw $\angle$ ABC whose measure is 120° , then draw the bisector of $\angle$ ABC (Don't remove the arcs)

14 Souhag Governorate

Akhmeem Educational Management
Future Generation International School

Answer the following questions :

1 **Choose the correct answer :**

1 The two adjacent angles formed by a straight line and a ray with a starting point on this straight line are ....

(a) equal in length. (b) supplementary. (c) complementary. (d) congruent.

2 If Δ ABC $\equiv$ Δ XYZ , then m ($\angle$ A) + m ($\angle$ B) = 100° , m ($\angle$ Z) = ...°

(a) 50 (b) 130 (c) 150 (d) 80

3 If L_1 , L_2 and L_3 are three coplanar straight lines , $L_1 // L_3$, $L_2 \perp L_3$, then ....

(a) $L_1 // L_2$ (b) $L_3 \perp L_1$ (c) $L_1 \cap L_2 = \varnothing$ (d) $L_2 \perp L_1$

4 The obtuse angle supplements .... angle.

(a) obtuse (b) right (c) acute (d) straight

5 The two straight lines that are perpendicular to a third , then the two straight lines are ....

(a) perpendicular. (b) intersecting. (c) coincident. (d) parallel.

6 If Δ ABC $\equiv$ Δ XYZ , then AB = ...

(a) XY (b) XZ (c) YZ (d) BC

2 **Complete :**

1 The perpendicular bisector of a line segment is called ....

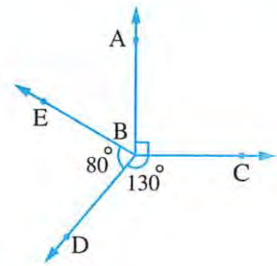
2 The angle whose measure is 35° complements an angles of measure ....°

3 The sum of measures of the accumulative angles at a point equals ....°

4 The two straight lines are parallel to a third are ....

5 If m ($\angle$ B) = 105° , then m (reflex $\angle$ B) = ...°

3 **[a] In the opposite figure :**

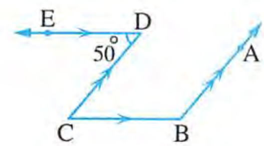


$\overrightarrow{BA} \perp \overrightarrow{BC}$, m (∠ CBD) = 130°

, m (∠ EBD) = 80°

Calculate : m (∠ ABE)

[b] In the opposite figure :

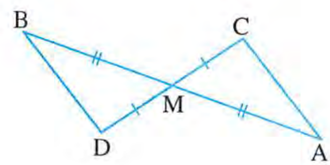


$\overrightarrow{BA}$ // $\overline{CD}$, $\overline{BC}$ // $\overrightarrow{DE}$

, m (∠ D) = 50°

, **find :** m (∠ C) and m (∠ B)

4 **[a] In the opposite figure :**



$\overline{AB} \cap \overline{CD} = \{M\}$

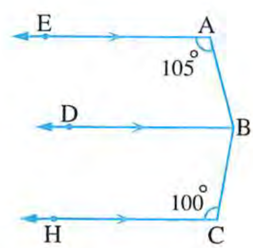
, AM = BM , CM = DM

and CD = 4 cm. , AC = 3 cm.

1 Does Δ AMC ≡ Δ BMD , why ?

2 **Find :** The length of $\overline{BD}$ and the length of $\overline{CM}$

[b] In the opposite figure :



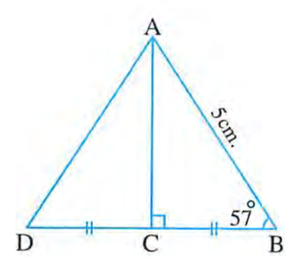
If $\overrightarrow{AE}$ // $\overrightarrow{CH}$ // $\overrightarrow{BD}$

, m (∠ C) = 100°

, m (∠ A) = 105°

Find : m (∠ ABC)

5 **[a] In the opposite figure :**



C is the midpoint of $\overline{BD}$, $\overline{AC} \perp \overline{BD}$

, AB = 5 cm. and m (∠ B) = 57°

Find : 1 The length of $\overline{AD}$

2 m (∠ DAC)

[b] By using your geometric instruments draw ∠ ABC whose measure is 120°

Draw $\overrightarrow{BF}$ to bisect the angle.

15 New Valley Governorate

Al-Dakhla Educational Zone
Mout official Language School

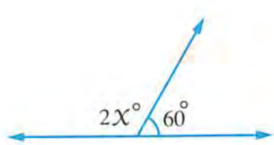
Answer the following questions :

1 Choose the correct answer :

1. If $\angle X \equiv \angle Y$, $\angle X$ and $\angle Y$ are supplementary angles , then m ($\angle X$) = ...°
   (a) 45 (b) 90 (c) 135 (d) 180
2. The two straight lines that are perpendicular to a third are ....
   (a) perpendicular. (b) parallel. (c) intersecting. (d) coincident.
3. The angle whose measure is more than 90° and less than 180° is .... angle.
   (a) an obtuse (b) an acute (c) a right (d) a straight
4. The sum of measures of the accumulative angles at a point equals ....°
   (a) 90 (b) 180 (c) 270 (d) 360
5. If $\Delta ABC \equiv \Delta XYZ$, then XZ = ...
   (a) AB (b) AC (c) BC (d) YZ
6. The axis of symmetry of a line segment is ....
   (a) perpendicular to it from its midpoint. (b) parallel to it.
   (c) congruent to it. (d) equal to it in length.

2 Complete each of the following :

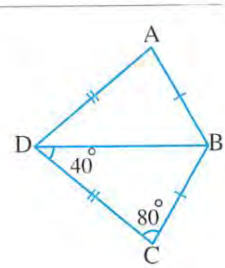
1. If m ($\angle$ A) = 110° , then m (reflex $\angle$ A) = ...°
2. The angle whose measure is 50° complements an angle of measure ....°
3. If $\overline{AB} \equiv \overline{XY}$, then AB – XY = ...
4. **In the opposite figure :**
   x = ...°



5. If two straight lines intersect , then each two vertically opposite angles are ....

3 [a] Mention two cases of congruency of two triangles.

[b] **In the opposite figure :**

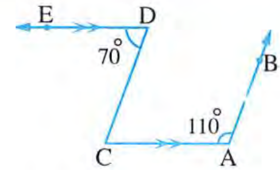


AB = BC , AD = DC

, m ($\angle$ C) = 80° , m ($\angle$ BDC) = 40°

prove that : $\Delta CBD \equiv \Delta ABD$ **, then find :** m ($\angle$ ABD)

4 [a] **In the opposite figure :**

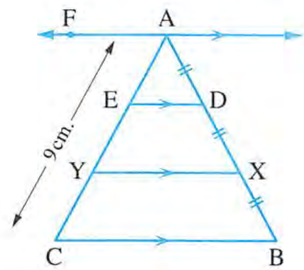


$\overline{AC} \parallel \overrightarrow{DE}$, m ($\angle$ A) = 110°

, m ($\angle$ D) = 70°

Prove that : $\overrightarrow{AB} \parallel \overline{CD}$

[b] **In the opposite figure :**

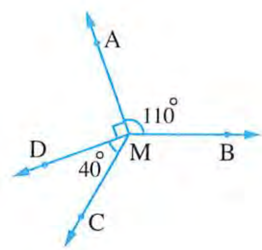


$\overleftrightarrow{AF} \parallel \overline{DE} \parallel \overline{XY} \parallel \overline{BC}$

, AD = DX = XB , AC = 9 cm.

Find : The length of $\overline{AY}$

5 [a] **In the opposite figure :**



m ($\angle$ AMB) = 110°

, m ($\angle$ AMD) = 90°

, m ($\angle$ DMC) = 40°

Find : m ($\angle$ BMC)

[b] Using the geometric instrument , draw $\angle$ ABC where m ($\angle$ ABC) = 80°

, then draw $\overrightarrow{BD}$ to bisect the angle. (Don't remove the arcs)

EL-MOASSER

By a group of supervisors

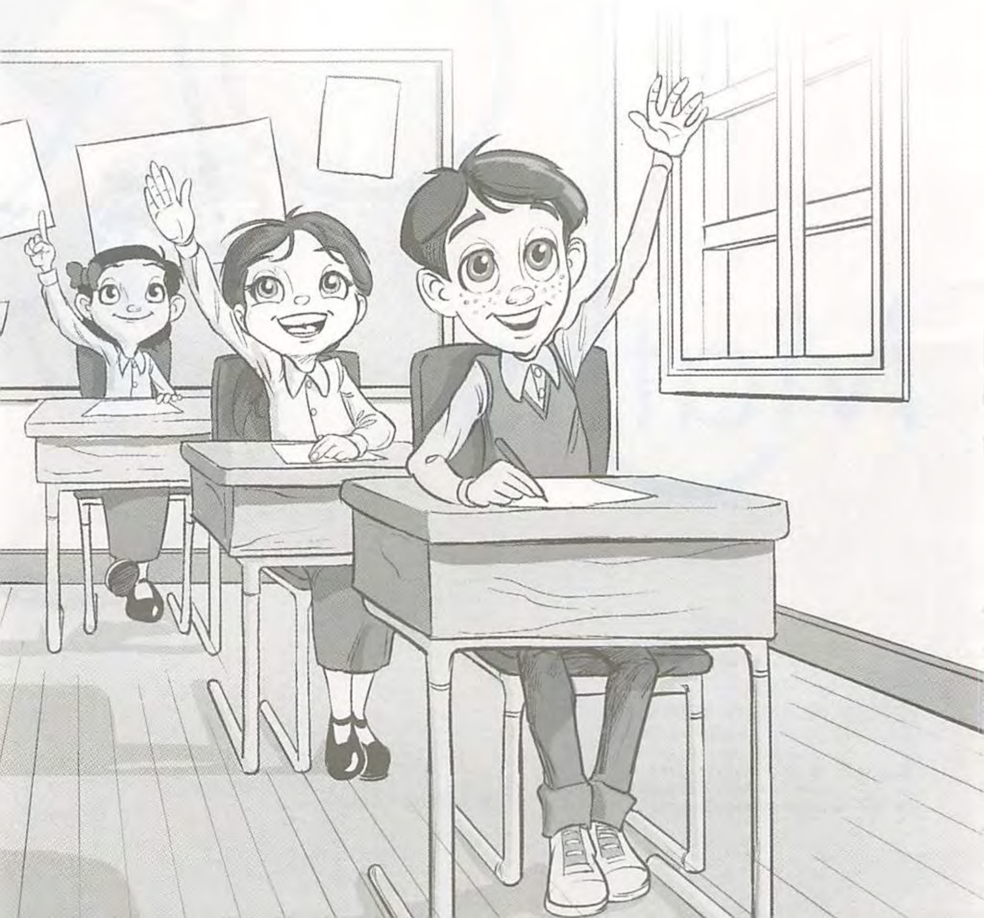
GUIDE ANSWERS

1st PREP. 2023

FIRST TERM

Maths

Guide Answers
Of The Exercises



Answers of unit one

Answers of Exercise 1

1

1 zero 2 2 3 zero 4 -2 5 4
6 5 7 4 8 -3 9 1 10 12
11 -8 12 35 13 2.1 14 40

2

1 (d) 2 (c) 3 (b) 4 (d) 5 (d)
6 (b) 7 (a) 8 (c) 9 (a) 10 (d)
11 (d) 12 (d) 13 (c)

3

1 $\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$ 2 $-\frac{24}{56} = -\frac{24 \div 8}{56 \div 8} = -\frac{3}{7}$

3 $\frac{45}{20} = \frac{45 \div 5}{20 \div 5} = \frac{9}{4}$ 4 $-\frac{132}{88} = -\frac{132 \div 44}{88 \div 44} = -\frac{3}{2}$

4 The required rational numbers are : $\frac{7}{20}$, $\frac{5}{8}$, $2\frac{2}{5}$

5

1 $0.\dot{5}\dot{4}$ 2 $-3.0\dot{6}$

6

1 $-\frac{5}{1}$ 2 $\frac{\text{zero}}{1}$ 3 $\frac{75}{100}$ 4 $-\frac{1}{100}$
5 $\frac{54}{10}$ 6 $\frac{30}{100}$ 7 $\frac{45}{1000}$ 8 $\frac{26}{3}$

(There are other solutions)

7

1 $2.5 = 250\,\%$ 2 $-0.15 = -15\%$
3 $7.1875 = 718.75\,\%$ 4 $0.1\dot{6} \simeq 16.7\,\%$

8 Because division by zero is meaningless.

9

1 (a) 2 (c)

10

Since : $\frac{3}{5} = \frac{9}{15}$, $9 + 15 = 24$

, then the number is : $\frac{9}{15}$

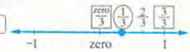
11

1 $X = 1$ ***or*** 3 ***or*** 5 ***or*** 15 ***or*** 25 ***or*** 75

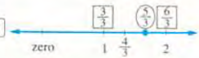
2 $X =$ zero ***or*** 2 ***or*** 4 ***or*** 14

Answers of Exercise 2

1

1 

2 

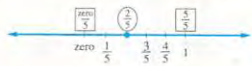
3 

4 

5 $1\frac{1}{5} = \frac{6}{5}$

6 $-3\frac{1}{2} = -\frac{7}{2}$

7 $0.4 = \frac{4}{10} = \frac{2}{5}$



8 $|-\frac{3}{5}| = \frac{3}{5}$

2

1 $<$ 2 $<$ 3 $>$
4 $<$ 5 $>$ 6 $=$

3

1 $>$ 2 $>$ 3 $>$
4 $<$ 5 $>$ 6 $=$

4

L.C.M. of denominators = 30 , then

$\frac{3}{10} = \frac{9}{30}$, $-\frac{1}{3} = -\frac{10}{30}$, $-\frac{1}{5} = -\frac{6}{30}$, $\frac{4}{15} = \frac{8}{30}$

The descending order is :

$\frac{3}{10}$, $\frac{4}{15}$, $\frac{7}{30}$, $-\frac{1}{5}$ and $-\frac{1}{3}$

5

L.C.M. of denominators = 24

$\frac{3}{4} = \frac{18}{24}$, $-\frac{5}{8} = -\frac{15}{24}$, $-\frac{7}{12} = -\frac{14}{24}$, $\frac{2}{3} = \frac{16}{24}$

The ascending order is : $-\frac{5}{8}$, $-\frac{7}{12}$, $\frac{2}{3}$ and $\frac{3}{4}$

6

1 (d) 2 (b) 3 (d) 4 (c) 5 (a)
6 (d) 7 (b) 8 (a) 9 (a) 10 (c)
11 (d) 12 (b) 13 (a) 14 (b)

7

1 $\frac{1}{2}$ 2 $-\frac{1}{2}$ 3 $\frac{3}{16}$ 4 $-\frac{7}{28} = -\frac{1}{4}$

(There are other solutions)

8

1 L.C.M. of the denominators = 10

$\frac{1}{2} = \frac{5}{10}$, $\frac{4}{5} = \frac{8}{10}$

The two numbers are : $\frac{6}{10}$ and $\frac{7}{10}$

2 L.C.M. of the denominators = 12

$-\frac{3}{4} = -\frac{9}{12}$, $-\frac{2}{3} = -\frac{8}{12}$

$-\frac{9}{12} = -\frac{27}{36}$, $-\frac{8}{12} = -\frac{24}{36}$

The two numbers are : $-\frac{25}{36}$ and $-\frac{26}{36}$

3 $0.3 = \frac{3}{10}$

L.C.M. of the denominators = 10

$\frac{3}{5} = \frac{6}{10}$

The two numbers are : $\frac{4}{10}$ and $\frac{5}{10}$

4 $75\ \% = \frac{75}{100} = \frac{3}{4}$, $0.\dot{6} = \frac{2}{3}$

Since : L.C.M. of the denominators = 12

then : $\frac{3}{4} = \frac{9}{12}$, $\frac{2}{3} = \frac{8}{12}$

, Since : $\frac{9}{12} = \frac{27}{36}$, $\frac{8}{12} = \frac{24}{36}$

Then the two numbers are : $\frac{25}{36}$, $\frac{26}{36}$

(There are other solutions)

9

1 L.C.M. of the denominators = 12

$\frac{1}{2} = \frac{6}{12}$

The numbers are : $\frac{7}{12}$, $\frac{8}{12}$, $\frac{9}{12}$ and $\frac{10}{12}$

2 L.C.M. of the denominators = 18

$-\frac{4}{9} = -\frac{8}{18}$, $-\frac{5}{6} = -\frac{15}{18}$

The numbers are : $-\frac{9}{18}$, $-\frac{10}{18}$, $-\frac{11}{18}$ and $-\frac{12}{18}$

3 $0 = \frac{0}{2}$, $3 = \frac{6}{2}$

The numbers are : $\frac{1}{2}$, 1 , $\frac{3}{2}$ and 2

(There are other solutions)

10

The left numbers are : $\frac{6}{15}$, $\frac{7}{15}$, $\frac{8}{15}$ and $\frac{9}{15}$

11

L.C.M. of the denominators = 4

, then $\frac{3}{2} = \frac{6}{4}$, then $\frac{6}{4} = \frac{12}{8}$, $\frac{3}{4} = \frac{6}{8}$

Then the numbers are : $\frac{7}{8}$, 1 , $\frac{9}{8}$ and $\frac{10}{8}$

(There are other solutions)

12

Since $\frac{X-3}{X+2} = 0$, then $X - 3 = 0$, then $X = 3$

, then $\frac{1}{X} = \frac{1}{3}$, $\frac{X-1}{X+2} = \frac{3-1}{3+2} = \frac{2}{5}$

L.C.M. of the denominators = 15

$\frac{1}{3} = \frac{5}{15}$, $\frac{2}{5} = \frac{6}{15}$

$\frac{5}{15} = \frac{20}{60}$, $\frac{6}{15} = \frac{24}{60}$

The numbers are : $\frac{21}{60}$, $\frac{22}{60}$, $\frac{23}{60}$

(There are other solutions)

13 (c)

14

L.C.M. of the numbers : 2 , 3 , 4 and 6 is 12

$\frac{11}{3} = \frac{44}{12}$, $\frac{11}{2} = \frac{66}{12}$

The integers between $\frac{11}{3}$ and $\frac{11}{2}$ are :

$\frac{48}{12} = 4$, $\frac{60}{12} = 5$ (1)

$\frac{9}{4} = \frac{27}{12}$, $\frac{25}{6} = \frac{50}{12}$

The integers between $\frac{9}{4}$ and $\frac{25}{6}$ are :

$\frac{36}{12} = 3$, $\frac{48}{12} = 4$ (2)

From (1) , (2) : The required integer = $\frac{48}{12} = 4$

Another solution :

The two numbers $\frac{11}{3}$, $\frac{11}{2}$ are $3\frac{2}{3}$, $5\frac{1}{2}$

The included integers between them are 4 and 5

The two numbers $\frac{9}{4}$, $\frac{25}{6}$ are $2\frac{1}{4}$, $4\frac{1}{6}$

and the included integers between them are 3 and 4

The required integer is 4

15

OA = OB

$\frac{x}{6} = -\frac{5}{3}$

$x = -\frac{5 \times 6}{3} = -10$

$\frac{x}{6} = -1\frac{2}{3}$

$3x = -5 \times 6$

Answers of Exercise 3

1

1 zero　2 $-\frac{3}{7}$　3 $\frac{4}{9}$　4 2.3　5 $-\frac{6}{11}$

6 -1　7 -1　8 8　9 $-\frac{4}{5}$　10 zero

2

1 $\frac{5}{7}$　2 zero　3 $\frac{4}{8} = \frac{1}{2}$

4 $-\frac{12}{5}$　5 $\frac{1}{6}$　6 $\frac{5}{9} + \frac{4}{9} = \frac{9}{9} = 1$

3

1 Since L.C.M. of the denominators is 8

therefore $\frac{1}{4} + \frac{25}{8} = \frac{2}{8} + \frac{25}{8} = \frac{27}{8}$

2 Since L.C.M. of the denominators is 15

therefore $\frac{1}{5} - \frac{2}{3} = \frac{3}{15} - \frac{10}{15} = -\frac{7}{15}$

3 $-\frac{9}{12} = -\frac{3}{4}$

Since L.C.M. of the denominators is 16

therefore $-\frac{3}{4} + \frac{3}{16} = -\frac{12}{16} + \frac{3}{16} = -\frac{9}{16}$

4 Since L.C.M. of the denominators is 10

therefore $-\frac{3}{10} + \left(-\frac{2}{5}\right) = -\frac{3}{10} + \left(-\frac{4}{10}\right) = -\frac{7}{10}$

5 $-\frac{15}{18} = -\frac{5}{6}$, $\frac{12}{16} = \frac{3}{4}$

Since L.C.M. of the denominators is 12

therefore $-\frac{5}{6} + \frac{3}{4} = -\frac{10}{12} + \frac{9}{12} = -\frac{1}{12}$

6 $\frac{3}{15} = \frac{1}{5}$

$-\frac{2}{5} - \frac{3}{15} = -\frac{2}{5} - \frac{1}{5} = -\frac{3}{5}$

7 Since L.C.M. of the denominators is 35

therefore $\frac{3}{7} - \left(-\frac{2}{5}\right) = \frac{15}{35} - \left(-\frac{14}{35}\right) = \frac{15}{35} + \frac{14}{35}$

"From the definition of subtraction operation" $= \frac{29}{35}$

8 Since L.C.M. of the denominators is 12

therefore $-\frac{5}{6} - \left(-\frac{3}{4}\right) = -\frac{10}{12} - \left(-\frac{9}{12}\right) = -\frac{10}{12} + \frac{9}{12}$

"From the defnition of subtracition operation" $= -\frac{1}{12}$

9 Since L.C.M. of the denominators is 100

therefore $\frac{19}{10} + \left(-\frac{39}{100}\right) = \frac{190}{100} + \left(-\frac{39}{100}\right) = \frac{151}{100}$

4

1 $5\frac{5}{7}$

2 $9\frac{1}{5} = \frac{46}{5}$, $7\frac{3}{5} = \frac{38}{5}$

$9\frac{1}{5} - 7\frac{3}{5} = \frac{46}{5} - \frac{38}{5} = \frac{8}{5} = 1\frac{3}{5}$

Another solution :

$9\frac{1}{5} = 8\frac{6}{5}$

$9\frac{1}{5} - 7\frac{3}{5} = 8\frac{6}{5} - 7\frac{3}{5} = 1\frac{3}{5}$

3 $-10\frac{7}{8} + 4\frac{5}{8} = -6\frac{2}{8} = -6\frac{1}{4}$

"From the definition of subtraction operation"

4 Since L.C.M. of the denominators is 8

therefore $\frac{1}{4} + 2\frac{3}{8} = \frac{2}{8} + 2\frac{3}{8} = 2\frac{5}{8}$

5 $6\frac{2}{3} = \frac{20}{3}$, $3\frac{1}{6} = \frac{19}{6}$

Since L.C.M. of the denominators is 6

therefore $\frac{20}{3} - \frac{19}{6} = \frac{40}{6} - \frac{19}{6} = \frac{21}{6} = \frac{7}{2}$

6 $-15\frac{1}{2} = -\frac{31}{2}$, $2\frac{3}{8} = \frac{19}{8}$

Since L.C.M. of the denominators is 8

therefore $-\frac{31}{2} + \frac{19}{8} = -\frac{124}{8} + \frac{19}{8}$

$= -\frac{105}{8}$

7 $-2\frac{1}{2} = -\frac{5}{2}$, $-12\frac{1}{16} = -\frac{193}{16}$

Since L.C.M. of the denominators is 16

therefore $-\frac{5}{2} - \frac{193}{16} = -\frac{40}{16} - \frac{193}{16} = -\frac{233}{16}$

8 $2\frac{3}{8} = \frac{19}{8}$

Since L.C.M. of the denominators is 8

therefore $\frac{19}{8} - \frac{1}{4} = \frac{19}{8} - \frac{2}{8} = \frac{17}{8}$

9 $13\frac{3}{7} = \frac{94}{7}$

therefore $-2 + 13\frac{3}{7} = -\frac{14}{7} + \frac{94}{7}$

$= \frac{80}{7}$

5

1 $0.2 = \frac{2}{10} = \frac{1}{5}$ therefore $\frac{2}{5} + 0.2 = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

2 $|-5\frac{1}{2}| = |-\frac{11}{2}| = \frac{11}{2}$

Since L.C.M. of the denominators is 4

therefore $\frac{11}{2} - \frac{1}{4} = \frac{22}{4} - \frac{1}{4} = \frac{21}{4}$

3 $25\,\% = \frac{1}{4}$

therefore $\frac{1}{4} + \left(-\frac{1}{4}\right) = \text{zero}$

4 $0.\dot{3} = \frac{1}{3}$

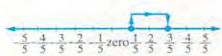
therefore $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

6

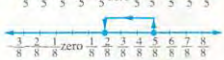
1 (c)	2 (c)	3 (c)	4 (d)	5 (b)
6 (c)	7 (c)	8 (a)	9 (c)	10 (b)
11 (a)	12 (b)	13 (c)	14 (c)	15 (d)
16 (d)	17 (a)	18 (c)	19 (c)	20 (c)

7

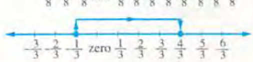
1 $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$



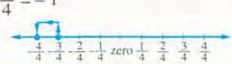
2 $\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$



3 $-\frac{1}{3} + \frac{5}{3} = \frac{4}{3}$



4 $-\frac{3}{4} + \left(-\frac{1}{4}\right) = -\frac{4}{4} = -1$



8

1 Commutative property. 2 Associative property.

3 Additive inverse 4 Additive identity

9

1 $\frac{4}{7} + \text{zero} = \frac{4}{7}$ 2 $\text{zero} + \left(-\frac{7}{10}\right) = -\frac{7}{10}$

3 $\text{zero} - \left(-\frac{17}{4}\right) = \text{zero} + \frac{17}{4}$

"From the definition of subtraction operation"

$= \frac{17}{4}$

4 $\left[\frac{1}{4} + \left(-\frac{1}{4}\right)\right] + \frac{3}{4} = \text{zero} + \frac{3}{4} = \frac{3}{4}$

5 $\frac{5}{6} + \left(-\frac{3}{6} + \frac{3}{6}\right) = \frac{5}{6} + \text{zero} = \frac{5}{6}$

6 $\left[\frac{2}{9} + \left(-\frac{4}{9}\right)\right] + \left(-\frac{3}{9}\right) = -\frac{2}{9} + \left(-\frac{3}{9}\right) = -\frac{5}{9}$

10

1 $\left(\frac{1}{4} + \frac{3}{4}\right) + \frac{1}{2} = 1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$

2 $\left(\frac{2}{7} + \frac{5}{7}\right) + \left(\frac{3}{4} + \frac{1}{4}\right) = \frac{7}{7} + \frac{4}{4} = 2$

3 $\left(\frac{5}{4} + \left(-\frac{25}{4}\right)\right) + \left(-\frac{13}{5} + \frac{28}{5}\right)$

$= -\frac{20}{4} + \frac{15}{5} = -5 + 3 = -2$

4 $\left(\frac{5}{8} + \frac{3}{8}\right) + \left(-\frac{3}{4} + \frac{3}{4}\right) = \frac{8}{8} + \text{zero} = 1$

5 $\left(\frac{2}{13} + \frac{11}{13}\right) + \left(\frac{1}{5} + \left(-\frac{6}{5}\right)\right) = \frac{13}{13} + \left(-\frac{5}{5}\right)$

$= 1 + (-1) = \text{zero}$

6 $\left(-\frac{3}{7} + \frac{1}{2}\right) + \left(-\frac{1}{14}\right) = \left(-\frac{6}{14} + \frac{7}{14}\right) + \left(-\frac{1}{14}\right)$

$= \frac{1}{14} + \left(-\frac{1}{14}\right) = \text{zero}$

7 $\frac{12}{18} = \frac{2}{3}$, $-\frac{15}{27} = -\frac{5}{9}$

$\frac{2}{3} + \frac{5}{9} + \frac{1}{3} + \left(-\frac{5}{9}\right) = \left(\frac{2}{3} + \frac{1}{3}\right) + \left(\frac{5}{9} + \left(-\frac{5}{9}\right)\right)$

$= \frac{3}{3} + \text{zero} = 1$

8 $\left(\frac{2}{3} + \frac{4}{5}\right) + \frac{3}{4} = \left(\frac{10}{15} + \frac{12}{15}\right) + \frac{3}{4}$

$= \frac{22}{15} + \frac{3}{4} = \frac{88}{60} + \frac{45}{60} = \frac{133}{60}$

9 $\frac{1}{4} + 7 + \left(-\frac{1}{4}\right) + (-11)$

$= \left(\frac{1}{4} + \left(-\frac{1}{4}\right)\right) + \left(7 + (-11)\right) = \text{zero} + (-4) = -4$

10 $-\frac{1}{8} - 13 + \frac{3}{8} + 7 = \left(-\frac{1}{8} + \frac{3}{8}\right) + (-13 + 7)$

$= \frac{2}{8} + (-6) = \frac{1}{4} + \left(-\frac{24}{4}\right)$

$= -\frac{23}{4}$

11

1 $\frac{5}{6} + \frac{1}{2} = \frac{5}{6} + \frac{3}{6} = \frac{8}{6} = \frac{4}{3}$

2 $\frac{5}{6} + \left(-\frac{1}{3}\right) = \frac{5}{6} + \left(-\frac{2}{6}\right) = \frac{3}{6} = \frac{1}{2}$

3 $\frac{5}{6} - \left(-\frac{1}{3}\right) = \frac{5}{6} + \frac{1}{3}$

"From the definition of subtraction operation"

$= \frac{5}{6} + \frac{2}{6} = \frac{7}{6}$

4 $\left(-\frac{1}{3} + \frac{1}{2}\right) - \frac{5}{6} = \left(-\frac{2}{6} + \frac{3}{6}\right) - \frac{5}{6}$

$= \frac{1}{6} - \frac{5}{6} = -\frac{4}{6} = -\frac{2}{3}$

12

$[\frac{1}{2}-(-\frac{3}{2})]^3=(\frac{1}{2}+\frac{3}{2})^3$

"From the definition of subtraction operation"

$=(\frac{4}{2})^3=(2)^3=8$

13

1 3 2 $-\frac{7}{16}$

14

1 $\frac{31}{32}$, $\frac{63}{64}$ 2 $3\frac{3}{4}$, 3 , $2\frac{1}{4}$, $1\frac{1}{2}$

15

1 $x+\frac{1}{5}=\frac{2}{5}$ therefore $x=\frac{1}{5}$

or $x+\frac{1}{5}=-\frac{2}{5}$ therefore $x=-\frac{3}{5}$

2 $\frac{3}{4}-x=\frac{1}{4}$ therefore $x=\frac{1}{2}$

or $\frac{3}{4}-x=-\frac{1}{4}$ therefore $x=1$

16

Since $(51\frac{1}{2}-1\frac{1}{2})=50$

and $(52\frac{1}{2}-2\frac{1}{2})=50$ .... and so on

Since the expression consists of 50 operation of subtraction , the result of each of them = 50

therefore the expression = 50 × 50 = 2500

Answers of Exercise 4

1

1 1 2 $\frac{7}{3}$ 3 $-\frac{9}{4}$ 4 $-\frac{1}{6}$ 5 $\frac{2}{7}$

6 2 7 1 8 -1 9 1 10 $\frac{5}{3}$

11 1 12 zero

2

1 $\frac{2}{3}$ 2 $-\frac{2}{3}$ 3 1 4 $\frac{4}{5}$ 5 $\frac{7}{2}$

6 1 7 1 8 $-\frac{11}{4}$ 9 $\frac{5}{13}$ 10 $\frac{5}{4}$

11 $-\frac{5}{4}$ 12 $\frac{2}{3}\times\frac{1}{2}$ or $\frac{1}{3}$

3

1 (b) 2 (a) 3 (c) 4 (d)

5 (b) 6 (b) 7 (a) 8 (d)

4

1 commutative 2 multiplicative inverse

3 commutative 4 multiplicative identity

5 multiplying by zero

5

1 $\frac{6}{35}$ 2 $-\frac{1}{3}$ 3 $\frac{5}{8}$

4 $-\frac{1}{4}$ 5 $-\frac{5}{12}$ 6 $-\frac{12}{35}$

7 $-\frac{4}{7}$ 8 6 9 $\frac{1}{3}$

6

1 $\frac{4}{5}\times\frac{7}{3}=\frac{28}{15}$ 2 $-\frac{1}{6}\times\frac{2}{5}=-\frac{1}{15}$

3 $-\frac{4}{11}\times(-\frac{11}{4})=1$ 4 $\frac{5}{27}\times 9=\frac{5}{3}$

5 $\frac{5}{6}\times(-\frac{2}{15})=-\frac{1}{9}$ 6 $-\frac{5}{16}\times(-\frac{8}{11})=\frac{5}{22}$

7 $-\frac{5}{8}\times\frac{8}{5}=-1$ 8 zero $\times\frac{5}{3}=$ zero

9 $\frac{3}{4}\times(-\frac{1}{9})=-\frac{1}{12}$

7

1 $\frac{7}{2}\times(-4)=-14$

2 $\frac{3}{2}\times(-\frac{3}{2})=-\frac{9}{4}$

3 $-\frac{30}{7}\times(-\frac{31}{6})=\frac{155}{7}=22\frac{1}{7}$

4 $\frac{25}{8}\times(-\frac{21}{5})=-\frac{105}{8}=-13\frac{1}{8}$

5 $-\frac{5}{10}\times\frac{2}{5}=-\frac{1}{5}$ 6 $\frac{5}{2}\times\frac{8}{10}=2$

7 $\frac{3}{2}\times\frac{5}{3}=\frac{5}{2}$ 8 $|-\frac{2}{3}|\times\frac{4}{3}=\frac{2}{3}\times\frac{4}{3}=\frac{8}{9}$

8

1 $\frac{11}{5}\times\frac{5}{11}=1$ 2 $\frac{11}{2}\div\frac{11}{5}=\frac{11}{2}\times\frac{5}{11}=\frac{5}{2}$

3 $-\frac{30}{7}\div\frac{15}{14}=-\frac{30}{7}\times\frac{14}{15}=-4$

4 $-1\div\frac{9}{4}=-1\times\frac{4}{9}=-\frac{4}{9}$

5 $-\frac{13}{3}\div(-\frac{13}{4})=-\frac{13}{3}\times-\frac{4}{13}=\frac{4}{3}$

6 $\frac{5}{10}\div\frac{11}{2}=\frac{5}{10}\times\frac{2}{11}=\frac{1}{11}$

7 $-\frac{11}{4}\div(-\frac{25}{8})=-\frac{11}{4}\times(-\frac{8}{25})=\frac{22}{25}$

8 $\frac{25}{4}\times(-\frac{1}{15})=-\frac{5}{12}$

9 $\frac{13}{5}\div(-\frac{26}{15})=\frac{13}{5}\times(-\frac{15}{26})=-\frac{3}{2}$

9

1 $\frac{5}{12}(3+9) = \frac{5}{12} \times 12 = 5$

2 $\frac{4}{9}(11+16) = \frac{4}{9} \times 27 = 12$

3 $(4+9+4) \times \frac{8}{17} = 17 \times \frac{8}{17} = 8$

4 $\frac{6}{37}(7+5-11) = \frac{6}{37} \times 1 = \frac{6}{37}$

5 $\frac{4}{5}(13-22+9) = \frac{4}{5} \times \text{zero} = \text{zero}$

6 $\frac{7}{12}(5+9-2) = \frac{7}{12} \times 12 = 7$

7 $\frac{27}{11}\left(\frac{9}{4} - \frac{1}{4} + 9\right) = \frac{27}{11} \times 11 = 27$

8 $\frac{7}{13}(6+8-1) = \frac{7}{13} \times 13 = 7$

9 $-\frac{3}{7}(8+5+1) = -\frac{3}{7} \times 14 = -6$

10 $\frac{22}{25}\left(\frac{7}{11} + \frac{5}{11} - 1\right) = \frac{22}{25} \times \frac{1}{11} = \frac{2}{25}$

11 $35\left(\frac{3}{4} + \frac{1}{2} - \frac{1}{4}\right) = 35 \times 1 = 35$

10

1 $\left(\frac{5}{6} + \frac{4}{6}\right) \div \frac{3}{5} = \frac{9}{6} \times \frac{5}{3} = \frac{5}{2}$

2 $\frac{3}{4} \times \left(\frac{3}{6} - \frac{2}{6}\right) = \frac{3}{4} \times \frac{1}{6} = \frac{1}{8}$

3 $\left(-\frac{18}{5} \times \frac{35}{9}\right) \times \left(-\frac{3}{7}\right) = -14 \times \left(-\frac{3}{7}\right) = 6$

4 $\frac{12}{35} \times \left(-\frac{14}{9}\right) = -\frac{8}{15}$

5 $\left(-\frac{5}{3} \times \frac{14}{3}\right) \div \frac{55}{9} = -\frac{70}{9} \times \frac{9}{55} = -\frac{14}{11}$

6 $\left(\frac{81}{16} \div \frac{27}{4}\right) \times \left(-\frac{68}{9}\right)$

$= \left(\frac{81}{16} \times \frac{4}{27}\right) \times \left(-\frac{68}{9}\right) = \frac{3}{4} \times \left(-\frac{68}{9}\right) = -\frac{17}{3}$

11

1 1 2 $\frac{3}{17}$ 3 zero

4 1 5 5

12

1 $x\,y\,z = -\frac{1}{3} \times \frac{3}{4} \times -3 = \frac{3}{4}$

2 $x\,y + y\,z = -\frac{1}{3} \times \frac{3}{4} + \frac{3}{4} \times -3$

$= -\frac{1}{4} - \frac{9}{4} = -\frac{10}{4} = -\frac{5}{2}$

13

1 $a\,b\,c + 3 = \frac{7}{4} \times \frac{12}{7} \times \frac{2}{3} + 3 = 2 + 3 = 5$

2 $a\,b - c = \frac{7}{4} \times \frac{12}{7} - \frac{2}{3} = 3 - \frac{2}{3} = \frac{9}{3} - \frac{2}{3} = \frac{7}{3}$

14

$x + y = \frac{5}{8} + \frac{1}{2} = \frac{5}{8} + \frac{4}{8} = \frac{9}{8}$

, $x - y = \frac{5}{8} - \frac{1}{2} = \frac{5}{8} - \frac{4}{8} = \frac{1}{8}$

, $\frac{x+y}{x-y} = \frac{9}{8} \div \frac{1}{8} = \frac{9}{8} \times 8 = 9$

15

1 $x\,y\,z = \frac{3}{2} \times \left(-\frac{1}{4}\right) \times (-2) = \frac{3}{4}$

$\frac{1}{x\,y\,z} = 1 \div \frac{3}{4} = 1 \times \frac{4}{3} = \frac{4}{3}$

2 $x - (z \div y) = \frac{3}{2} - \left(-2 \div \left(-\frac{1}{4}\right)\right)$

$= \frac{3}{2} - \left(-2 \times \left(-\frac{4}{1}\right)\right) = \frac{3}{2} - 8$

$= \frac{3}{2} - \frac{16}{2} = -\frac{13}{2}$

3 $\frac{x}{y} = \frac{3}{2} \div \left(-\frac{1}{4}\right) = \frac{3}{2} \times \left(-\frac{4}{1}\right) = -6$

$\frac{z}{y} = -2 \div \left(-\frac{1}{4}\right) = -2 \times \left(-\frac{4}{1}\right) = 8$

$\frac{x}{y} - \frac{z}{y} = -6 - 8 = -14$

4 $x + z = \frac{3}{2} + (-2) = \frac{3}{2} - \frac{4}{2} = -\frac{1}{2}$

$y - z = -\frac{1}{4} - (-2) = -\frac{1}{4} + \frac{8}{4} = \frac{7}{4}$

$(x+z) \div (y-z) = -\frac{1}{2} \div \frac{7}{4} = -\frac{1}{2} \times \frac{4}{7} = -\frac{2}{7}$

5 $x + y = \frac{3}{2} + \left(-\frac{1}{4}\right) = \frac{6}{4} - \frac{1}{4} = \frac{5}{4}$

$\frac{x+y}{z} = \frac{5}{4} \div (-2) = \frac{5}{4} \times \left(-\frac{1}{2}\right) = -\frac{5}{8}$

16

The weight of the man on the moon $= \frac{1}{6} \times 76\frac{4}{5}$

$= \frac{1}{6} \times \frac{384}{5} = \frac{64}{5}$

$= 12\frac{4}{5}$ kg.

17

The capacity of three containers $= 3 \times 20 = 60$ litres

the number of minutes needed to fill the 3 containers

$= 60 \div 2\frac{1}{2} = 60 \div \frac{5}{2} = 60 \times \frac{2}{5} = 24$ minutes

18

The number of pieces

$= 60 \div 3\frac{3}{4} = 60 \div \frac{15}{4} = 60 \times \frac{4}{15} = 16$ pieces

There is not any wire left over.

19

1 $\frac{7}{15}\left(\frac{4}{25} + \frac{1}{5}\right) + \frac{16}{25}\left(\frac{2}{3} - \frac{1}{5}\right)$

$= \frac{7}{15} \times \frac{9}{25} + \frac{16}{25} \times \frac{7}{15}$

$= \frac{7}{15}\left(\frac{9}{25} + \frac{16}{25}\right) = \frac{7}{15} \times 1 = \frac{7}{15}$

2 $\frac{2}{13} \times 3 + \frac{2}{13} \times 8 + \frac{2}{13} \times 2$

$= \frac{2}{13}(3 + 8 + 2) = \frac{2}{13} \times 13 = 2$

20

$\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{5} \times \text{........} \times \frac{99}{100} = \frac{1}{100}$

and if the last rational number is $\frac{n-1}{n}$

the result will be $\frac{1}{n}$

Answers of Exercise 5

1 **Let the required number be L :**

1 $L = \frac{3}{8} + \frac{1}{2}\left|\frac{5}{8} - \frac{3}{8}\right| = \frac{3}{8} + \frac{1}{2} \times \frac{2}{8}$

$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

2 $L = \frac{2}{5} + \frac{1}{2}\left|\frac{4}{5} - \frac{2}{5}\right| = \frac{2}{5} + \frac{1}{2} \times \frac{2}{5}$

$= \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

3 $L = \frac{3}{4} - \frac{1}{2}\left|\frac{3}{4} - \left(-\frac{3}{4}\right)\right| = \frac{3}{4} - \frac{1}{2}\left|\frac{3}{4} + \frac{3}{4}\right|$

$= \frac{3}{4} - \frac{1}{2} \times \frac{6}{4} = \frac{3}{4} - \frac{3}{4} = \text{zero}$

4 The distance between the two numbers

$= \left|\frac{1}{2} - \frac{7}{8}\right| = \left|\frac{4}{8} - \frac{7}{8}\right| = \left|-\frac{3}{8}\right| = \frac{3}{8}$

Then $L = \frac{4}{8} + \frac{1}{2} \times \frac{3}{8} = \frac{4}{8} + \frac{3}{16} = \frac{8+3}{16} = \frac{11}{16}$

5 The distance between the two numbers

$= \left|-\frac{1}{2} - \left(-\frac{3}{4}\right)\right| = \left|-\frac{2}{4} + \frac{3}{4}\right| = \left|\frac{1}{4}\right| = \frac{1}{4}$

Then $L = -\frac{2}{4} - \frac{1}{2} \times \frac{1}{4} = -\frac{2}{4} - \frac{1}{8} = \frac{-4-1}{8} = -\frac{5}{8}$

6 The distance between the two numbers

$= \left|0.1 - \left(-\frac{2}{5}\right)\right| = \left|\frac{1}{10} + \frac{4}{10}\right| = \left|\frac{5}{10}\right| = \frac{1}{2}$

Then $L = \frac{1}{10} - \frac{1}{2} \times \frac{1}{2} = \frac{1}{10} - \frac{1}{4} = \frac{2-5}{20} = -\frac{3}{20}$

7 The distance between the two numbers

$= \left|-\frac{11}{9} - \left(-\frac{13}{35}\right)\right| = \left|-\frac{385}{315} + \frac{117}{315}\right| = \left|-\frac{268}{315}\right| = \frac{268}{315}$

Then $L = -\frac{385}{315} + \frac{1}{2} \times \frac{268}{315} = -\frac{385}{315} + \frac{134}{315} = -\frac{251}{315}$

8 The distance between the two numbers

$= \left|-4\frac{3}{7} - 8\frac{1}{3}\right| = \left|-\frac{31}{7} - \frac{25}{3}\right| = \left|-\frac{93}{21} - \frac{175}{21}\right|$

$= \left|-\frac{268}{21}\right| = \frac{268}{21}$

Then $L = -\frac{93}{21} + \frac{1}{2} \times \frac{268}{21} = -\frac{93}{21} + \frac{134}{21} = \frac{41}{21}$

9 $L = \text{zero} + \frac{1}{2}\left|\frac{2}{5} - \text{zero}\right| = \text{zero} + \frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$

2

1 The distance between the two numbers

$= \left|\frac{5}{7} - \left(-\frac{3}{7}\right)\right| = \left|\frac{5}{7} + \frac{3}{7}\right| = \frac{8}{7}$

Then the number $= -\frac{3}{7} + \frac{1}{4} \times \frac{8}{7} = -\frac{3}{7} + \frac{2}{7} = -\frac{1}{7}$

2 The distance between the two numbers

$= \left|\frac{1}{3} - 1\right| = \left|\frac{1}{3} - \frac{3}{3}\right| = \left|-\frac{2}{3}\right| = \frac{2}{3}$

Then the number $= 1 - \frac{1}{4} \times \frac{2}{3} = 1 - \frac{1}{6}$

$= \frac{6-1}{6} = \frac{5}{6}$

3 The distance between the two numbers

$= \left|-\frac{3}{5} - \left(-\frac{4}{5}\right)\right| = \left|-\frac{3}{5} + \frac{4}{5}\right| = \frac{1}{5}$

Then the number $= -\frac{3}{5} - \frac{1}{3} \times \frac{1}{5} = -\frac{3}{5} - \frac{1}{15} = \frac{-9-1}{15}$

$= -\frac{10}{15} = -\frac{2}{3}$

4 The distance between the two numbers

$= \left|\frac{4}{7} - 1\frac{3}{4}\right| = \left|\frac{4}{7} - \frac{7}{4}\right| = \left|\frac{16}{28} - \frac{49}{28}\right| = \frac{33}{28}$

Then the number $= \frac{16}{28} + \frac{1}{3} \times \frac{33}{28} = \frac{16}{28} + \frac{11}{28} = \frac{27}{28}$

5 The distance between the two numbers

$= \left|-\frac{1}{2} - \left(-\frac{2}{5}\right)\right| = \left|-\frac{1}{2} + \frac{2}{5}\right| = \left|-\frac{5}{10} + \frac{4}{10}\right| = \frac{1}{10}$

Then the number

$= -\frac{4}{10} - \frac{1}{5} \times \frac{1}{10} = -\frac{4}{10} - \frac{1}{50} = \frac{-20-1}{50} = -\frac{21}{50}$

6 The distance between the two numbers

$= |-\frac{2}{3} - (-\frac{3}{5})| = |-\frac{2}{3} + \frac{3}{5}| = |-\frac{10}{15} + \frac{9}{15}| = \frac{1}{15}$

Then the number $= -\frac{10}{15} + \frac{1}{5} \times \frac{1}{15} = -\frac{10}{15} + \frac{1}{75}$

$= \frac{-50+1}{75} = -\frac{49}{75}$

7 The distance between the two numbers

$= |\frac{5}{6} - \frac{2}{3}| = |\frac{5}{6} - \frac{4}{6}| = \frac{1}{6}$

Then the number $= \frac{4}{6} + \frac{1}{10} \times \frac{1}{6} = \frac{4}{6} + \frac{1}{60}$

$= \frac{40+1}{60} = \frac{41}{60}$

8 The distance between the two numbers

$= |\text{zero} - (-1\frac{1}{2})| = |\text{zero} + \frac{3}{2}| = \frac{3}{2}$

, then the number from the side of the greater

$= \text{zero} - \frac{1}{8} \times \frac{3}{2} = -\frac{3}{16}$

, the number from the side of the smaller

$= -\frac{3}{2} + \frac{1}{8} \times \frac{3}{2} = -\frac{3}{2} + \frac{3}{16} = \frac{-24+3}{16} = -\frac{21}{16}$

3

1 (c)	2 (a)	3 (c)	4 (d)
5 (d)	6 (b)	7 (a)	

4 The distance between the tree and the lamp post

$= |7\frac{1}{2} - 3.3| = |\frac{15}{2} - \frac{33}{10}|$

$= |\frac{75}{10} - \frac{33}{10}| = \frac{42}{10} = \frac{21}{5}$

Then the distance where the flower

bed should be put at $= 3.3 + \frac{1}{3} \times \frac{21}{5} = \frac{33}{10} + \frac{7}{5}$

$= \frac{33}{10} + \frac{14}{10} = \frac{47}{10} = 4.7$ m.

Answers of unit two

Answers of Exercise 6

1

Coefficient	3	7	– 8	1
Degree	zero	5	3	3

2

Number of terms	Name	Degree
3	trinomial	3
3	trinomial	4
2	binomial	5
4	4 terms	5

3

1 third , 3 2 $\frac{1}{2}$, sixth 3 1 , first 4 zero
5 – 8 , zero 6 second 7 3 , second

4

1 (b) 2 (c) 3 (c) 4 (b)
5 (c) 6 (b) 7 (d)

5

1 $5a^5b^3 - 3a^2b^5 + 7ab$ 2 $-7 + 5x + x^2 + x^3$

6

The expression = $(x \times y) - (\frac{1}{2} \times 1 \times 2) = xy - 1$ of the second degree.

7 The expression = $\frac{1}{2}$ bh – πr^2 of second degree

8

1 5 2 3 , 5 3 3
4 2 5 {zero , 1 , 2 , 3}

Answers of Exercise 7

1

1 $5x$ 2 $3x$ 3 $-7x$ 4 $-10x$
5 $-2a^2$ 6 x^2y 7 a 8 zero
9 $2x$ 10 $\frac{2x}{7}$

2

1 $-3y^2 - y^2 = -4y^2$
2 $9x^2y - (-6x^2y) = 9x^2y + 6x^2y = 15x^2y$
3 $-2x - (-5x) = -2x + 5x = 3x$
4 $3a^2b - a^2b = 2a^2b$
5 $2ab - (-3ab) = 2ab + 3ab = 5ab$
6 $-7x^2y - 6x^2y = -13x^2y$

3

1 $4a$ 2 $8x^2$ 3 $-2m$
4 $-5x$ 5 $2a$ 6 $10x$
7 $3x$ 8 $-2x$ 9 $2x , -2x$

4

1 (c) 2 (b) 3 (b)
4 (d) 5 (d) 6 (b)

5 1 $5a^2$ 2 $2x^2$ 3 $-2m^2$
4 $-2a^2b$ 5 $7a^2b$ 6 11
7 65 8 $22x$ 9 $14x$

6 The other term = $12x^2y - 4x^2y = 8x^2y$

7

1 $8a + 6b$ 2 $2x - 3y$ 3 $-7x - 7y$
4 $30m - 12n$ 5 $-4a + 3$ 6 $-5b$
7 $-6y - 7x$ 8 $6a + 13b$

8

1 $-10x^2 - x + 3$ 2 $x^2y - xy^2 + 2x^2y^2$
3 $4a^2 - 2a - 4$ 4 $6x^2 - 9x + 5$

9

1 The expression $= 3x + 1 + 3x^2 + x = 3x^2 + 4x + 1$
2 The expression $= 2x^2 + x + 4x + 2 = 2x^2 + 5x + 2$
3 The expression $= 5x^2 + 2x + 15x + 6 = 5x^2 + 17x + 6$

10

1 The expression $= x + 3 + 5 + x + 3 + x + x + 5$
$= 4x + 16$
2 The expression $= y + 3 + x + 2 + 3 + y + 2 + x$
$= 2y + 2x + 10$
3 The expression $= y + x + y + \frac{1}{2}x + x + \frac{1}{2}x + y + y$
$= 4y + 3x$

11 The perimeter $= 4 + 4 + 4 - x + x + x + 4 - x$
$= 16$ cm.

12 [1] 3 [2] 1 , 2 [3] 12

Answers of Exercise 8

1
[1] $8a + 2b + 4c$ [2] $4a - 3b - c$
[3] $10x - 2y + 4$ [4] $3a^3 - 2a^2b - ab^2$

2
[1] $4x + 3$ [2] 2ℓ
[3] $2n^2 + 2n - 3$ [4] $2m^2 + \ell^2$
[5] $a^2b - 3ab^2 + 2b^3$ [6] $4a^3 - 2ab^2 + 4a^2b$

3
[1] $a + 2$ [2] $7x + 3y + 3z$
[3] 1 [4] zero
[5] $-2x^2 + 4x + 9$ [6] $x^2 + xy - y^2$

4
[1] $x - 3$ [2] $-11y + 9$
[3] $3x^2 + 2$ [4] $a^3 + 2a^2b + 3b^3 - 3ab^2$

5
[1] $2a + 9b$ [2] $5x + 6y$
[3] $-2x^2 - 7x + 2$ [4] $7x^2y - 8x$

6
[1] $-5a + 2b$ [2] $2x^2 - 3xy - 2y^2$
[3] $a^2 + 4ab + 9b^2$ [4] $2x^2 - 3x + 3$

7
[1] (c) [2] (c) [3] (a)
[4] (b) [5] (d)

8 The expression $= 6 + x^2 - x - (2x - 3x^2 + 5)$
$= 4x^2 - 3x + 1$

9 The expression $= 2x - 3y + 6z - \ell$
$- (5z - 4y + 3x - 2\ell) = -x + y + z + \ell$

10 The expression $= 0 - (3a^2 - 5ab + 2b^2)$
$= -3a^2 + 5ab - 2b^2$

11 The other expression $= 5x - 7y + 9$
$- (2y + 3x - 4) = 2x - 9y + 13$

12 The remainder $= a + 5b - 2$
The numerical value $= 2 + 5 \times 1 - 2 = 5$

13 The sum $= 4x - 5y - 6z$
The remainder $= 5x + 5y - z - (4x - 5y - 6z)$
$= x + 10y + 5z$

14 The sum $= 5a - 7b - 7c$
The decrease $= 5a - 7b - 7c - (2a - 8b - c)$
$= 3a + b - 6c$

15 The sum $= \ell + 2m + 2n$
The remainder $= 2\ell - 4m + 5n - (\ell + 2m + 2n)$
$= \ell - 6m + 3n$

16 The sum $= 7x^2 - x - 3$
The increase $= 3x^2 - 5 + 2x - (7x^2 - x - 3)$
$= -4x^2 + 3x - 2$

17 The sum $= x^2 - xy + x - 5$
The numerical value $= (-1)^2 - (-1) \times 2 + (-1) - 5$
$= 1 + 2 - 1 - 5 = -3$

18 $x + y - z = (a - 2b + c) + (2a + 3b - 4c)$
$- (b - 4a + c) = 7a - 4c$

19 The total surface area of the first solid
= The area of six faces
$= 2 \times a \times b + 2 \times 5 \times b + 2 \times 5 \times a$
$= 2ab + 10b + 10a$
The total surface area of the second solid
= The area of six faces
$= 2 \times a \times b + 2 \times 3 \times b + 2 \times 3 \times a$
$= 2ab + 6b + 6a$
The sum of the two areas
$= (2ab + 10b + 10a) + (2ab + 6b + 6a)$
$= 4ab + 16b + 16a$

20 1 Since , $a + b + b + c = \frac{5}{4} + \frac{3}{4}$

Therefore , $a + 2b + c = \frac{8}{4} = 2$

2 Since , $a + b + b + c - a - c = \frac{5}{4} + \frac{3}{4} - \frac{1}{2}$

Therefore , $2b = \frac{8}{4} - \frac{2}{4} = \frac{6}{4}$

Therefore , $b = \frac{6}{4} \times \frac{1}{2} = \frac{3}{4}$

21 $5x$

Answers of Exercise 9

1

1 $15xy$ 2 $-21ac$ 3 $-6x^2$

4 $56y^9$ 5 $-6x^3y$ 6 $10x^4y^6$

7 $-10a^3b^3$ 8 $2x^3$ 9 $-40a^2$

10 $6a^2b^2$ 11 $30x^9$ 12 $24x^6y^8$

2

1 $3a$ 2 -12 3 5 4 $-2x$

5 $5a^4$ 6 -1 7 $\frac{3}{2}x^2y^3$ 8 $8b^4$

9 $-2m^3n$ 10 $3x^2y^3$

3

1 t^8 2 $6a^7$ 3 $6a^4b^3$

4 $\frac{1}{2}x^5$ 5 $6h^4k^8$ 6 $-7m^6$

4

1 (d) 2 (d) 3 (d)

4 (d) 5 (d) 6 (b)

7 (a) 8 (a) 9 (b)

5

1 $6y^2$ 2 $3x^2 - 2x$ 3 $3x$

4 -5 5 3ℓ 6 $-35a^5$

7 $5xy$ 8 $-64x^7y^6$ 9 $4a^3$

6 1 $3a^2b^6$ 2 $3a^4$ 3 $-2c^2d$

4 $7b^3$ 5 $2a^3b^2$ 6 $7xy^3$

7 1 $9y$ 2 $-\frac{2}{3}xy^n$

8

The volume of the cuboid $= x \times 2x \times 4x = 8x^3$ cm^3.

The volume of the small cube $= x \times x \times x = x^3$ cm^3.

The number of the small cubes $= \frac{8x^3}{x^3} = 8$ cubes.

9

1 The perimeter $= 2(3ab + 2ab) = 10ab$

The area $= 3ab \times 2ab = 6a^2b^2$

2 The perimeter $= 2x + x + x + 3x + 3x + 4x = 14x$

The area $= 4x \times 3x - x \times x$

$= 12x^2 - x^2 = 11x^2$

3 The perimeter $= 4a + a + a + a + a + a + 4a + a$

$+ a + a + a + a = 18a$

The area $= (4a \times a) + (2a \times a) + (4a \times a)$

$= 4a^2 + 2a^2 + 4a^2 = 10a^2$

10 The total surface area of the first solid = the sum of areas of its six faces

$= (3y \times 2x \times 2) + (3y \times a \times 2) + (2x \times a \times 2)$

$= 12xy + 6ya + 4xa$

The total surface area of the second solid

= the sum of areas of its six faces

$= (x \times y \times 2) + (y \times 3a \times 2) + (x \times 3a \times 2)$

$= 2xy + 6ay + 6xa$

The total surface area of the two solids

$= 12xy + 6ya + 4xa + 2xy + 6ay + 6xa$

$= 14xy + 12ay + 10xa$

11 Let the radius length of the ball be r

Then , the dimensions of the box are :

6 r , 2 r and 2 r

Then , $\frac{\text{Volume of the three balls}}{\text{Volume of the box}} = \frac{3 \times \frac{4}{3} \times \pi r^3}{6r \times 2r \times 2r}$

$= \frac{4\pi r^3}{24r^3} = \frac{\pi}{6} \simeq \frac{3.14}{6} = \frac{157}{300}$

12 The solid consists of 12 lateral faces equal in area beside the area of two bases.

The lateral area $= (12) \times (3x) \times (x) = 36x^2$

The area of the base = the sum of areas of 5 squares (equal in area) $= 5x^2$

The total area $= 36x^2 + 10x^2 = 46x^2$

The volume = the sum of volumes of 5 cuboids equal in volume where the dimensions of each are x , x and $3x$

$= 5 \times x \times x \times 3x = 15x^3$

Answers of Exercise 10

1

1. $a^2 + a$
2. $a^2 - 2a$
3. $21xy - 12xz$
4. $-3y - 9$
5. $-14c + 6c^2$
6. $6x^3 + 8xy^2$
7. $-10x^2 - 5xy + 15xz$
8. $6x^3y - 15x^3y^2 - 12xy^3$
9. $\ell^3m^2 - 3\ell^2m^3 - 4\ell m^4$
10. $2x^4 - 3x^3y - x^2y^2$

2

1. $4y^3 - 2y^2 - 10y$
2. $-4xy^3 - 3x^2y^2 + 5y^2$
3. $-20x^2y + 16xy^2 - 4x^2y^2$
4. $-2xy$, $-2xy^2$

3

1. 6 , $2x^2$
2. $2x$, $15xy$
3. $4x^2$, $10xy$
4. $5x^2y$, $12x^2y^2$
5. $3ab^2$, $4a^3b^2$
6. 5 , $6x^2$
7. $2b^2$, $-8a^2b$
8. $2x$, $2xy$
9. $5y$, $2x$
10. a , b , c
11. $2x$, $4y$, $15x^3y$
12. $2mn^2$, $5m$

4

1. $3a^2 - 3ab + 8a^2 + 4ab = 11a^2 + ab$
2. $12a^2 - 6a - 12a^2 + 8a = 2a$
3. $12a^2 - 3a + 2a^2 + 6a - 10a^2 + 5a = 4a^2 + 8a$
4. $2x^2 + 2xy - 2xy + y^2 + 2y^2 - 2x^2 = 3y^2$

5 The expression $= 6a^2 - 2a + 3a^2 + 6a = 9a^2 + 4a$

The numerical value $= 9(1)^2 + 4(1) = 9 + 4 = 13$

6 The expression $= 6a^2 + 2ab - 3ab - 3b^2$

$= 6a^2 - ab - 3b^2$

The numerical value $= 6 \times 1^2 - 1 \times 1 - 3 \times (1)^2$

$= 6 - 1 - 3 = 2$

7 The expression $= 2x^2 - xy - 2xy + 2y^2$

$= 2x^2 - 3xy + 2y^2$

The numerical value $= 2 \times 2^2 - 3 \times 2 \times (-1) + 2 \times (-1)^2$

$= 8 + 6 + 2 = 16$

8 The expression $= 2x(3x - 2y) + y(x + y) + (x^2 - y^2)$

$= 6x^2 - 4xy + xy + y^2 + x^2 - y^2$

$= 7x^2 - 3xy$

The numerical value $= 7 \times (-2)^2 - 3 \times (-2) \times (-1)$

$= 28 - 6 = 22$

9 The expression $= 3 - 6x - x^2 + 5x - 3 + 2x^2 + 6x$

$= x^2 + 5x$

The numerical value $= (-2)^2 + 5(-2) = 4 - 10 = -6$

10 The expression $= 3a^2b - 2ab^2 - 2a^2b + 2ab^2$

$+ 4ab^2 - a^2b = 4ab^2$

The numerical value $= 4 \times 1 \times (-3)^2$

$= 4 \times 1 \times 9 = 36$

11 The expression =

$2x[x - 2y + 2x] - 3y[y - 2x + 2y]$

$= 2x(3x - 2y) - 3y(-2x + 3y)$

$= 6x^2 - 4xy + 6xy - 9y^2$

$= 6x^2 + 2xy - 9y^2$

The numerical value $= 6 \times 1^2 + 2 \times 1 \times 1 - 9 \times 1^2$

$= 6 + 2 - 9 = -1$

12 $a + 3(b + c) = a + 3b + 3c$

$= (a + 3b) + 3c = 7 + 3 \times 3 = 7 + 9 = 16$

13 The perimeter of the triangle

$= 3(22x - 3y + 5z)$

$= (66x - 9y + 15z)$ cm.

14 The perimeter of the rectangle

$= 2(2a + b + 4a - 2b) = 2(6a - b)$

$= (12a - 2b)$ cm.

15

1. The expression $= x(3x + 2y) = 3x^2 + 2xy$
2. The expression $= 2y(2y + 2) = 4y^2 + 4y$
3. The expression $= 3x(x + 9) - 4x$
   $= 3x^2 + 27x - 4x = 3x^2 + 23x$
4. The expression $= 4x(2y + 5) - (5 \times 2x)$
   $= 8xy + 20x - 10x = 8xy + 10x$
5. The expression $= 6x(5x - 1) - 4(2x^2 - 2)$
   $= 30x^2 - 6x - 8x^2 + 8 = 22x^2 - 6x + 8$

6 The expression

$= 5x(3x+5) - (x \times x + x \times x + 2 \times x)$

$= 15x^2 + 25x - x^2 - x^2 - 2x$

$= 13x^2 + 23x$

7 The expression $= 3x(2x+8) - \frac{1}{2} \times 3x(2x+8)$

$= 6x^2 + 24x - 3x^2 - 12x$

$= 3x^2 + 12x$

8 The expression $= 4y(3x+4) + xy - xy$

$= 12xy + 16y$

9 The expression $= x(x+5) + (x \times x)$

$= x^2 + 5x + x^2 = 2x^2 + 5x$

10 The expression $= (x \times 3x) + x(x+8)$

$= 3x^2 + x^2 + 8x = 4x^2 + 8x$

16 The width of the rectangle $= x$ cm. , hence

The length of the rectangle $= (2x+3)$ cm.

The area of the rectangle $= x(2x+3)$

$= (2x^2 + 3x)$ cm^2.

17 The volume = length × width × height

$= 3x \times 3x \times (2x^2+3) = 9x^2(2x^2+3)$

$= (18x^4 + 27x^2)$ cm^3.

Answers of Exercise 11

1

1 x^2 2 $-3x$ 3 y^2 , 20

4 $10a$, 21 5 $2x^2$, $9x$ 6 $14xy$

2

1 $x^2 + 6x + 8$ 2 $y^2 - 3y - 10$

3 $30m^2 - 7m - 2$ 4 $8x^2 + 14x + 3$

5 $6a^2 + 11ab - 35b^2$ 6 $6x^2 + 5xy - 4y^2$

7 $b^4 - 2b^2 - 8$ 8 $6m^4 + 7m^2 - 24$

9 $-x^2 + 8xy - 7y^2$ 10 $\frac{9}{4}a^2 - 3ab - 24b^2$

3

1 $a^2 + 6a + 9$ 2 $4y^2 + 12y + 9$

3 $16m^2 - 56m + 49$ 4 $9x^2 + 6xy + y^2$

5 $x^2 - 6xy + 9y^2$ 6 $\ell^2 + 2\ell m + m^2$

7 $16a^2 + 56a + 49$ 8 $4x^2 + 12xy + 9y^2$

9 $16x^4 - 4x^2y^2 + \frac{1}{4}y^4$

4

1 $a^2 - 9$ 2 $16m^2 - 49$

3 $36x^2 - 4y^2$ 4 $a^4 - 81$

5 $144m^2 - 81$ 6 $9x^4 - 25y^4$

7 $\ell^2 m^2 - 36n^2$ 8 $\frac{1}{4}x^2 - \frac{1}{9}y^2$

9 $4x^2 - 9y^2$ 10 $y^4 - 81$

11 $x^4 - 16y^4$

5

1 $x^3 + 4x^2 + 4x + 3$ 2 $x^3 + 1$

3 $2y^3 + 3y^2 + 11y + 5$ 4 $8x^3 - 4x + 21$

5 $4x^3 - 8x^2y + 5xy^2 - y^3$

6 $3a^6 - 11a^4b^2 + 11a^2b^4 - 15b^6$

7 $2a^4 + 4a^3 - 11a^2 - 2a + 5$

8 $-3a^3 + 4a^2 + 8$

9 $3x^3 + 26x^2 + 64x + 32$

10 $27x^3 + 54x^2y + 36xy^2 + 8y^3$

6

1 (b)	2 (a)	3 (d)	4 (a)
5 (c)	6 (d)	7 (c)	8 (a)
9 (d)	10 (c)	11 (b)	12 (c)
13 (d)	14 (a)	15 (d)	16 (b)

7

1 $4x^2$ 2 49

3 $x + 5$ 4 y , $3x$

5 3 , $8x$ 6 4 , a^2 , $8a$

7 2 , $6x^2$, $19x$ 8 1 , $2x$, $18x$

9 $3b$, $4a$, $2ab$ 10 x , 3 , 12

11 $4a$, $16a^2$, $9b^2$

8

1 $x^2 - 6x$ 2 $50a^3 - 32ab^2$

3 $3m^2 - 9m - 30$ 4 $-4 - x$ 5 -11

6 $5x^2 + 15x - 1$ 7 1 8 $-4x + 8$

9 $3x^2 + 4xy - 5y^2$ 10 $7a^2 + b^2$

11 $-40xy$ 12 $24x^2y^2 - 32x^2y + 8x^2$

13 $67x + 11$

9

1 The expression $= x^2 - 25y^2$

The numerical value $= 1^2 - 25 \times (-2)^2$

$= 1 - 100 = -99$

2 The expression $= 3x^2 + 10xy + 3y^2$

The numerical value

$= 3 \times 1^2 + 10 \times 1 \times (-2) + 3 \times (-2)^2$

$= 3 - 20 + 12 = -5$

3 The expression $= 3x^2 + 14x + 8$

The numerical value $= 3 \times (1)^2 + 14 \times (1) + 8$

$= 3 + 14 + 8 = 25$

4 The expression $= 6y^2 + 29y + 28$

The numerical value $= 6 \times (-2)^2 + 29 \times (-2) + 28$

$= 24 - 58 + 28 = -6$

5 The expression $= |x^2 - 4y^2|$

The numerical value $= |1^2 - 4 \times (-2)^2|$

$= |1 - 16| = |-15| = 15$

6 The expression $= 4x^2 + 4xy + y^2$

The numerical value $= 4 \times 1^2 + 4 \times 1 \times (-2) + (-2)^2$

$= 4 - 8 + 4 = 0$

10

The expression $= 4x^2 - 25 + 25 = 4x^2$

The numerical value $= 4 \times (2)^2 = 4 \times 4 = 16$

11

The expression $= x^2 - 2xy + y^2 + 2xy = x^2 + y^2$

The numerical value $= (-1)^2 + (2)^2 = 5$

12

The expression $= 4x^2 - 8x + 4 + x^2 - 4 = 5x^2 - 8x$

The numerical value $= 5 \times (-1)^2 - 8 \times (-1) = 5 + 8 = 13$

13

The expression $= x^2 + 7x + 10 - 6x + x^2$

$= 2x^2 + x + 10$

The numerical value $= 2 \times (-1)^2 + (-1) + 10$

$= 2 - 1 + 10 = 11$

14

The remainder $= (2x^2 + 19x + 9) - (x^2 - 6x + 9)$

$= 2x^2 + 19x + 9 - x^2 + 6x - 9 = x^2 + 25x$

15 The expression $= (3x - 4)(x + 2) - (2x - 3)^2$

$= 3x^2 + 2x - 8 - (4x^2 - 12x + 9) = -x^2 + 14x - 17$

The numerical value $= -17$

16 Since : $2a^2 = 2(16x^2 - 24x + 9)$

$= 32x^2 - 48x + 18$,

$-3b^2 = -3(4x^2 + 4x + 1)$

$= -12x^2 - 12x - 3$,

$bc = (2x + 1)(3x - 2) = 6x^2 - x - 2$

Therefore : $2a^2 - 3b^2 + bc = 32x^2 - 48x$

$+ 18 - 12x^2 - 12x - 3 + 6x^2 - x - 2$

$= 26x^2 - 61x + 13$

17

1 The area of the coloured part

$= 2(x - y)(2x + 3y) - (x - y)(x + y)$

$= 4x^2 + 2xy - 6y^2 - x^2 + y^2$

$= (3x^2 + 2xy - 5y^2)$ cm^2.

2 The area of the coloured part

$= 2x(x + 5) - (x - 1)(x + 2)$

$= 2x^2 + 10x - x^2 - x + 2$

$= (x^2 + 9x + 2)$ cm^2.

18

1 The perimeter of the coloured region

$= 2x + 5 + 3x - 1 + x + 2x + 5$

$+ 3x + 5 + 5x + 4$

$= 16x + 18$

The area of the coloured region

$= (5x + 4)(3x + 5) - x(3x - 1)$

$= 15x^2 + 37x + 20 - 3x^2 + x$

$= 12x^2 + 38x + 20$

2 The perimeter of the coloured region

$= x + y + x - 2y + 2x + y + 3x + y + 3x$

$+ 2y + 4x - y$

$= 14x + 2y$

The area of the coloured region

$= (3x + 2y)(4x - y) - (2x + y)(x - 2y)$

$= 12x^2 + 5xy - 2y^2 - 2x^2 + 3xy + 2y^2$

$= 10x^2 + 8xy$

3 The perimeter of the coloured region

$= x - y + x + 2y + 2x - y + x + y + 3x - 2y + 2x + 3y$

$= 10x + 2y$

The area of the coloured region

$= (2x + 3y)(3x - 2y) - (2x - y)(x + 2y)$

$= 6x^2 + 5xy - 6y^2 - 2x^2 - 3xy + 2y^2$

$= 4x^2 + 2xy - 4y^2$

19

1 $(101)^2 = (100 + 1)^2 = 10000 + 200 + 1 = 10201$

2 $\left(10\frac{1}{2}\right)^2 = \left(10 + \frac{1}{2}\right)^2 = 100 + 10 + \frac{1}{4} = 110\frac{1}{4}$

3 $(99)^2 = (100 - 1)^2 = 10000 - 200 + 1 = 9801$

4 $64 \times 56 = (60 + 4)(60 - 4) = 3600 - 16 = 3584$

5 $98 \times 102 = (100 - 2)(100 + 2) = 10000 - 4 = 9996$

6 $19 \times 21 = (20 - 1)(20 + 1) = 400 - 1 = 399$

7 $201 \times 199 = (200 + 1)(200 - 1) = 40000 - 1 = 39999$

8 $(49)^2 = (50 - 1)^2 = 2500 - 100 + 1 = 2401$

9 $(41)^2 = (40 + 1)^2 = 1600 + 80 + 1 = 1681$

20

$(2 - y)^3 = 8 - 12y + 6y^2 - y^3$

$(2 - y)^4 = (2 - y)^3(2 - y) = (8 - 12y + 6y^2 - y^3)(2 - y)$

$= 16 - 24y + 12y^2 - 2y^3 - 8y + 12y^2 - 6y^3 + y^4$

$= 16 - 32y + 24y^2 - 8y^3 + y^4$

21 Since : $(x + 8)(x + 2) = 100$

Therefore : $x^2 + 10x + 16 = 100$

Therefore : $x^2 + 10x = 84$

Therefore : $(x + 4)(x + 6) = x^2 + 10x + 24$

$= 84 + 24 = 108$

22 The area of the square

$= (2x + 5)^2 = (4x^2 + 20x + 25)$ cm².

The length of the rectangle

$= 2x + 5 + x - 1 = (3x + 4)$ cm.

The width of the rectangle

$= 2x + 5 - (x - 1) = (x + 6)$ cm.

The area of the rectangle $= (3x + 4)(x + 6)$

$= (3x^2 + 22x + 24)$ cm².

Answers of Exercise 12

1

1 $a - 2$ 2 $-4x - 5y$ 3 $2a + 3$

4 $-4x + 3$ 5 $3a + 5b$ 6 $4ab - 6b$

7 $7x^2 + 2x - 4$ 8 $-a + 2b - 4$ 9 $a - 2b + 3$

2

1 $13x + 7x^3$ 2 $-9m^2 - 16$ 3 $6x - 10$

4 $3\ell^2 m^2 - 6$ 5 $-4x^2 + 6 - 9x^4$

6 $-\frac{5}{3}x^2y - \frac{2}{3}y + \frac{1}{3}$

7 $-\ell^2 m^3 + 4m^2n^2 + 3mn^4$

8 $-3x^2 + 7x^3y^2 - 5x^4y^3$

3 1 (d) 2 (c) 3 (b)

4 (c) 5 (b) 6 (b)

7 (d)

4

1 $-5n$, $3m^4$ 2 $2a + 1$ 3 $-2x + y$, zero

4 $12x^3y^3$, $24x^2y^4$, $2x^2y$, $\frac{3}{2}xy^2$, $3y^3$

5 $3a^2b^2$, a^3b^2 6 $-3a^2b^2$, $-3ab$

7 3 8 6 9 15

5 The product $= 12x^5y^3 - 24x^4y^3$

The quotient $= xy - 2y$

6 The quotient $= -x^2 - 2xy + 7$,

The sum $= -6x^2 + 3y^2 + 7$

7 The quotient $= 3y^2 - 2y$

The absolute value $= \left|3\left(\frac{1}{2}\right)^2 - 2 \times \frac{1}{2}\right|$

$= \left|\frac{3}{4} - 1\right| = \left|-\frac{1}{4}\right| = \frac{1}{4}$

8 The quotient $= 3x - y$

The numerical value $= 3 \times 1 - (-1) = 4$

9 The quotient $= 4x^2 - 3x + 2$

The sum $= 3x^2 + 9$

The numerical value $= 3(1)^2 + 9 = 12$

10 The length of the rectangle

$$= \frac{\text{The area of the rectangle}}{\text{The width}} = \frac{24x^3 + 18x^2 + 42x}{6x}$$

$$= (4x^2 + 3x + 7) \text{ cm.}$$

11 The width of the rectangle

$$= \frac{\text{The area of the rectangle}}{\text{The length}} = \frac{8a^4b^3 + 12a^3b^4 - 8a^2b^2}{4a^2b^2}$$

$= 2a^2b + 3ab^2 - 2$

When : $a = 1$, $b = 2$

Therefore , the width $= 2 \times 1^2 \times 2 + 3 \times 1 \times 2^2 - 2$

$= 14$ cm.

12 The height of the triangle

$$= \frac{2 \times (\text{The area of the triangle})}{\text{The length of the base}}$$

$$= \frac{2(12x^2 + 9x)}{3x} = \frac{24x^2 + 18x}{3x} = (8x + 6) \text{ cm.}$$

13 The area of the base $= (2x) \times (2x) = 4x^2$ cm².

$$\text{The height} = \frac{\text{Volume}}{\text{Base's area}} = \frac{12x^3 + 8x^2y}{4x^2}$$

$= (3x + 2y)$ cm.

When : $x = 1$ and $y = 2$

Therefore , the height $= 3 \times 1 + 2 \times 2 = 7$ cm.

14

AD $= (xy + 10)$ cm.

The area of the rectangle ABCD

$= 4xy(xy + 10) = 4x^2y^2 + 40xy$ and

the area of the rectangle MNEF

= The area of the rectangle ABCD – the area of the coloured part $= 4x^2y^2 + 40xy - (3x^2y^2 + 35xy)$

$= 4x^2y^2 + 40xy - 3x^2y^2 - 35xy$

$= x^2y^2 + 5xy$

$$\text{FE} = \frac{\text{The area of the rectangle MNEF}}{\text{EN}} = \frac{x^2y^2 + 5xy}{xy}$$

$= (xy + 5)$ cm.

Answers of Exercise 13

1

1

$$\begin{array}{c|r}
x+2 & x^2+5x+6 \\
\hline
x+3 & \ominus \quad \ominus \quad \\
 & x^2+2x \quad \\
\hline
 & 3x+6 \\
 & \ominus \quad \ominus \\
 & 3x+6 \\
\hline
 & 00 \quad 00
\end{array}$$

Then , the quotient $= x + 3$

2

$$\begin{array}{c|r}
y-4 & y^2-9y+20 \\
\hline
y-5 & \ominus \quad \oplus \quad \\
 & y^2-4y \quad \\
\hline
 & -5y+20 \\
 & \oplus \quad \ominus \\
 & -5y+20 \\
\hline
 & 00 \quad 00
\end{array}$$

Then , the quotient $= y - 5$

3

$$\begin{array}{c|r}
x-7 & x^2-5x-14 \\
\hline
x+2 & \ominus \quad \oplus \quad \\
 & x^2-7x \quad \\
\hline
 & 2x-14 \\
 & \ominus \quad \oplus \\
 & 2x-14 \\
\hline
 & 00 \quad 00
\end{array}$$

Then , the quotient $= x + 2$

4

$$\begin{array}{c|r}
x+5 & 2x^2+13x+15 \\
\hline
2x+3 & \ominus \quad \ominus \quad \\
 & 2x^2+10x \quad \\
\hline
 & 3x+15 \\
 & \ominus \quad \ominus \\
 & 3x+15 \\
\hline
 & 00 \quad 00
\end{array}$$

Then , the quotient $= 2x + 3$

5

$$\begin{array}{c|r}
3x-4 & 3x^2+2x-8 \\
\hline
x+2 & \ominus \quad \oplus \quad \\
 & 3x^2-4x \quad \\
\hline
 & 6x-8 \\
 & \ominus \quad \oplus \\
 & 6x-8 \\
\hline
 & 00 \quad 00
\end{array}$$

Then , the quotient $= x + 2$

6

$$\begin{array}{c|r}
x+2 & x^2-x-6 \\
\hline
x-3 & \ominus \quad \ominus \quad \\
 & x^2+2x \quad \\
\hline
 & -3x-6 \\
 & \oplus \quad \oplus \\
 & -3x-6 \\
\hline
 & 00 \quad 00
\end{array}$$

Then , the quotient $= x - 3$

7 $7+2x \;\big|\; 14-17x-6x^2$

$2-3x$

$14+4x$

$-21x-6x^2$

$-21x-6x^2$

$00 \quad 00$

Then , the quotient $= 2-3x$

8 $4x-3y \;\big|\; 8x^2+6xy-9y^2$

$2x+3y$

$8x^2-6xy$

$12xy-9y^2$

$12xy-9y^2$

$00 \quad 00$

Then , the quotient $= 2x+3y$

9 $2x-4y \;\big|\; 4x^2-16xy+16y^2$

$2x-4y$

$4x^2-8xy$

$-8xy+16y^2$

$-8xy+16y^2$

$00 \quad 00$

Then , the quotient $= 2x-4y$

10 $x+1 \;\big|\; x^2 \quad -1$

$x-1$

x^2+x

$-x-1$

$-x-1$

$00 \quad 00$

Then , the quotient $= x-1$

11 $4y-2x \;\big|\; 16y^2 \quad -4x^2$

$4y+2x$

$16y^2-8xy$

$8xy-4x^2$

$8xy-4x^2$

$00 \quad 00$

Then , the quotient $= 4y+2x$

2

1 $x^2+3x+1 \;\big|\; x^3+5x^2+7x+2$

$x+2$

x^3+3x^2+x

$2x^2+6x+2$

$2x^2+6x+2$

$00 \quad 00 \quad 00$

Then , the quotient $= x+2$

2 $3x^2-4x+1 \;\big|\; 6x^3+7x^2-18x+5$

$2x+5$

$6x^3-8x^2+2x$

$15x^2-20x+5$

$15x^2-20x+5$

$00 \quad 00 \quad 00$

Then , the quotient $= 2x+5$

3 $x^2-7x-4 \;\big|\; 2x^3-9x^2-43x-20$

$2x+5$

$2x^3-14x^2-8x$

$5x^2-35x-20$

$5x^2-35x-20$

$00 \quad 00 \quad 00$

Then , the quotient $= 2x+5$

4 $x^2-1 \;\big|\; x^3+3x^2-x-3$

$x+3$

$x^3 \quad -x$

$3x^2 \quad -3$

$3x^2 \quad -3$

$00 \quad 00$

Then , the quotient $= x+3$

5 $4x^2+2 \;\big|\; 8x^3-20x^2+4x-10$

$2x-5$

$8x^3 \quad +4x$

$-20x^2 \quad -10$

$-20x^2 \quad -10$

$00 \quad 00$

Then , the quotient $= 2x-5$

6 $x^2+1 \;\big|\; x^4+3x^2+2$

x^2+2

x^4+x^2

$2x^2+2$

$2x^2+2$

$00 \quad 00$

Then , the quotient $= x^2+2$

7 $x-1 \;\big|\; x^3 \quad -x$

x^2+x

x^3-x^2

x^2-x

x^2-x

$00 \quad 00$

Then , the quotient $= x^2+x$

8
$$\begin{array}{r|l} 4x^2+2x+1 & 8x^3 \qquad\qquad -1 \\ \hline 2x-1 & 8x^3+4x^2+2x \\ \hline & -4x^2-2x-1 \\ & -4x^2-2x-1 \\ \hline & 00 \qquad 00 \quad 00 \end{array}$$

Then , the quotient $= 2x-1$

3

1
$$\begin{array}{r|l} x+2 & x^3+5x^2+7x+2 \\ \hline x^2+3x+1 & x^3+2x^2 \\ \hline & 3x^2+7x+2 \\ & 3x^2+6x \\ \hline & x+2 \\ & x+2 \\ \hline & 00 \quad 00 \end{array}$$

Then , the quotient $= x^2+3x+1$

2
$$\begin{array}{r|l} x-4 & x^3-x^2-9x-12 \\ \hline 3x^2+3x+3 & x^3-4x^2 \\ \hline & 3x^2-9x-12 \\ & 3x^2-12x \\ \hline & 3x-12 \\ & 3x-12 \\ \hline & 00 \quad 00 \end{array}$$

Then , the quotient $= x^2+3x+3$

3
$$\begin{array}{r|l} 2x-3 & 6x^3-5x^2-14x+12 \\ \hline 3x^2+2x-4 & 6x^3-9x^2 \\ \hline & 4x^2-14x+12 \\ & 4x^2-6x \\ \hline & -8x+12 \\ & -8x+12 \\ \hline & 00 \quad 00 \end{array}$$

Then , the quoient $= 3x^2+2x-4$

4
$$\begin{array}{r|l} 3x+2 & 6x^3-5x^2+9x+10 \\ \hline 2x^2-3x+5 & 6x^3+4x^2 \\ \hline & -9x^2+9x+10 \\ & -9x^2-6x \\ \hline & 15x+10 \\ & 15x+10 \\ \hline & 00 \quad 00 \end{array}$$

Then , the quotient $= 2x^2-3x+5$

5
$$\begin{array}{r|l} 5-4x & 15+3x-7x^2-4x^3 \\ \hline 3+3x+x^2 & 15-12x \\ \hline & 15x-7x^2-4x^3 \\ & 15x-12x^2 \\ \hline & 5x^2-4x^3 \\ & 5x^2-4x^3 \\ \hline & 00 \quad 00 \end{array}$$

Then , the quotient $= 3+3x+x^2$

6
$$\begin{array}{r|l} x-1 & 3x^3 \qquad -4x+1 \\ \hline 3x^2+3x-1 & 3x^3-3x^2 \\ \hline & 3x^2-4x+1 \\ & 3x^2-3x \\ \hline & -x+1 \\ & -x+1 \\ \hline & 00 \quad 00 \end{array}$$

Then , the quotient $= 3x^2+3x-1$

7
$$\begin{array}{r|l} x-3 & x^3 \qquad\qquad -27 \\ \hline x^2+3x+9 & x^3-3x^2 \\ \hline & 3x^2 \qquad -27 \\ & 3x^2-9x \\ \hline & 9x-27 \\ & 9x-27 \\ \hline & 00 \quad 00 \end{array}$$

Then , the quotient $= x^2+3x+9$

8
$$\begin{array}{r|l} 3a-2 & 27a^3 \qquad\qquad -8 \\ \hline 9a^2+6a+4 & 27a^3-18a^2 \\ \hline & 18a^2 \qquad -8 \\ & 18a^2-12a \\ \hline & 12a-8 \\ & 12a-8 \\ \hline & 00 \quad 00 \end{array}$$

Then , the quotient $= 9a^2+6a+4$

9
$$\begin{array}{r|l} x^2+2x-7 & x^4 \qquad -18x^2 \qquad +49 \\ \hline x^2-2x-7 & x^4+2x^3-7x^2 \\ \hline & -2x^3-11x^2 \qquad +49 \\ & -2x^3-4x^2+14x \\ \hline & -7x^2-14x+49 \\ & -7x^2-14x+49 \\ \hline & 00 \quad 00 \quad 00 \end{array}$$

Then , the quotient $= x^2-2x-7$

10

$$\begin{array}{r|l} 3x^2+5x-2 & -9x^4 \qquad +37x^2 \qquad -4 \\ \hline -3x^2+5x+2 & -9x^4-15x^3+6x^2 \\ & \overline{\quad 15x^3+31x^2 \qquad -4} \\ & \quad 15x^3+25x^2-10x \\ & \overline{\qquad 6x^2+10x-4} \\ & \qquad 6x^2+10x-4 \\ & \overline{\qquad 00 \quad 00 \quad 00} \end{array}$$

Then , the quotient $= -3x^2+5x+2$

4

1 Since , $13xy+6(x^2+y^2)$

$= 13xy+6x^2+6y^2$

$= 6x^2+13xy+6y^2$

Then ,
$$\begin{array}{r|l} 2x+3y & 6x^2+13xy+6y^2 \\ \hline 3x+2y & 6x^2+9xy \\ & \overline{\quad 4xy+6y^2} \\ & \quad 4xy+6y^2 \\ & \overline{\quad 00 \qquad 00} \end{array}$$

Then , the quotient $= 3x+2y$

2

$$\begin{array}{r|l} a^2-2ab-5b^2 & a^4 \qquad -6a^2b^2-16ab^3-15b^4 \\ \hline a^2+2ab+3b^2 & a^4-2a^3b-5a^2b^2 \\ & \overline{\quad 2a^3b-a^2b^2-16ab^3-15b^4} \\ & \quad 2a^3b-4a^2b^2-10ab^3 \\ & \overline{\qquad 3a^2b^2-6ab^3-15b^4} \\ & \qquad 3a^2b^2-6ab^3-15b^4 \\ & \overline{\qquad 00 \quad 00 \quad 00} \end{array}$$

Then , the quotient $= a^2+2ab+3b^2$

5

$$\begin{array}{r|l} x+3 & 2x^2+3x-9 \\ \hline 2x-3 & 2x^2+6x \\ & \overline{\quad -3x-9} \\ & \quad -3x-9 \\ & \overline{\quad 00 \quad 00} \end{array}$$

Then , the other factor $= 2x-3$

6

$$\begin{array}{r|l} x^2+3x+3 & x^3-x^2-9x-12 \\ \hline x-4 & x^3+3x^2+3x \\ & \overline{\quad -4x^2-12x-12} \\ & \quad -4x^2-12x-12 \\ & \overline{\quad 00 \quad 00 \quad 00} \end{array}$$

Then , the other factor $= x-4$

7

$$\begin{array}{r} 3x^3-5x^2+7x+1 \\ 3x^3 \qquad -x+7 \\ \hline 6x^3-5x^2+6x+8 \end{array}$$

$$\begin{array}{r|l} 3x+2 & 6x^3-5x^2+6x+8 \\ \hline 2x^2-3x+4 & 6x^3+4x^2 \\ & \overline{\quad -9x^2+6x+8} \\ & \quad -9x^2-6x \\ & \overline{\qquad 12x+8} \\ & \qquad 12x+8 \\ & \overline{\qquad 00 \quad 00} \end{array}$$

Then , the quotient $= 2x^2-3x+4$

8

$$\begin{array}{r|l} 2x+3 & 2x^3-x^2-2x+6 \\ \hline x^2-2x+2 & 2x^3+3x^2 \\ & \overline{\quad -4x^2-2x+6} \\ & \quad -4x^2-6x \\ & \overline{\qquad 4x+6} \\ & \qquad 4x+6 \\ & \overline{\qquad 00 \quad 00} \end{array}$$

Then , the quotient $= x^2-2x+2$

, the numerical value $= (1)^2-2(1)+2 = 1$

9

$$\begin{array}{r|l} x-2 & 2x^2-7x+m \\ \hline 2x-3 & 2x^2-4x \\ & \overline{\quad -3x+m} \\ & \quad -3x+6 \\ & \overline{\qquad m-6} \end{array}$$

Then , $m-6=0$ Then , $m=6$

10
$$\begin{array}{l|l} x^2+4x+3 & x^3-3x^2-25x+k \\ \hline x-7 & x^3+4x^2+3x \\ \hline & -7x^2-28x+k \\ & -7x^2-28x-21 \\ \hline & \qquad\qquad k+21 \end{array}$$

Then , $k+21=0$

So , $k=-21$

11
$$\begin{array}{l|l} 3x-5 & 6x^3-13x^2-13x+k \\ \hline 2x^2-x-6 & 6x^3-10x^2 \\ \hline & -3x^2-13x+k \\ & -3x^2+5x \\ \hline & -18x+k \\ & -18x+30 \\ \hline & \qquad\qquad k-30 \end{array}$$

Then , $k-30=0$ So , $k=30$

12
$$\begin{array}{l|l} x^2+x+2 & x^3+2x^2+3x+2 \\ \hline x+1 & x^3+x^2+2x \\ \hline & x^2+x+2 \\ & x^2+x+2 \\ \hline & 00 \quad 00 \quad 00 \end{array}$$

Then , the required expression $= x+1$

13
$$\begin{array}{l|l} 3x-2 & 15x^2+11x-14 \\ \hline 5x+7 & 15x^2-10x \\ \hline & 21x-14 \\ & 21x-14 \\ \hline & 00 \quad 00 \end{array}$$

Then , the length $= (5x+7)$ cm.

14
$$\begin{array}{l|l} x+5 & 2x^2+7x-15 \\ \hline 2x-3 & 2x^2+10x \\ \hline & -3x-15 \\ & -3x-15 \\ \hline & 00 \quad 00 \end{array}$$

Then , the width $= (2x-3)$ length units

When , $x=3$

Then , the length $= 3+5=8$ length units

, the width $= 2\times 3-3=3$ length units

Then , the perimeter $= (8+3)\times 2=22$ length units

15
$$\begin{array}{l|l} x-4 & x^2-kx+12 \\ \hline x+(4-k) & x^2-4x \\ \hline & (4-k)x+12 \\ & (4-k)x-16+4k \\ \hline & 12+16-4k \end{array}$$

Then , $28-4k=0$

Then , $28=4k$ then , $k=7$

16 Let the number that will be added be m

$$\begin{array}{l|l} 2x-5 & 6x^2-11x-17+m \\ \hline 3x+2 & 6x^2-15x \\ \hline & 4x-17+m \\ & 4x-10 \\ \hline & -7+m \end{array}$$

Then , $-7+m=0$ Then , $m=7$

17 Since , the area $= \frac{1}{2}\times$ the length of $\overline{BC}$

$\times$ the corresponding height of $\overline{BC}$

Then , the corresponding height of

$\overline{BC} = 2\times \dfrac{\text{The area}}{\text{The length of } \overline{BC}}$

, since
$$\begin{array}{l|l} 2x+1 & 6x^2+7x+2 \\ \hline 3x+2 & 6x^2+3x \\ \hline & 4x+2 \\ & 4x+2 \\ \hline & 00 \quad 00 \end{array}$$

Then , the corresponding height of $\overline{BC}$

$= 2(3x+2) = (6x+4)$ cm.

Answers of Exercise 14

1

1 $5(a+b)$	2 $3(x-y)$
3 $5(y-2)$	4 $4(2y^3-x^2)$
5 $7y(x+z)$	6 $5b(a-3c)$
7 $3x(x+2)$	8 $5a(7+2a)$
9 $2a^2(3a-2b^2)$	10 $7b^2(7-b)$
11 $5xy(7x^2+y)$	12 $5a^2b(3a-b)$

2

1. $5(a-b+c)$
2. $2(3a+4b+5c)$
3. $3(x^2+4x-2)$
4. $2a(4a^2-2a+3)$
5. $2y(x^2+3xy-1)$
6. $3m^2n^2(3m^2-2mn+4n^2)$
7. $2x(-x^4+2x-3+x^2)$
8. $8xy(4x^2y^2+2xy+1)$
9. $6abc(3a-1+5c-4bc)$
10. $3a^2b^2(5ab^2+2a^3b-1)$

3

1. $(a+b)(3x+7)$
2. $(a+3)(a+b)$
3. $(x+4)(x^2+y^2)$
4. $7(x+y)(2a-3b)$
5. $2a(x-1)(3a-4)$
6. $4x(x+1)(3x-2y)$
7. $24a^2b^3(a-2)+36a^3b^2(a-2)$
   $=12a^2b^2(a-2)(2b+3a)$
8. $(x-7)(3x^2+2x+5)$
9. $(2x+y)(4m^2-3m-7)$
10. $8a^2b(a+b+2)(2b-1)$

4

1. $48(45+55)=48\times 100=4800$
2. $52(43-33)=52\times 10=520$
3. $7(123+35-18)=7\times 140=980$
4. $15(17+13-30)=15\times$ zero $=$ zero
5. $12(5+4+1)=12\times 10=120$
6. $(1+14-5)\times 35=10\times 35=350$
7. $\frac{5}{18}\times 11+\frac{5}{18}\times 7=\frac{5}{18}\times(11+7)=\frac{5}{18}\times 18=5$
8. $58(58+42)=58\times 100=5800$
9. $256(256-156)=256\times 100=25600$
10. $2\times 15(3\times 15+9-4)=30(45+5)$
    $=30\times 50=1500$
11. $(5\times 48+7+53)\times 48=(240+60)\times 48$
    $=300\times 48=14400$
12. $31(31+23-54)=31\times$ zero $=$ zero
13. $51(17+33)+49(21+29)=51\times 50+49\times 50$
    $=(51+49)\times 50$
    $=100\times 50=5000$
14. $49(49+1)+50(50+1)=49\times 50+50\times 51$
    $=50(49+51)=50\times 100=5000$

5

1. $2a$, $4b$
2. ab
3. $4xy$, $4y$
4. x , y
5. 7
6. x , y
7. 15
8. 3
9. $4x$, 1
10. 25
11. 16

6 1 (d) 2 (b) 3 (b) 4 (d) 5 (c) 6 (b) 7 (d) 8 (b)

7 The expression $=(2a+b)(2a+b)$

The numerical value $=3\times 3=9$

8 The expression $=2(a+c)(a+c)$

The absolute value $=|2\times -3\times -3|=18$

9 The expression $=(a-b)(x+y)=-(b-a)(x+y)$

The numerical value $=-4\times 3=-12$

10

1. $\frac{19(19-2+1)}{9}=\frac{19\times 18}{9}=38$
2. $\frac{(5\times 9+11-1)\times 9}{45}=\frac{45+10}{5}=\frac{55}{5}=11$
3. $\left|\frac{(36)^2(5-3)}{-2\times(36)^2}\right|=\left|\frac{5-3}{-2}\right|=|-1|=1$

11 The other factor $=4c-2bc^3+3$

12 The area $=(8a^2b+12a^2bc+16a^3b)$ cm^2.

or $4a^2b(2+3c+4a)$ cm^2.

13 The expression $=b(x-1+c-2x-c+x)=-b$

The numerical value $=-8$

14 $2m(2x+3y)+2n(2x+3y)=16$

$2(m+n)(2x+3y)=16$

Therefore : $4(m+n)=16$

Therefore : $m+n=4$

15 The expression $=abc(a+b+c-1)$

The numerical value $=12\times(8-1)=84$

Answers of unit three

Answers of Exercise 15

1

1 The mean $= \frac{4+6}{2} = 5$

2 The mean $= \frac{3+5}{2} = 4$

3 The mean $= \frac{3+4}{2} = 3\frac{1}{2}$

4 The mean $= \frac{2+4+6}{3} = 4$

5 The mean $= \frac{1+3+5}{3} = 3$

6 The mean $= \frac{1+2+3+4+5}{5} = 3$

7 The mean $= \frac{6+10}{2} = 8$

8 The mean $= \frac{\frac{1}{2}+1}{2} = \frac{3}{4}$

9 The mean $= \frac{35+50+60+55}{4} = 50$

2

The mean $= \frac{124+130+122+126+128}{5}$

$= \frac{630}{5} = 126$ cm.

3

The mean $= \frac{89+91+96}{3} = 92$ marks.

4

The mean $= \frac{25^\circ+27^\circ+31^\circ+23^\circ+22^\circ+22^\circ+18^\circ}{7}$

$= 24^\circ$

5

The mean $= \frac{3+2+0+6+1+6}{6} = 3$ goals.

6

The mean $= \frac{3\frac{1}{2}+3+2\frac{1}{2}+3+4}{5} = 3\frac{1}{5}$ hours.

7

1 20.75 2 3 3 3

4 4 5 6

8

1 (d) 2 (a) 3 (a) 4 (d)

5 (d) 6 (a) 7 (c)

9

Let the required number be L

1 $L = \left(\frac{1}{3}+\frac{2}{3}\right) \div 2 = \frac{3}{3} \times \frac{1}{2} = \frac{1}{2}$

2 $L = \left(-\frac{3}{5}+\left(-\frac{1}{5}\right)\right) \div 2$

$= -\frac{4}{5} \times \frac{1}{2} = -\frac{2}{5}$

3 $L = \left(1\frac{1}{2}+2\right) \div 2 = 3\frac{1}{2} \times \frac{1}{2}$

$= \frac{7}{2} \times \frac{1}{2} = \frac{7}{4} = 1\frac{3}{4}$

10

The sum of Youssif's marks in the first 3 tests

$= 3 \times 16 = 48$ marks

The sum of Youssif's marks in the next 2 tests

$= 2 \times 18 = 36$ marks

The sum of Youssif's marks in the 5 tests

$= 48 + 36 = 84$ marks

The mean = sum of marks ÷ number of tests

$= \frac{84}{5} = 16.8$ marks

11

The sum of Magdi's marks in 4 tests $= 4 \times 16$

$= 64$ marks

Let the mark of Magdi in the 5^{th} test $= x$

$\frac{64+x}{5} = 18$ $\qquad 64 + x = 90$

$x = 90 - 64 = 26$ marks

The mark of Magdi in the 5^{th} test should be 26 marks

12

The mean

$= \frac{(6\times4)+(9\times7)+(12\times8)+(15\times5)+(17\times6)}{30}$

$= \frac{24+63+96+75+102}{30} = 12$ marks

Answers of Exercise 16

1

1 (b) 2 (c) 3 (c) 4 (b) 5 (a)
6 (c) 7 (c) 8 (c) 9 (d) 10 (c)

2

1 The order is : $-2, -1, 0, 1, 5$

The median = 0

2 The order is : $-12, -2, -2, 8, 10, 18$

The median = $\frac{-2+8}{2} = 3$

3 The order is : $1, \frac{1}{2}, \frac{1}{4}$

The median = $\frac{1}{2}$

4 The order is : $\frac{5}{6}, \frac{7}{15}, \frac{2}{5}, \frac{3}{10}$

The median = $\frac{\frac{7}{15}+\frac{2}{5}}{2} = \frac{13}{30}$

5 The order is : 0.2 , 2.3 , 2.8 , 2.9 , 3.2

The median = 2.8

6 The order is : $0.8, \frac{3}{5}, \frac{1}{2}, 0.4, 0.25, \frac{5}{25}$

The median = 0.45

3

The order is : 6 , 6 , 7 , 8 , 10

The median of the number of absent pupils = 7 pupils

4

• The ascending order of studying hours of Sally is :

2 , 3 , 3.5 , 4.5 , 5 , 7

The median of Sally = $\frac{3.5+4.5}{2} = 4$ hours.

• The ascending order of studying hours of Basma is :

2 , 3 , 3 , 4 , 4.5 , 6

The median of Basma = $\frac{3+4}{2} = 3.5$ hours.

5

The ascending order of heights is : 116 , 117 , 118 , 120 , 120 , 121 , 122 , 123 , 124 , 125 , 126 , 127 , 128 , 128 , 131 , 133 , 133 , 134 , 135 , 135

The median = $\frac{125+126}{2} = 125.5$ cm.

6

1 The descending order of the marks is :

48 , 47 , 44 , 41 , 37 , 35

The median = $\frac{44+41}{2} = 42.5$ marks.

2 The mean = $\frac{41+35+47+37+44+48}{6}$

= 42 marks.

7

1 2 2 7

Answers of Exercise 17

1

1 the most common data 2 2 3 11
4 4 5 red 6 pen 7 3 8 7
9 8 10 10

2 The mode mark = 18 marks.

3 The mode number of studying hours = 27 hours.

4 The mode temperature degree = 21 degrees.

5

1 • The mean = $\frac{2+5+8+12+13+5+4}{7} = 7$

• The ascending order of the values is :

2 , 4 , 5 , 5 , 8 , 12 , 13

The median = 5

• The mode = 5

2 • The mean = $\frac{5+4+10+3+3+4+7+4+6+5}{10}$

= 5.1

• The descending order of the values is :

10 , 7 , 6 , 5 , 5 , 4 , 4 , 4 , 3 , 3

The median = $\frac{5+4}{2} = 4.5$

• The mode = 4

6

The mode mark = 7 marks

1 The number of pupils who obtained a mark more than the mode mark = the number of pupils who obtained 8 , 9 and 10 marks = 6 + 3 + 2 = 11pupils

2 The number of pupils who obtained a mark less than the mode mark = the number of pupils who obtained 6 and 5 marks = 4 + 8 = 12 pupils

Answers of activities on unit three

1 30

2

The sum of marks of Karem in 5 tests = $5 \times 84 = 420$

The sum of marks of Karem in the first three tests

$= 3 \times 80 = 240$

The sum of marks of Karem in the last two tests

$= 420 - 240 = 180$

, Then the mean of his marks in the last two tests

$= \frac{180}{2} = 90$ marks.

3

1 The mean = 5.5	The median = 5.5
2 The mean = 6	The median = 6
3 The mean = 50.5	The median = 50.5
4 The mean = 51	The median = 51
5 The mean = 5	The median = 5
6 The mean = 50	The median = 50

* There is not a mode for any one from the previous.

Answers of accumulative basic skills

1

1 45	2 $\{0\}$	3 70
4 $3x$	5 $x + 1$	6 4
7 1 : 3	8 200	9 21 kg. , 33 kg.
10 2	11 $\frac{6}{7}$, $\frac{50}{51}$	12 13

2

1 (c)	2 (c)	3 (a)	4 (c)
5 (d)	6 (d)	7 (a)	8 (b)
9 (c)	10 (d)	11 (c)	12 (b)

Answers of unit four

Answers of Exercise 1

1

1 $\in$ 2 $\notin$ 3 $\in$ 4 $\notin$
5 $\subset$ 6 $\not\subset$ 7 $\subset$ 8 $\subset$

2

1 acute 2 obtuse 3 right
4 reflex 5 straight 6 acute
7 right 8 reflex

3

1 30° 2 45° 3 53° 4 42°
5 $67\frac{1}{2}^\circ$ 6 zero° 7 64° 8 90°

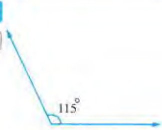
4

1 170° 2 90° 3 98° 4 63°
5 $87\frac{1}{2}^\circ$ 6 180° 7 zero° 8 38°

5

1 the union of two rays with the same starting point
2 180° , 0° 3 90° 4 90° , 0°
5 90° 6 180° 7 supplementary
8 complementary 9 supplementary
10 on the same straight line 11 310°
12 123° 13 180° , a straight
14 40° , 130° 15 60° , 120°
16 30° , 60° 17 an acute , an obtuse
18 a right , a straight 19 a zero , a right
20 an acute

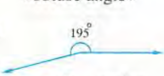
6

1


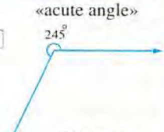
«obtuse angle»

2


«acute angle»

3


«reflex angle»

4


«reflex angle»

5


«straight angle»

7 1 240° 2 80° 3 120°

8

1 $\angle$ AFC 2 $\angle$ CFB
3 CFB 4 CFA , EFD
5 a straight , a right 6 AFC , CFD

9 1 25° 2 60° 3 25°

10

1 114° 2 50° 3 80°
4 40° 5 120° 6 140°

11

1 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.
The reason : $m(\angle ACD) + m(\angle DCB) = 114^\circ + 66^\circ = 180^\circ$

2 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are not on the same straight line.
The reason : $m(\angle ACD) + m(\angle DCB) = 62^\circ + 116^\circ = 178^\circ$

3 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are not on the same straight line.
The reason : $m(\angle ACD) + m(\angle DCE) + m(\angle ECB)$
$= 58^\circ + 85^\circ + 39^\circ = 182^\circ$

4 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.
The reason : $m(\angle ACF) = m(\angle FCE) = m(\angle ECB) = 60^\circ$
, $m(\angle ACF) + m(\angle FCE) + m(\angle ECB)$
$= 60^\circ + 60^\circ + 60^\circ = 180^\circ$

5 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are on the same straight line.
The reason : $m(\angle ACD) + m(\angle DCE)$
$+ m(\angle ECF) + m(\angle FCB)$
$= 28^\circ + 32^\circ + 64^\circ + 56^\circ = 180^\circ$

6 $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are not on the same straight line.
The reason : $m(\angle ACD) = m(\angle DCE) = 40^\circ$,
$m(\angle ECF) = m(\angle FCB) = 51^\circ$
, $m(\angle ACD) + m(\angle DCE) + m(\angle ECF)$
$+ m(\angle FCB)$
$= 40^\circ + 40^\circ + 51^\circ + 51^\circ = 182^\circ$

12

1 (b) 2 (c) 3 (c) 4 (b)
5 (a) 6 (a) 7 (d) 8 (b)
9 (d) 10 (d)

13

1 300° 2 45° 3 60°
4 complementary 5 54° 6 140°
7 90° 8 122° 9 35° 10 100°

14

Since m (∠ ACD) + m (∠ ECB) = 180° − 85° = 95°

, then m (∠ ACD) : m (∠ ECB) : The sum

2	3	5
?	?	95°

, then m (∠ ACD) = $\frac{2 \times 95°}{5}$ = 38°

, then m (∠ ACE) = 38° + 85° = 123°

, m (∠ DCB) = 180° − 38° = 142°

15

$\overrightarrow{CA}$ and $\overrightarrow{CE}$ are on the same straight line

The reason :

m (∠ ACB)	m (∠ BCD)	m (∠ DCE)
2	3	4
?	60°	?

, then m (∠ ACB) = $\frac{2 \times 60°}{3}$ = 40°

, then m (∠ DCE) = $\frac{4 \times 60°}{3}$ = 80°

, then m (∠ ACB) + m (∠ BCD) + m (∠ DCE)

= 40° + 60° + 80° = 180°

Answers of Exercise 2

1

1 70° 2 60° 3 110° 4 115°
5 60° 6 120° 7 140° 8 110°
9 120° 10 120° 11 50° 12 80°
13 38° 14 130° 15 90°

2

1 equal in measure. 2 360° 3 50°
4 135° 5 70° 6 50°

3

1 (b) 2 (c) 3 (a)
4 (a) 5 (d) 6 (c)
7 (b) 8 (b) 9 (d)

4 m (∠ ABD) = 45° , m (∠ DBE) = 90°
, m (∠ CBE) = 135°

5 m (∠ BME) = 45° , m (∠ DME) = 90°
, m (∠ AMC) = 45° , m (∠ AME) = 135°

6 1 m (∠ CMD) = 50° 2 m (∠ AMC) = 110°

7 m (∠ DMX) = 110°

8 1 m (∠ AMD) = 125° 2 m (∠ DMY) = 45°
3 m (∠ BMY) = 100°

9 m (∠ AMD) = 101°

10

$\overrightarrow{OA}$ and $\overrightarrow{OD}$ are on the same straight line.

The reason :

m (∠ AOH) + m (∠ HOD) = 140° + 40° = 180°

, m (∠ BOC) = 60°

11 m (∠ FBC) = 125°

12 m (∠ DCY) = 124°

13 m (∠ BAC) = 50° , m (∠ ABC) = 70°
, m (∠ C) = 60°

14

m (∠ AOB) + m (∠ AOH) = 80°

, m (∠ AOB) = $\frac{2}{5}$ × 80° = 32°

, m (∠ AOH) = $\frac{3}{5}$ × 80° = 48°

15

Since : $\overleftrightarrow{AB} \cap \overleftrightarrow{HO} = \{M\}$

, then m (∠ AMO)

= m (∠ BMH) (V.O.A.)

, then m (∠ AMO) = 140° ÷ 2 = 70°

, since M $\in \overleftrightarrow{CD}$

, then m (∠ AMC) + m (∠ AMO) +

m (∠ DMO) = 180°

, then m (∠ AMC) + 70° + m (∠ DMO) = 180°

, then m (∠ AMC) + m (∠ DMO) = 180° − 70° = 110°

, since m (∠ AMC) : m (∠ DMO) = 2 : 3

, then the sum of the parts = 2 + 3 = 5

, then the value of one part = 110° ÷ 5 = 22°

, then m ($\angle$ DMO) = 3 × 22° = 66°

, since $\overleftrightarrow{CD} \cap \overleftrightarrow{HO} = \{M\}$

, then m ($\angle$ CMH) = m ($\angle$ DMO) (V.O.A.)

, then m ($\angle$ CMH) = 66°

Answers of Exercise 3

1

1 they are equal in length.

2 they are equal in measure.

3 equal in measure , equal in length

4 congruent 5 10 6 zero 7 1

8 50° 9 5 10 60° 11 90°

12 45° 13 ≡ 14 $\overline{LX}$, YZL

15 $\overline{AD}$ 16 their sides , their dimensions.

17 20

2

1 R 2 ROHES 3 4

4 C 5 5 6 H

3

1 ED 2 EF 3 A 4 D

5 FCB 6 110° 7 10 8 90°

9 140° 10 $\overleftrightarrow{CF}$

4

1 XC 2 AD 3 BY 4 a common

5 120° 6 85° 7 65° 8 90°

5

1 90° 2 80° 3 6 4 AFDE , BFDC

6 32 cm. **7** 40 cm.

8

1 M 2 MDE 3 F

4 125° 5 55° 6 9

9

1 3 2 180°

Answers of Exercise 4

1

1 the included angle in one of them are congruent to their corresponding elements in the other.

2 the side drawn between their vertices.

3 side in one of the two triangles , side.

4 the hypotenuse and one side in one of them are congruent to their corresponding elements in the other.

5 congruent 6 XY , C 7 ABC , LMN

2

1 The two triangles are congruent (Two sides and included angle).

2 The two triangles are congruent (Three sides).

3 The two triangles are congruent (Two sides and included angle).

4 The two triangles are congruent (Three sides).

5 The two triangles are congruent (Two sides and included angle).

6 The two triangles are not congruent because the two congruent sides are not corresponding.

7 The two triangles are congruent (Hypotenuse and one side).

8 The two triangles are congruent (Hypotenuse and one side).

9 The two triangles are not congruent because the two congruent sides are not corresponding.

10 The two triangles are congruent (Two angles and one side).

11 The two triangles are congruent (Hypotenuse and one side).

12 The two triangles are congruent (Two angles and one side).

13 The data is not enough to prove the congruence of the two triangles.

14 The data is not enough to prove the congruence of the two triangles.

15 The two triangles are congruent (Two sides and included angle).

16 The two triangles are congruent (Three sides).

17 The two triangles are not congruent because the given angle is not included between the two sides.

18 The two triangles are congruent (Hypotenuse and one side).

19 The two triangles are congruent (Three sides).

20 The two triangles are not congruent because the two congruent sides are not corresponding.

21 The two triangles are congruent (Two sides and included angle).

22 The two triangles are congruent (Two angles and one side).

23 The two triangles are congruent (Two sides and included angle).

24 The two triangles are congruent (Hypotenuse and one side).

25 The two triangles are congruent (Two angles and one side).

26 The data in not enough to prove the congruence of the two triangles.

3 52°

4 1 ACD 2 30° 3 7 4 125°

5 45°

6

1 50° 2 35° 3 Δ EFD 4 $\overline{ED}$ 5 7

7 1 ABC 2 100° 3 6

8 1 DCB 2 30° 3 ACB

9 1 55° 2 55° 3 110°

10 1 (d) 2 (d) 3 (b) 4 (d) 5 (b)

11

Δ ABC $\equiv$ Δ ADC (Two sides and included angle).

i.e. AD = AB = 5 cm. , m ($\angle$ D) = m ($\angle$ B) = 57°

m ($\angle$ DAC) = 180° – (90° + 57°) = 33°

12

From Δ CDB : m ($\angle$ CDB) = 180° – (110° + 30°) = 40°

i.e. Δ ADB $\equiv$ Δ CDB (Two sides and included angle).

The length of $\overline{BC}$ = BA = 7 cm.

m ($\angle$ BAD) = m ($\angle$ BCD) = 110°

13

From Δ ADB : m ($\angle$ ABD) = 180° – (50° + 110°) = 20°

Δ ABD $\equiv$ Δ CBD (Three sides).

i.e. m ($\angle$ ABD) = m ($\angle$ CBD) = 20°

, m ($\angle$ ABC) = 20° + 20° = 40°

14

Δ ABD $\equiv$ Δ CBD (Two angles and a side).

i.e. The length of $\overline{CB}$ = AB = 8 cm.

The length of $\overline{AD}$ = CD = 6 cm.

15

Yes : Δ AMC $\equiv$ Δ BMD

Because : $\begin{cases} AM = BM \\ CM = DM \\ m(\angle AMC) = m(\angle BMD)\ (V.O.A.) \end{cases}$

16

Yes : Δ ACE $\equiv$ Δ DBE

Because : $\begin{cases} AE = ED \\ m(\angle A) = m(\angle D) \\ m(\angle AEC) = m(\angle DEB)\ (V.O.A.) \end{cases}$

and we deduce that : CE = EB

17

From Δ ABC : m ($\angle$ ACB) = 180° – (90° + 57°) = 33°

Δ ABC $\equiv$ Δ EDA (Hypotenuse and one side).

m ($\angle$ E) = m ($\angle$ BAC) = 57° ,

m ($\angle$ EAD) = m ($\angle$ ACB) = 33°

18

Δ AEC $\equiv$ Δ ADC (Three sides).

m ($\angle$ AEC) = m ($\angle$ ADC) = 90°

i.e. $\overline{AE} \perp \overline{BC}$

From Δ ABE : m ($\angle$ BAE) = 180° – (30° + 90°) = 60°

19

Since $\angle$ ABD supplements $\angle$ ABC ,

$\angle$ ACE supplements $\angle$ ACB ,

Where m ($\angle$ ABC) = m ($\angle$ ACB) ,

then m ($\angle$ ABD) = m ($\angle$ ACE)

In $\Delta\Delta$ ABD , ACE :

Since m ($\angle$ DAB) = m ($\angle$ EAC)

, m ($\angle$ ABD) = m ($\angle$ ACE)

, then m ($\angle$ D) = m ($\angle$ E)

In $\Delta\Delta$ ABD , ACE :

$\begin{cases} BD = CE \\ m(\angle ABD) = m(\angle ACE) \\ m(\angle D) = m(\angle E) \end{cases}$

, then Δ ABD $\equiv$ Δ ACE

, then we deduce that : AD = AE

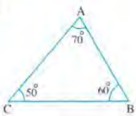
20

[1] 70° [2] 50° [3] 60°

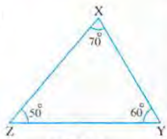
[4] 90° [5] 160° [6] 3 cm.

21

[a]



[b]



Yes since Δ ABC is not congruent to Δ XYZ

22

[1] Δ RPQ $\equiv$ Δ BCA (Two sides and included angle)

X = QR = AB = 4.8 cm. , Y° = m ($\angle$ R) = m ($\angle$ B) = 42°

[2] Δ DEF $\equiv$ Δ CAB (Two sides and included angle)

X = DE = CA = 16 cm. , Y° = m ($\angle$ D)

but : DF = DE = 16 cm. i.e. m ($\angle$ E) = m ($\angle$ F) = 75°

Therefore : Y° = 180° – (75° + 75°) = 30°

[3] Δ ABC $\equiv$ Δ RST (Two angles and a side).

X = BC = ST = 69 cm. , Y° = m ($\angle$ A) = m ($\angle$ R) = 83°

[4] Δ ABC $\equiv$ Δ EDB (Three sides).

Y° = m ($\angle$ A) = m ($\angle$ BED) = 64°

X° = m ($\angle$ C) = 180° – (64° + 30°) = 86°

[5] Δ ABC $\equiv$ Δ PNA (Two angles and a side).

X = AN = CB = 22 cm. , Y = 39 – 22 = 17 cm.

23

[1] The two triangles are congruent (Two sides and included angle).

[2] The data is not enough.

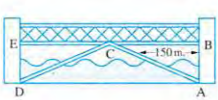
[3] The two triangles are congruent (Three sides).

[4] The data is not enough because the given angle is not included between the two sides.

[5] The two triangles are congruent (Two angles and a side).

[6] The data is not enough because the two sides $\overline{AC}$ and $\overline{PG}$ are not corresponding sides.

24



In $\Delta\Delta$ ABC and DEC :

$$\begin{cases} AB = DE \\ AC = CD \\ m(\angle ABC) = m(\angle DEC) \end{cases}$$

, then Δ ABC $\equiv$ Δ DEC , then we deduce that :

BC = CE = 150 m.

Then , the length of the bridge = 300 m.

25

Since $\overline{AE} \cap \overline{DB} = \{C\}$

Then , m ($\angle$ DCE) = m ($\angle$ ACB) (V.O.A.)

In $\Delta\Delta$ ABC and EDC :

$$\begin{cases} DC = BC \\ m(\angle DCE) = m(\angle BCA) \\ m(\angle D) = m(\angle B) = 90° \end{cases}$$

Then Δ ABC $\equiv$ Δ EDC , then we deduce that :

AB = DE = 400 m.

Then , the width of the river = 400 m.

26

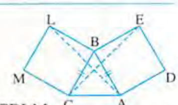
From the square ABCD :

m ($\angle$ XAB) = 90° – 70° = 20°

, Δ ABX $\equiv$ Δ BCY "Two sides and included angle"

Then : m ($\angle$ YBC) = m ($\angle$ XAB) = 20°

27



Since : AB = CB

Then :

the square ABED $\equiv$ the square CBLM

, Since m ($\angle$ ABE) = m ($\angle$ CBL) = 90°

By adding m ($\angle$ ABC) to both sides

then : m ($\angle$ CBE) = m ($\angle$ ABL)

In $\Delta\Delta$ CBE , ABL :

$$\begin{cases} CB = AB \\ m(\angle CBE) = m(\angle ABL) \\ BE = BL \end{cases}$$

Then : Δ CBE $\equiv$ Δ ABL

and we deduce that : CE = AL

Answers of Exercise 5

1

[1] perpendicular. [2] parallel.

[3] parallel [4] equal in measure.

[5] equal in measure. [6] supplementary.

[7] parallel. [8] parallel.

[9] parallel. [10] equal in length.

2

Fig. 1 : m (∠ CFE) = 110°

Fig. 2 : m (∠ DFY) = 63°

Fig. 3 : m (∠ XEB) = 116° , m (∠ EFD) = 116°

3

Fig. 1 : m (∠ B) = 60° , m (∠ D) = 60°

Fig. 2 : m (∠ D) = 51°

4

Fig. 1 : 2 Fig. 2 : 15 Fig. 3 : 6

5

Fig. 1 : m (∠ AEM) = 180° – m (∠ MEB)

= 180° – 122° = 58°

i.e. m (∠ AEM) = m (∠ CFE) = 58°

and they are two corresponding angles

, then $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

Fig. 2 : m (∠ BEF) = 180° – m (∠ BEM)

= 180° – 100° = 80°

i.e. m (∠ BEF) = m (∠ DFN) = 80°

and they are two corresponding angles

, then $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

Fig. 3 : m (∠ BEF) = m (∠ AEM) = 132° (V.O.A)

i.e. m (∠ BEF) = m (∠ DFN) = 132°

and they are two corresponding angles

, then $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$

6

Fig. 1 : m (∠ B) = m (∠ BCY) = 60°

i.e. m (∠ B) = m (∠ XAD) = 60°

and they are two corresponding angles

, then $\overline{AD}$ // $\overline{BC}$

Fig. 2 : m (∠ C) = m (∠ EBC) = 110°

i.e. m (∠ C) + m (∠ D) = 180°

and they are two interior angles in the same side of the transversal.

, then $\overline{AD}$ // $\overline{BC}$

Fig. 3 : m (∠ C) = 180° – m (∠ B) = 180° – 124° = 56°

i.e. m (∠ C) = m (∠ CDE) = 56°

and they are two alternate angles

, then $\overline{AD}$ // $\overline{BC}$

7

1 (c)	2 (c)	3 (a)	4 (c)
5 (a)	6 (c)	7 (b)	8 (d)
9 (c)	10 (c)	11 (a)	12 (b)
13 (c)	14 (a)	15 (a)	16 (b)
17 (b)			

8

Since : $\overleftrightarrow{AO}$ // $\overleftrightarrow{HD}$ // $\overleftrightarrow{YX}$ // $\overline{CB}$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them.

, AD = DX = XB

, then : AH = HY = YC = $\frac{18}{3}$ = 6 cm.

, then : AY = 12 cm.

9

Since : $\overleftrightarrow{AB}$ // $\overleftrightarrow{EF}$ // $\overleftrightarrow{CD}$, $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ are transversals to them

, AE = ED

, then : BE = EC

, then : BE = 4 cm.

10

m (∠ AEF) = m (∠ A) = 42° (alternate angles)

, m (∠ CEF) = 180° – m (∠ ECD)

= 180° – 117° = 63°

, then : m (∠ AEC) = 42° + 63° = 105°

11

m (∠ ACD) = m (∠ A) = 40° (alternate angles)

m (∠ DCE) = m (∠ E) = 55° (alternate angles)

, then : m (∠ ACE) = 40° + 55° = 95°

12

m (∠ B) = m (∠ DAB) = 50° (alternate angles)

m (∠ C) = m (∠ EAD) = 70° (corresponding angles)

m (∠ BAC) = 180° – (50° + 70°) = 60°

13

1 m (∠ ACD) = m (∠ A) = 35°

Since $\overrightarrow{CD}$ bisects ∠ ACE

, then : m (∠ DCE) = m (∠ ACD) = 35°

2 m (∠ CEF) + m (∠ DCE) = 180°

(Two interior angles in the same side of the transversal)

, then : m (∠ CEF) = 180° – 35° = 145°

14

m (∠ ABC) = m (∠ BAE) = 2 × 56° = 112° (alternate angles)

, then m (∠ C) + m (∠ ABC) = 180°

(Two interior angles in the same side of the transversal)

, then m (∠ C) = 180° − 112° = 68°

15

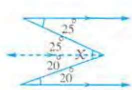
1. m (∠ X) = m (∠ XYM) = 100° (alternate angles)
2. m (∠ Z) = m (∠ XYM) = 100° (corresponding angles)
3. m (∠ L) + m (∠ X) = 180°

   (Two interior angles in the same side of the transversal)

   m (∠ L) = 180° − 100° = 80°

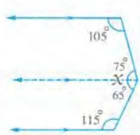
16

1. $X = 60°$
2. $X = 20°$
3. $X = 80°$
4.



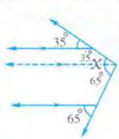
, then : $X = 25° + 20° = 45°$

5.



, then : $X = 75° + 65° = 140°$

6.

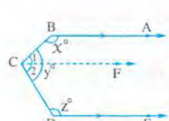


, then : $X = 35° + 65° = 100°$

17

Construction :

Draw $\overleftrightarrow{CF}$ // $\overrightarrow{BA}$ // $\overrightarrow{DE}$



From the figure

m (∠ B) + m (∠ 1) = 180°

(Two interior angles in the same side of the transversal)

m (∠ 2) + m (∠ D) = 180°

(Two interior angles in the same side of the transversal)

i.e. m (∠ B) + m (∠ 1) + m (∠ 2) + m (∠ D)

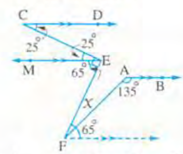
= 180° + 180° = 360°

i.e. m (∠ B) + m (∠ C) + m (∠ D) = 360°

, then : $X + y + z = 360°$

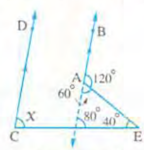
18

1. $X = 70°$ 2. $X = 40°$ 3. $X = 30°$
4. $X = 240°$ 5. $X = 105°$
6.



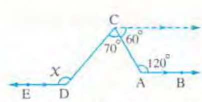
, then $X = 65° − 45° = 20°$

7.



, then $X = 80°$

8. $X = 112.5°$
9.



, then $X = 60° + 70° = 130°$

19

1. $\overleftrightarrow{OX}$ // $\overleftrightarrow{EZ}$ 2. $\overleftrightarrow{KC}$ // $\overleftrightarrow{DE}$ 3. $\overleftrightarrow{AS}$ // $\overleftrightarrow{LT}$

20 Yes ,

m (∠ DCE) = m (∠ D) = 125° (alternate angles)

Then : m (∠ FCE) = 125° − 50° = 75°

i.e. m (∠ FCE) = m (∠ B) = 75°

and they are two corresponding angles.

Then : $\overline{AB}$ // $\overrightarrow{CF}$

21

Since : m (∠ XAB) = m (∠ B) = 60°

and they are two alternate angles

, then : $\overleftrightarrow{XY}$ // $\overleftrightarrow{BC}$

m (∠ EDB) + m (∠ B) = 120° + 60° = 180°

and they are interior angles in the same side of the transversal

, then : $\overleftrightarrow{ZL} // \overleftrightarrow{BC}$

i.e. : $\overleftrightarrow{XY} // \overleftrightarrow{ZL} // \overleftrightarrow{BC}$

, AD = DB

, then : AE = $\frac{18}{2}$ = 9 cm.

---

22 Yes ,

since A ∈ $\overline{BE}$

i.e. m (∠ BAC) = 180° – 100° = 80°

, since $\overrightarrow{AD}$ bisects ∠ BAC

Then : m (∠ DAC) = 80° ÷ 2 = 40°

Then : m (∠ DAC) = m (∠ ACE) = 40°

and they are two alternate angles.

Then : $\overrightarrow{AD} // \overline{CE}$

---

23 Yes ,

i.e. m (∠ B) = m (∠ FAB) = 60° (alternate angles)

, since C ∈ $\overline{BD}$

, then : m (∠ ECD) = 180° ÷ 3 = 60°

i.e. m (∠ B) = m (∠ ECD) = 60°

and they are two corresponding angles.

, then : $\overline{AB} // \overrightarrow{CE}$

---

24 Yes ,

m (∠ A) = m (∠ C) = 45°

and they are two alternate angles.

Then : $\overrightarrow{AB} // \overline{DC}$

m (∠ D) + m (∠ E) = 72° + 108° = 180°

and they are two interior angles in the same side of the transversal.

, then : $\overline{DC} // \overrightarrow{EF}$ Then : $\overrightarrow{AB} // \overline{DC} // \overrightarrow{EF}$

25

[1] Yes ,

Δ AMB ≡ Δ CMD

Because $\begin{cases} MB = MD \\ MA = MC \\ m (\angle AMB) = m (\angle CMD) \quad \text{(V.O.A.)} \end{cases}$

[2] Yes ,

Δ AMB ≡ Δ CMD

We deduce from the congruence that :

m (∠ B) = m (∠ D)

and they are two alternate angles

, then : $\overline{AB} // \overline{CD}$

---

26 Yes ,

Δ ADB ≡ Δ CBD

Because $\begin{cases} AD = CB \\ \overline{DB} \text{ is a common side} \\ m (\angle ADB) = m (\angle CBD) = 90° \end{cases}$

We deduce from the congruence that :

m (∠ ABD) = m (∠ CDB)

and they are two alternate angles

, then : $\overline{AB} // \overline{CD}$

---

27 Yes ,

AB = CD

AB + BC = CD + BC

AC = BD

Δ ACL ≡ Δ BDM

Because $\begin{cases} AL = BM \\ LC = MD \\ AC = BD \end{cases}$

We deduce from the congruence that :

m (∠ A) = m (∠ MBD) and they are two corresponding angles

, then : $\overline{AL} // \overline{BM}$

, m (∠ D) = m (∠ LCA) and they are two corresponding angles

, then : $\overline{CL} // \overline{DM}$

---

28 Yes ,

BF = CE

BF + FC = CE + FC

BC = EF

Δ ABC ≡ Δ DEF

Because $\begin{cases} m (\angle B) = m (\angle E) \text{ (alternate angles)} \\ m (\angle ACB) = m (\angle DFE) \text{ (alternate angles)} \\ BC = EF \end{cases}$

We deduce from the congruence that :

$\overline{AB} \equiv \overline{DE}$

29

m (∠ AFG) = m (∠ B)

and they are two corresponding angles

, then : $\overleftrightarrow{FG} \parallel \overleftrightarrow{BK}$

, m (∠ B) = m (∠ K)

and they are two alternate angles.

, then : $\overleftrightarrow{BF} \parallel \overleftrightarrow{KM}$

, m (∠ K) = m (∠ M)

and they are two alternate angles

, then : $\overleftrightarrow{BK} \parallel \overleftrightarrow{ME}$

Since $\overleftrightarrow{FG} \parallel \overleftrightarrow{BK}$, $\overleftrightarrow{BK} \parallel \overleftrightarrow{ME}$

, then : $\overleftrightarrow{FG} \parallel \overleftrightarrow{ME}$

30 Yes ,

i.e. $\overleftrightarrow{BC} \parallel \overleftrightarrow{ED}$

, then : m (∠ 2) = m (∠ 3) (alternate angles)

, m (∠ 1) = m (∠ 4)

, then : m (∠ 1) + m (∠ 2) = m (∠ 3) + m (∠ 4)

, then : m (∠ ABD) = m (∠ FDB)

and they are alternate angles.

, then : $\overleftrightarrow{BA} \parallel \overleftrightarrow{DF}$

31 Yes ,

i.e. $\overleftrightarrow{AM} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$, AD = DB

, then : FE = EC , ME = EL

, then : Δ FEM ≡ Δ CEL

Because $\begin{cases} FE = CE \\ EM = EL \\ m(\angle FEM) = m(\angle CEL) \end{cases}$ (V.O.A.)

, then : FM = LC

32

[1] $\overleftrightarrow{YQ} \parallel \overleftrightarrow{XD}$, $\overleftrightarrow{YR} \parallel \overleftrightarrow{XB}$

[2] $\overleftrightarrow{EF} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{GH} \parallel \overleftrightarrow{AB}$ and J // P

33

Since : $\overline{DE} \parallel \overline{BC}$

Then : m (∠ C) + m (∠ D) = 180°

(Two interior angles in the same side of the transversal)

, Since : $\overline{BC} \parallel \overline{FL}$

then : m (∠ B) + m (∠ F) = 180°

(Two interior angles in the same side of the transversal)

then : m (∠ C) + m (∠ D) + m (∠ B) + m (∠ F)

= 180° + 180° = 360°

Since : m (∠ D) + m (∠ F) = 220°

then : m (∠ C) + m (∠ B) + 220° = 360°

then : m (∠ C) + m (∠ B) = 360° – 220° = 140°

From Δ ABC :

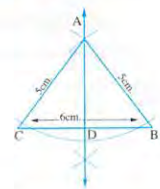
then : m (∠ BAC) = 180° – 140° = 40°

Answers of Exercise 6

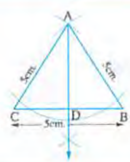
1

From drawing and by measuring

AD = 4 cm.



2



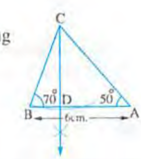
3

From the drawing and by measuring the length of $\overline{CD}$ = 5 cm.

the area of

Δ ABC = $\frac{1}{2}$ × 6 × 5

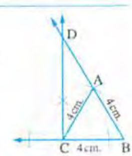
= 15 cm².



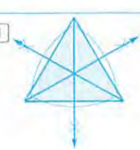
4

From the figure and by measuring :

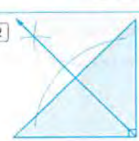
DA = 4 cm.



5 [1]



[2]

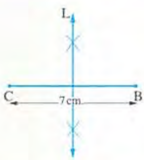


3

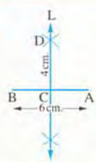
We notice that the straight lines carrying the altitudes of the triangle are concurrent at a point. This point lies :

- Inside the acute-angled triangle.
- At the vertex of the right angle in the right-angled triangle.
- Outside the obtuse-angled triangle.

6

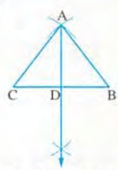


7



From the figure DA = DB = 5 cm.

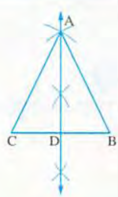
8



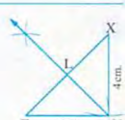
From the figure we notice that AB = AC

9

Yes , $\overline{AD} \perp \overline{BC}$

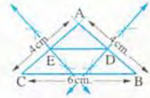


10 By measuring we find that m (∠ XLY) = 90°



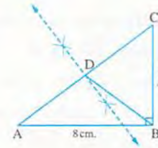
11

From the figure DE = 3 cm.



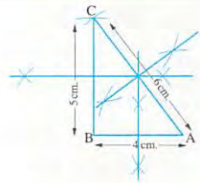
12

Yes , BD = $\frac{1}{2}$ AC



13

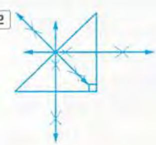
From the figure :
We notice that the bisectors of the sides of the triangle meet at one point.



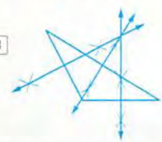
14

1

2



3

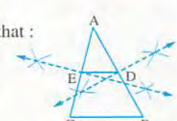


The symmetry axes of the sides of the triangle are meeting at one point and this point lies inside the triangle if it is acute-angled. At the midpoint of the hypotenuse if it is right-angled and outside the triangle if it is obtuse-angled.

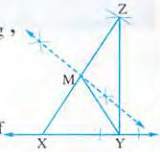
15

1 From the figure , we find that : BC = 2 DE

2 Yes , ∠ ABC ≡ ∠ ADE , $\overline{DE} \parallel \overline{BC}$



16



From the figure and by measuring ,

we find that :

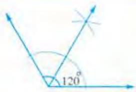
MX = MY = MZ

Draw other triangles by yourself

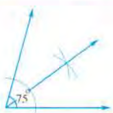
and notice that in each time :

MX = MY = MZ

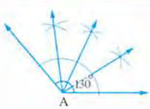
17



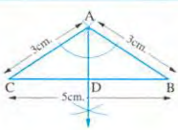
18



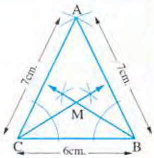
19



20



21

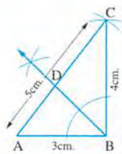


From drawing

and by measuring :

MB = MC

22

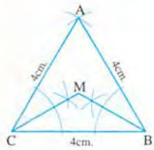


From drawing

and by measuring :

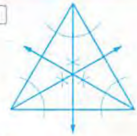
BD ≃ 2.4 cm.

23

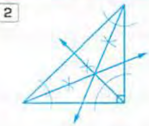


From the drawing and by measuring m (∠ BMC) = 120°

24 1



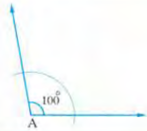
2

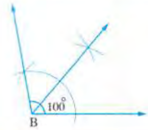


3

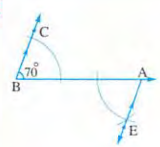
We notice that the three bisectors of the angles of the triangle are concurrent.

25

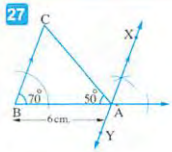




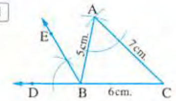
26



27

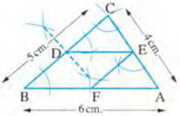


28 1



2 C

29



From the figure and by measure

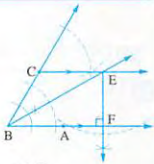
ED = 3 cm. , EF = 2.5 cm.

The figure DEFB is a parallelogram

Its perimeter = 2.5 + 3 + 2.5 + 3 = 11 cm.

30

31



Since , m (∠ ABC) = 60°

Then , m (∠ ABE) = 30°

, since m (∠ EFB) = 90°

Then , in Δ EFB :

m (∠ FEB) = 180° – (90° + 30°) = 60°

Then , m (∠ ABC) = m (∠ FEB)

Answers of accumulative basic skills

1

1 22	2 170
3 5	4 5 length units
5 4 : 1	6 (– 3 , 2)
7 30°	8 1 : π
9 28	10 14
11 42	12 17.85

2

1 (a)	2 (c)	3 (a)	4 (c)
5 (c)	6 (b)	7 (c)	8 (d)
9 (c)	10 (b)	11 (d)	12 (a)

Guide Answers Of The Notebook

Answers of the accumulative tests on Algebra and Statistics

Accumulative test 1

1 1 c 2 d 3 c 4 c
5 a 6 b 7 a 8 c

2 1 $\frac{9}{4}$ 2 $\frac{7}{20}$

3
1 $\frac{5}{7} = \frac{10}{14} = \frac{15}{21} = \frac{20}{28}$ (There are other solutions)
2 $\frac{2}{9} = \frac{4}{18} = \frac{6}{27} = \frac{8}{36}$ (There are other solutions)

Accumulative test 2

1 1 a 2 b 3 b 4 b
5 d 6 d 7 a 8 c

2 $\frac{13}{60}$, $\frac{14}{60}$ (There are other solutions)

3 $\frac{13}{12}$, 1 , $\frac{11}{12}$ (There are other solutions)

Accumulative test 3

1 1 a 2 c 3 d 4 c
5 b 6 b 7 d 8 a

2 $x = 2$, the numbers are : $\frac{13}{24}$, $\frac{14}{24}$, $\frac{15}{24}$
(There are other solutions)

3 $-\frac{7}{8}$

Accumulative test 4

1 1 b 2 c 3 b 4 c
5 d 6 b 7 b 8 b

2 $\frac{9}{5}$

3 6

Accumulative test 5

1 1 a 2 b 3 c 4 c
5 d 6 c 7 c 8 d

2 $\frac{-13}{30}$

3 $\frac{3}{2}$

Accumulative test 6

1 1 c 2 b 3 d 4 b
5 a 6 b 7 c 8 d

2 $\frac{9}{2}$

3 $\frac{1}{2} a^2 b - x y$ from the third degree.

Accumulative test 7

1 1 b 2 d 3 b 4 c
5 a 6 c 7 a 8 b

2 $6x^2 - 9x + 5$

3 5

Accumulative test 8

1 1 c 2 d 3 c 4 b
5 c 6 c 7 d 8 b

2 $x + 10y + 5z$

3 $2y^2 + xy + x^2$

Accumulative test 9

1 1 c 2 c 3 b 4 c
5 c 6 c 7 d 8 c

2 $7x^2 - 5xy$, 17

3 $\frac{33}{48}$, $\frac{34}{48}$, $\frac{35}{48}$ (There are other solutions)

Accumulative test 10

1 1 c 2 c 3 b 4 d
5 b 6 b 7 b 8 b

2 $3x^2 + 11x$

3 16

Accumulative test 11

1 1 d, 2 b, 3 b, 4 b, 5 b, 6 b, 7 c, 8 b

2 $4x^2+2x-46$

3 3

Accumulative test 12

1 1 c, 2 b, 3 d, 4 d, 5 d, 6 c, 7 c, 8 d

2 18

3 $(4x^2+3x+7)$ cm.

Accumulative test 13

1 1 d, 2 a, 3 c, 4 b, 5 a, 6 a, 7 a, 8 b

2 x^2-4x-5

3 x^2 , 4

Accumulative test 14

1 1 c, 2 a, 3 c, 4 d, 5 a, 6 c, 7 a, 8 b

2 $3x^2y^2(6y+2x-1)$

3 25

Accumulative test 15

1 1 d, 2 b, 3 a, 4 b, 5 c, 6 c, 7 b, 8 c

2 2

3 $4x+8$, 12

Accumulative test 16

1 1 c, 2 c, 3 b, 4 b, 5 b, 6 b, 7 b, 8 c

2 98 marks

3 -1

Accumulative test 17

1 1 c, 2 d, 3 c, 4 d, 5 d, 6 b, 7 c, 8 c

2 2

3 9

Answers of important questions on Algebra and Statistics

Unit one

First Answers of multiple choice questions

1 (d)	2 (b)	3 (c)	4 (c)	5 (c)
6 (b)	7 (c)	8 (b)	9 (c)	10 (c)
11 (d)	12 (d)	13 (b)	14 (d)	15 (a)
16 (a)	17 (c)	18 (d)	19 (d)	20 (a)
21 (a)	22 (a)	23 (c)	24 (d)	

Second Answers of complete questions

1 -1	2 $\frac{-3}{4}$	3 $\frac{5}{2}$	4 zero	5 5
6 $0.\dot{3}$	7 -1	8 3	9 7	10 50
11 $\frac{7}{5}$	12 1	13 2	14 1	15 $0.\dot{4}$
16 zero	17 identity	18 $\frac{-4}{5}$	19 zero	20 $\frac{28}{3}$

Third Answers of essay questions

1 L.C.M. of the denominators = 6

Then : $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{3} = \frac{2}{6}$

, since : $\frac{3}{6} = \frac{12}{24}$, $\frac{2}{6} = \frac{8}{24}$

, then the three numbers are : $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$

(There are other numbers)

2 L.C.M. of the denominators = 21

Then : $\frac{2}{3} = \frac{14}{21}$, $\frac{3}{7} = \frac{9}{21}$

, then : The four numbers are : $\frac{10}{21}$, $\frac{11}{21}$, $\frac{12}{21}$, $\frac{13}{21}$

(There are other numbers)

3 $\frac{3}{8}(3+9-4) = \frac{3}{8} \times 8 = 3$

4 $\frac{5}{7}(3+5-1) = \frac{5}{7} \times 7 = 5$

5 $\frac{4}{9}\left(\frac{5}{6}+\frac{7}{6}-1\right) = \frac{4}{9}(2-1) = \frac{4}{9}$

6 The expression = $\frac{1}{4} \times \frac{2}{5} \times 4 = \frac{2}{5}$

7 $\left(\frac{3}{5} \div \frac{2}{5}\right) \times \frac{2}{3} = \left(\frac{3}{5} \times \frac{5}{2}\right) \times \frac{2}{3}$

$= \frac{3}{2} \times \frac{2}{3} = 1$

8 The distance between the two numbers

$= \left|\frac{4}{7} - 1\frac{3}{4}\right| = \left|\frac{4}{7} - \frac{7}{4}\right| = \left|\frac{16-49}{28}\right| = \frac{33}{28}$

, then the number = $\frac{4}{7} + \frac{1}{3} \times \frac{33}{28} = \frac{27}{28}$

9 The expression = $3 \times \frac{1}{2} \times \frac{2}{3} + 9 = 1 + 9 = 10$

10 $X + y = \frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$

, $X - y = \frac{1}{2} - \frac{3}{4} = \frac{2}{4} - \frac{3}{4} = \frac{-1}{4}$

The expression = $\frac{5}{4} \div \left(\frac{-1}{4}\right) = \frac{5}{4} \times \left(\frac{-4}{1}\right)$

$= -5$

11 The expression = $\frac{1}{2} \times \frac{-3}{4} - \frac{1}{3} = \frac{-3}{8} - \frac{1}{3}$

$= \frac{-9-8}{24} = \frac{-17}{24}$

12 Since : $\frac{1}{3} = \frac{4}{12}$, $\frac{1}{2} = \frac{6}{12}$

Then : $\frac{5}{12}$ lies between the two numbers : $\frac{1}{3}$, $\frac{1}{2}$

13 The expression = $\frac{3}{2} - \left(-2 \div \frac{-1}{4}\right)$

$= \frac{3}{2} - (-2 \times -4) = \frac{3}{2} - 8$

$= \frac{-13}{2}$

14 Since : $\frac{X-2}{X+3} = 0$

Then : $X - 2 = 0$, then $X = 2$

, then : the two numbers are : $\frac{1}{2}$, $\frac{2}{3}$

, then : L.C.M. = 6

, then : $\frac{1}{2} = \frac{3}{6}$, $\frac{2}{3} = \frac{4}{6}$

, then : $\frac{3}{6} = \frac{12}{24}$, $\frac{4}{6} = \frac{16}{24}$

, then : the numbers are : $\frac{13}{24}$, $\frac{14}{24}$, $\frac{15}{24}$

(There are other numbers)

Unit two

First Answers of multiple choice questions

1 (c)	2 (a)	3 (b)	4 (d)	5 (d)
6 (d)	7 (d)	8 (a)	9 (b)	10 (b)
11 (d)	12 (a)	13 (d)	14 (d)	15 (a)
16 (a)	17 (c)	18 (b)	19 (d)	20 (d)

Second Answers of complete questions

1 $-5y^2$	2 $10x^4y^3$	3 $5x^2y$
4 second	5 1	6 $8xy$
7 $x, 3y$	8 $7x$	9 b
10 $3x$	11 x^3	12 $2y^2$
13 $12x$	14 zero	15 49
16 14	17 6	18 3

Third Answers of essay questions

1 $10x^2 - 3x - 18$

2 The expression $= x^2 + 4x + 4 - (x^2 - 6x - 16)$
$= x^2 + 4x + 4 - x^2 + 6x + 16$
$= 10x + 20$

3 $5x + 3y$

4 $4a^2b + 3ab^2 + 2$

5 The expression $= 9x^2 - 25 + 8 = 9x^2 - 17$
The numerical value $= 9(-2)^2 - 17 = 36 - 17$
$= 19$

6 $7x - y + 2$

7 $2x^2 + 2xy - 9y^2$

8 $-2x^2 - 7x + 2$

9 $-4x^2 + xy + 2y^2$

10 $5x(x^2 + 3x + 2)$

11 $2xy(2x^2y^2 - 3xy + 1)$

12 $(4a + 5b)(3a - 2b)$

13
$$\begin{array}{l|l} x-2 & x^2 - 5x + 6 \\ \hline x-3 & \ominus x^2 \oplus 2x \\ & \quad -3x + 6 \\ & \quad \oplus 3x \ominus 6 \\ \hline & \quad\; 0 \quad 0 \end{array}$$

The quotient $= x - 3$

14
$$\begin{array}{l|l} x+2 & x^2 + 5x + 6 \\ \hline x+3 & \ominus x^2 \ominus 2x \\ & \quad 3x + 6 \\ & \quad \ominus 3x \ominus 6 \\ \hline & \quad\; 0 \quad 0 \end{array}$$

The length of the rectangle $= (x + 3)$ metres

15 $2x^2 + 4$

16 $x + 2y + z = x + y + y + z$
$= \frac{5}{3} + \frac{1}{3} = \frac{6}{3} = 2$

17 The expression $= x^2 + 4x + 4 - 4x - 4 = x^2$
The numerical value $= 3^2 = 9$

18 The expression $= x^2 + 4x + 4 - (x^2 - 4)$
$= x^2 + 4x + 4 - x^2 + 4$
$= 4x + 8$

19 The expression $= x^2 + 3x + 2 - x^2 = 3x + 2$
The numerical value $= 3 \times \frac{2}{3} + 2 = 2 + 2 = 4$

20
$$\begin{array}{l|l} x-4 & x^2 - 9x + k \\ \hline x-5 & \ominus x^2 \oplus 4x \\ & \quad -5x + k \\ & \quad \oplus 5x \ominus 20 \\ \hline & \quad\; k \quad -20 \end{array}$$

Then : $k - 20 = 0$

Then : $k = 20$

21 The area of the rectangle $= (2x + 3)(x + 2)$
$= (2x^2 + 7x + 6)$ cm^2

When $x = 5$

Then : the area $= 2 \times 5^2 + 7 \times 5 + 6 = 91$ cm^2

Unit three

First Answers of multiple choice questions

1 (b)	2 (c)	3 (b)	4 (a)
5 (c)	6 (d)	7 (c)	8 (d)
9 (d)	10 (d)	11 (b)	12 (b)

Second Answers of complete questions

1 27	2 13	3 the mode	4 7
5 2	6 100	7 8	8 4

Third Answers of essay questions

1 The mode = a + 2 = 18

Then : a = 16

2 The arithmetic mean = $\frac{25 + 37 + 34 + 48 + 44 + 52}{6}$

= 40 marks

3 The mode mark = 9 marks

4 **First :** The ascending order of Mahmoud's marks : 10 , 18 , 25 , 27 , 30

Then the median = 25 marks

Second : The mean of Hassan's mark

$= \frac{24 + 21 + 29 + 26 + 20}{5}$ = 24 marks

Third : The mode of Ahmed's marks = 23 marks

5 The mean = $\frac{42 + 30 + 51 + 35 + 42 + 40}{6} = 40$

The ascending order of the values is :

30 , 35 , 40 , 42 , 42 , 51

Then : The median = $\frac{40 + 42}{2} = 41$

The mode = 42

6 The mean = $\frac{8 + 7 + 5 + 9 + 4 + 3 + k + 4}{7} = 6$

Then : $\frac{40 + k}{7} = 6$

Then : 40 + k = 42

Then : k = 2

7 The mode mark = 6 marks

The number of students who got less than 6 marks = 12 students

8 The mean of studying hours

$= \frac{3\frac{1}{2} + 2 + 2\frac{1}{2} + 3 + 4 + 3}{6}$ = 3 hours.

9 The mean = $\frac{9 + 2k + 5 + 4k}{4} = 8$

Then : 6 k + 14 = 32

Then : 6 k = 32 – 14

Then : 6 k = 18

Then : k = $\frac{18}{6} = 3$

10 The ascending order of the values is :

$X + 3$, $X + 5$, $X + 8$

Then : the median = $X + 5 = 9$

Then : $X = 4$

Answers of the school book models on Algebra and Statistics

Model 1

1

1 $\frac{5}{11}$ 2 27 3 -0.12
4 $3y^3$ 5 $7x$

2

1 c 2 a 3 a 4 b 5 b 6 d

3

[a] $x^2 + xy + 2y^2 - 1$ [b] $\frac{27}{7}$

4

[a] $4x^2 - 2$, the numerical value $= 2$

[b] $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$ (There are other solutions)

5

[a] $x^2 - 2$ [b] $39\ \frac{2}{3}$

Model 2

1

1 $4x^2y^3$ 2 $5x$ 3 13
4 4 5 $x + 3y$

2

1 c 2 c 3 c 4 a 5 b 6 a

3

[a] 3

[b] $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$ (There are other solutions)

4

[a] $5x - y$ [b] $2x - 5y + 1$

5

[a] The expression $= x^2$
, the numerical value $= 25$

[b] $k = 2$

Model examination for the merge students

1

1 second 2 $x + 3$ 3 zero
4 4 5 zero

2

1 a 2 c 3 b 4 a 5 d

3

[a] $\frac{5}{7}(8 + 5 + 1) = \frac{5}{7}(14) = 10$

[b] $b \div a = (-2) \div \left(\frac{1}{2}\right) = (-2) \times (2) = -4$

4

1 ✓ 2 ✗ 3 ✓ 4 ✓ 5 ✗

5

1 7 2 3 3 $7x$ 4 50 5 1

Answers of schools examinations on Algebra and Statistics

1 Cairo

1 [1] c [2] c [3] b
[4] d [5] a [6] b

2 [1] 22 [2] $\frac{1}{4}$ [3] $x^2 - 16$
[4] 21 [5] $35 x y$

3

[a] $\frac{5}{9}\left(\frac{2}{7}+\frac{1}{7}+\frac{4}{7}\right)=\frac{5}{9}\times\frac{7}{7}=\frac{5}{9}$

[b] $3 x^2 - 5 x + 8$

[c] $10 x^2 - 3 x + 8$

4

[a] $c^2 - 6 a b = (3)^2 - 6 \times \left(\frac{1}{2}\right) \times \left(\frac{-2}{3}\right) = 9 - (-2)$
$= 9 + 2 = 11$

[b]

$$\begin{array}{r|l} x+5 & x^2 + 12 x + 35 \\ \hline x+7 & \ominus\ x^2 + 5 x \\ & \quad\ 7 x + 35 \\ & \ominus\ 7 x + 35 \\ & \quad\ 0 \quad 0 \end{array}$$

The quotient $= x + 7$

5

[a] $2 x^3 y^3 (3 x - 6 y + 1)$

[b] L.C.M. of the denominators = 6

$\frac{1}{2}=\frac{3}{6}$, $\frac{1}{3}=\frac{2}{6}$

$\frac{3}{6}=\frac{12}{24}$, $\frac{2}{6}=\frac{8}{24}$

The numbers are : $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$

(There are other solutions)

[c] The mode = 9 marks.

2 Cairo

1 [1] a [2] c [3] a
[4] a [5] a [6] c

2 [1] -4 [2] $-10 x$ [3] $\frac{1}{2}$
[4] 8 [5] 45

3

[a] $- x y + 2 y^2 - 4 y$

[b] L.C.M. of the denominators = 28

$\frac{5}{7}=\frac{20}{28}$, $\frac{1}{4}=\frac{7}{28}$

The numbers are : $\frac{8}{28}$, $\frac{11}{28}$, $\frac{13}{28}$

(There are other solutions)

4

[a] $\frac{5}{9}(2+6+1)=\frac{5}{9}\times 9=5$

[b] The expression $= x^2 - 6 x + 9 + 9$
$= x^2 - 6 x + 18$

The numerical value $= (2)^2 - 6 \times 2 + 18$
$= 4 - 12 + 18 = 10$

5

[a] The expression $= (a - b)(x + y)$

The numerical value $= 5 \times 2 = 10$

[b] The mean $= \frac{15+7+4+5+8+7}{6}$

$= \frac{46}{6} = \frac{23}{3} = 7.\dot{6} \simeq 7.7$

The mode = 7

3 Cairo

1 [1] 15 [2] 5 , -5 [3] $-7 x$
[4] $\frac{5}{11}$ [5] 7

2 [1] d [2] d [3] b
[4] c [5] d [6] b

3

[a] The expression $= 4 x^2 - 9 + 7 = 4 x^2 - 2$

The numerical value $= 4 \times (1)^2 - 2 = 4 - 2 = 2$

[b] $\frac{22}{25}\left(\frac{6}{11}+\frac{5}{11}-1\right)=\frac{22}{25}\times 0=0$

4

[a] $7 x - 2 y - 3$

[b] $(a-b) \div (a+b) = \left(\frac{7}{4} - \left(-\frac{1}{2}\right)\right) \div \left(\frac{7}{4} + \left(-\frac{1}{2}\right)\right)$

$= \left(\frac{7}{4} + \frac{1}{2}\right) \div \left(\frac{7}{4} - \frac{1}{2}\right)$

$= \left(\frac{7}{4} + \frac{2}{4}\right) \div \left(\frac{7}{4} - \frac{2}{4}\right)$

$= \frac{9}{4} \div \frac{5}{4} = \frac{9}{4} \times \frac{4}{5} = \frac{9}{5}$

5

[a] $3a^2 + 2b^2 - 1$

[b] 1 The mode = 14 marks.

2 10 students.

4 Giza

1

1 d	2 a	3 c
4 c	5 c	6 a

2

1 $-4a$	2 $3x$	3 5
4 2	5 second	

3

[a] L.C.M. of the denominators = 6

$\frac{1}{3} = \frac{2}{6}$, $\frac{1}{2} = \frac{3}{6}$

$\frac{2}{6} = \frac{8}{24}$, $\frac{3}{6} = \frac{12}{24}$

The numbers are : $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$

(There are other solutions)

[b] $2x^2 - 8x + 5$

[c] $4x^2 + 3xy - 6y^2$

4

[a] 1 The expression $= 4x^2 + 12x + 9$

The numerical value

$= 4 \times (-2)^2 + 12 \times (-2) + 9$

$= 4 \times 4 - 24 + 9 = 16 - 24 + 9 = 1$

2 The expression $= 2x^2 + 5x - 3$

The numerical value

$= 2 \times (-2)^2 + 5(-2) - 3$

$= 2 \times 4 - 10 - 3 = 8 - 10 - 3 = -5$

[b]

$$\begin{array}{r|l} x+2 & x^2 + 5x + 6 \\ \hline x+3 & \ominus \; \ominus \\ & x^2 + 2x \\ \hline & 3x + 6 \\ & \ominus \; \ominus \\ & 3x + 6 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= x + 3$

5

[a] $3xy(xy - 3x^2y^3 + 4)$

[b] The mode = 5

, then : $x + 3 = 5$, then : $x = 5 - 3 = 2$

[c] The expression $= x^2 - 4 + 4 = x^2$

5 Giza

1

1 d	2 b	3 c
4 b	5 b	6 d

2

1 -6	2 x , $3y$	3 3
4 $\frac{4}{9}$	5 zero	

3

[a] $\frac{8}{13}(11 + 9 - 7) = \frac{8}{13} \times 13 = 8$

[b] $5a + b - 3c$

4

[a] The expression $= a^2 - 8a + 16 + 8a = a^2 + 16$

[b]

$$\begin{array}{r|l} x-3 & x^2 - 5x + 6 \\ \hline x-2 & \ominus \; \oplus \\ & x^2 - 3x \\ \hline & -2x + 6 \\ & \oplus \; \ominus \\ & -2x + 6 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= x - 2$

5

[a] $6a^2b(2 + 3ab)$

[b] 1 The mean $= \frac{2+5+3+6+9}{5} = \frac{25}{5} = 5$

2 The ascending order is : 6 , 7 , 8 , 9 , 13

The median = 8

6 Alexandria

1 1 a 2 c 3 d
4 d 5 b 6 a

2 1 4 2 $x+1$ 3 second
4 x^2 , $2y^2$ 5 5

3

[a] $4x+3$

[b] $x+6y-3z$

4

[a] The expression $= 6x^2-9x+5$

[b] $a-b = \frac{3}{4}-\left(\frac{-5}{2}\right) = \frac{3}{4}+\frac{5}{2} = \frac{3}{4}+\frac{10}{4} = \frac{13}{4}$

, $a+b = \frac{3}{4}-\frac{5}{2} = \frac{3}{4}-\frac{10}{4} = \frac{-7}{4}$

Then : $\frac{a-b}{a+b} = \frac{13}{4} \div \frac{-7}{4} = \frac{13}{4} \times \frac{-4}{7} = \frac{-13}{7}$

5

[a] $6y^2+29y+28$

[b]
$$\begin{array}{r|l} x+5 & 2x^2+13x+15 \\ \hline 2x+3 & \ominus \quad \ominus \\ & 2x^2+10x \\ \cline{2-2} & \quad 3x+15 \\ & \quad \ominus \quad \ominus \\ & \quad 3x+15 \\ \cline{2-2} & \quad\ 0 \quad 0 \end{array}$$

The quotient $= 2x+3$

[c] The arithmetic mean

$= \frac{25^\circ+27^\circ+31^\circ+23^\circ+22^\circ+22^\circ+18^\circ}{7} = \frac{168^\circ}{7} = 24^\circ$

7 Alexandria

1 1 b 2 a 3 d
4 c 5 c 6 c

2 1 zero 2 third 3 15
4 $x+1$ 5 5

3

[a] $(5+7-1)\times\frac{3}{11} = 11\times\frac{3}{11} = 3$

[b] $3x(x+5y)$

4

[a] $5a-2b+2c$

[b] L.C.M. of the denominators = 20

$\frac{3}{5} = \frac{12}{20}$, $\frac{1}{4} = \frac{5}{20}$

The numbers are : $\frac{6}{20}$, $\frac{7}{20}$, $\frac{8}{20}$

(there are other solutions)

5

[a] The expression $= 9x^2-25$

[b] The mode = 9 marks.

8 El-Kalyoubia

1 1 d 2 b 3 d
4 a 5 b 6 c

2 1 The most common data. 2 52
3 $\frac{1}{5}$ 4 5 5 500

3

[a] $\frac{23}{45}\left(\frac{15}{17}+\frac{19}{17}-1\right) = \frac{23}{45}\times 1 = \frac{23}{45}$

[b] $(x+y)\div z = \left(\frac{-2}{5}+\frac{2}{3}\right)\div\frac{8}{15} = \left(\frac{-6}{15}+\frac{10}{15}\right)\times\frac{15}{8}$

$= \frac{4}{15}\times\frac{15}{8} = \frac{1}{2}$

[c] The number $= \left(\frac{9}{4}+\frac{17}{6}\right)\div 2 = \left(\frac{27}{12}+\frac{34}{12}\right)\times\frac{1}{2}$

$= \frac{61}{12}\times\frac{1}{2} = \frac{61}{24}$

4

[a] $2x^2y(2x^2-3xy+y^2)$

[b] $5x+3y-5z+4$

[c]
$$\begin{array}{r|l} x-5 & x^2-10x+25 \\ \hline x-5 & \ominus \quad \oplus \\ & x^2-5x \\ \cline{2-2} & \quad -5x+25 \\ & \quad \oplus \quad \ominus \\ & \quad -5x+25 \\ \cline{2-2} & \quad\ 0 \quad 0 \end{array}$$

The quotient $= x-5$

5

[a] The expression $= x^2 + 6x + 9 - (x^2 - 9)$

$= x^2 + 6x + 9 - x^2 + 9$

$= 6x + 18$

[b] 1 The arithmetic mean

$= \frac{28 + 34 + 42 + 38 + 48 + 50}{6} = 40$ marks.

2 The ascending order is : 28 , 34 , 38 , 42 , 48 , 50

The median $= \frac{38 + 42}{2} = 40$

9 El-Sharkia

1 1 c 2 a 3 b
4 c 5 c 6 d

2 1 $3x^2 y$ 2 third 3 9
4 5 5 x^2

3

[a] L.C.M. of the denominators = 6

$\frac{1}{2} = \frac{3}{6}$, $\frac{1}{3} = \frac{2}{6}$

$\frac{3}{6} = \frac{12}{24}$, $\frac{2}{6} = \frac{8}{24}$

The numbers are : $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$

(there are other solutions)

[b] $5a + b - 3c$

4

[a] $6xy(2y + 3x - xy)$

[b] The expression $= x^2 - 36 + 36 = x^2$

5

[a] $\frac{13}{15}(17 + 14 - 1) = \frac{13}{15} \times 30 = 26$

[b]

$$\begin{array}{r|l} 2x+3 & 6x^2 + 13x + 6 \\ \hline 3x+2 & \ominus \quad \ominus \\ & 6x^2 + 9x \\ \hline & 4x + 6 \\ & \ominus \quad \ominus \\ & 4x + 6 \\ \hline & 0 \quad 0 \end{array}$$

The quotient $= 3x + 2$

10 El-Monofia

1 1 c 2 b 3 c
4 a 5 d 6 d

2 1 $4x^2 y^3$ 2 $5x$ 3 x , 3
4 zero 5 zero

3

[a] $\frac{5}{7}(8 + 5 + 1) = \frac{5}{7} \times 14 = 10$

[b] Since $\frac{x-2}{x+3} = 0$, then $x - 2 = 0$

, then $x = 2$, then $\frac{1}{x} = \frac{1}{2}$, $\frac{2}{1+x} = \frac{2}{1+2} = \frac{2}{3}$

L.C.M. of the denominators = 6

$\frac{1}{2} = \frac{3}{6}$, $\frac{2}{3} = \frac{4}{6}$

, $\frac{3}{6} = \frac{12}{24}$, $\frac{4}{6} = \frac{16}{24}$

The numbers are : $\frac{13}{24}$, $\frac{14}{24}$, $\frac{15}{24}$

(there are other solutions)

4

[a] $\frac{y-z}{x} = \frac{\frac{-2}{3} - 2}{\frac{1}{2}} = \frac{\frac{-2-6}{3}}{\frac{1}{2}} = \frac{-8}{3} \times \frac{2}{1} = \frac{-16}{3}$

[b] The expression $= (a - 2b)(3a - 6b)$

$= 3(a - 2b)(a - 2b)$

The numerical value $= 3 \times \left|-\frac{1}{3}\right| \times \left|-\frac{1}{3}\right|$

$= 3 \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{3}$

5

[a]

$$\begin{array}{r|l} 2a^2 - 4a + 3 & 6a^3 - 10a^2 + 5a + 3 \\ \hline 3a + 1 & \ominus \quad \oplus \quad \ominus \\ & 6a^3 - 12a^2 + 9a \\ \hline & 2a^2 - 4a + 3 \\ & \ominus \quad \oplus \quad \ominus \\ & 2a^2 - 4a + 3 \\ \hline & 0 \quad 0 \quad 0 \end{array}$$

The quotient $= 3a + 1$

[b] $6x^2 + 28xy - 10y^2$

11 El-Dakahlia

1

1 b	2 b	3 b
4 d	5 c	6 b

2

1 $\frac{2}{7}$	2 4	3 2
4 $5x$	5 $7y$	

3

[a] $\frac{27}{16}(11+7-2)=\frac{27}{16}\times 16=27$

[b] $7x^2y^2+5xy-3$

4

[a] L.C.M. of the denominators = 6

$\frac{1}{2}=\frac{3}{6}$, $\frac{1}{3}=\frac{2}{6}$

$\frac{3}{6}=\frac{12}{24}$, $\frac{2}{6}=\frac{8}{24}$

The numbers are : $\frac{9}{24}$, $\frac{10}{24}$, $\frac{11}{24}$

(There are other solutions)

[b] x^2-x+1

5

[a] 1 $3x(2x+5y-1)$

2 $-3a-7b+4$

[b] 1 The arithmetic mean $=\frac{2+5+9+6+3}{5}=5$

2 The ascending order is : 2 , 3 , 5 , 6 , 9

The median = 5

12 Ismailia

1

1 b	2 a	3 c
4 b	5 d	6 c

2

1 -1	2 36	3 7
4 $\{5, 6\}$	5 $\frac{7}{3}$	

3

[a] $9a^2b+7ab-3b$

[b] The expression $=x^2+7x+12-12$

$=x^2+7x$

The numerical value $=(-1)^2+7\times(-1)$

$=1-7=-6$

4

[a] $\frac{4}{9}(8+11-1)=\frac{4}{9}\times 18=8$

[b] $3a-9b+5$

5

[a] L.C.M. of the denominators = 28

$\frac{2}{7}=\frac{8}{28}$, $\frac{3}{4}=\frac{21}{28}$

The numbers are : $\frac{9}{28}$, $\frac{10}{28}$, $\frac{11}{28}$

(There are other solutions)

[b] The mode height = 110

[c] The mean $=\frac{25+30+35+45+40}{5}=35$ marks.

13 Damietta

1

1 a	2 b	3 b
4 c	5 d	6 d

2

1 $-\frac{3}{4}$	2 7100	3 5 A
4 6	5 $5y^3$	

3

[a] $7x+y-2$

[b] $\frac{5}{12}(7+6-1)=\frac{5}{12}\times 12=5$

4

[a] The expression $=x^2+4x+4-4x=x^2+4$

The numerical value $=(3)^2+4=9+4=13$

[b] $5y(y^2+7x)$

5

[a]

$$\begin{array}{r|l} x+5 & x^2+7x+10 \\ \hline x+2 & \ominus\ \ominus \\ & x^2+5x \\ \hline & 2x+10 \\ & \ominus\ \ \ominus \\ & 2x+10 \\ \hline & 0\ \ \ 0 \end{array}$$

The quotient $=x+2$

[b] 1 The mode mark = 25 mark.

2 The arithmetic mean

$=\frac{25+25+29+25+28+30}{6}=27$ marks.

14 Beni Suef

1 1 d 2 b 3 d
4 a 5 d 6 a

2 1 $\frac{7}{10}$ 2 4 3 second
4 -20 5 2

3

[a] 1 $\left[\frac{3}{2}+\left(-\frac{1}{4}\right)\right]\div\left[-\frac{1}{4}+2\right]$

$=\left[\frac{6}{4}-\frac{1}{4}\right]\div\left[-\frac{1}{4}+\frac{8}{4}\right]$

$=\frac{5}{4}\div\frac{7}{4}=\frac{5}{4}\times\frac{4}{7}=\frac{5}{7}$

2 L.C.M. of the denominators = 45

$-\frac{1}{5}=-\frac{9}{45}$, $-\frac{1}{9}=-\frac{5}{45}$

The number is : $-\frac{7}{45}$

(There are other solutions)

[b] $\frac{5}{9}(4+6-1)=\frac{5}{9}\times 9=5$

4

[a] $-2y^2-3xy+2x^2$

[b] $5a^3x^2(2a^2+3ax^2-6x)$

5

[a]

$$\begin{array}{r|l} x+3 & x^2+5x+6 \\ \hline x+2 & \ominus\ \ominus \\ & x^2+3x \\ \cline{2-2} & \quad 2x+6 \\ & \ominus\quad\ominus \\ & \quad 2x+6 \\ \cline{2-2} & \quad\ 0\quad 0 \end{array}$$

The quotient $= x+2$

[b] The ascending order is :

35 , 37 , 40 , 44 , 47 , 48

The median $=\frac{40+44}{2}=42$ marks.

15 Aswan

1 1 a 2 c 3 a
4 a 5 d 6 a

2 1 $-\frac{3}{2}$ 2 5 3 -0.12
4 5 5 13

3

[a] $\frac{27}{16}\left(\frac{11}{7}+\frac{11}{7}-\frac{6}{7}\right)=\frac{27}{16}\times\frac{16}{7}=\frac{27}{7}$

[b] $5a^2b^2(a+7b)$

4

[a] L.C.M. of the denominators = 15

$\frac{4}{5}=\frac{12}{15}$, $\frac{2}{3}=\frac{10}{15}$

$\frac{12}{15}=\frac{24}{30}$, $\frac{10}{15}=\frac{20}{30}$

The numbers are : $\frac{21}{30}$, $\frac{22}{30}$, $\frac{23}{30}$

(There are other solutions)

[b] $2x-7y+4$

5

[a] $5x-2y+1$

[b] The mode mark = 9 marks.

Answers of the accumulative tests on Geometry

Accumulative test 1

1

1 d	2 b	3 d	4 b
5 a	6 a	7 c	8 c

2 m (∠ EBC) = 47° , m (∠ DBC) = 137°

3

Yes , $\overrightarrow{AD}$, $\overrightarrow{AB}$ are on the same straight line

, mention by yourself.

Accumulative test 2

1

1 c	2 b	3 b	4 c
5 b	6 c	7 c	8 b

2 m (∠ AMC) = 45° , m (∠ AMD) = 135°

3 50°

Accumulative test 3

1

1 a	2 d	3 c	4 a
5 c	6 b	7 d	8 c

2

1 $\overleftrightarrow{FE}$	2 3	3 A
4 110°	5 90°	6 24

3 m (∠ A) = 110°

Accumulative test 4

1

1 d	2 b	3 d	4 c
5 b	6 b	7 c	8 b

2 1 mention by yourself.

2 m (∠ X) = 110° , m (∠ XYZ) = 50°

3 m (∠ CED) = 130°

Accumulative test 5

1

1 d	2 d	3 d	4 b
5 c	6 c	7 b	8 b

2 m (∠ A) = 60° , m (∠ B) = 50° , m (∠ C) = 70°

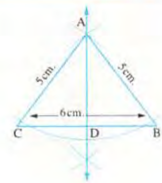
3 Prove by yourself.

Accumulative test 6

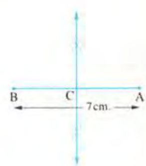
1

1 c	2 d	3 d	4 c

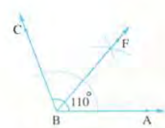
2 From the drawing by measuring the length of $\overline{AD}$ = 4 cm.



3



4



5 m (∠ C) = 65° , prove by yourself.

Answers of important questions on Geometry

Unit four

First Answers of multiple choice questions

1 (b)	2 (b)	3 (b)	4 (c)	5 (d)
6 (b)	7 (a)	8 (a)	9 (d)	10 (a)
11 (a)	12 (b)	13 (c)	14 (a)	15 (c)
16 (b)	17 (c)	18 (b)	19 (b)	20 (c)
21 (a)	22 (b)	23 (b)	24 (b)	25 (d)
26 (b)	27 (a)	28 (c)	29 (a)	30 (c)
31 (a)	32 (c)	33 (c)	34 (b)	35 (c)
36 (b)	37 (c)	38 (c)	39 (a)	40 (b)

Second Answers of complete questions

1 perpendicular

2 on the same straight line

3 240°

4 its axis of symmetry

5 The hypotenuse and a side in one triangle are congruent to the corresponding parts of the other triangle

6 equal in measure

7 70°

8 a straight line

9 an acute

10 supplementary

11 $\overline{BC}$

12 two sides

13 The side drawn between their vertices

14 each two interior angles in the same side of the transversal

15 parallel

16 equal in measure

17 35°

18 supplementary

19 perpendicular

20 the perpendicular bisector to it

21 zero

22 110°

23 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

24 8

25 11°

Third Answers of essay questions

1 $m(\angle BMC) = 360° - (110° + 90° + 40°) = 120°$

2 The points B , A and E are on the same straight line

, because : $m(\angle BAC) = m(\angle CAD) = 45°$

, $m(\angle BAC) + m(\angle CAD) + m(\angle DAE)$

$= 45° + 45° + 90° = 180°$

3 $m(\angle BAD) = m(\angle B) = 52°$ (alternate angles)

, since : $\overrightarrow{AD}$ bisects $\angle BAE$

Then : $m(\angle DAE) = m(\angle DAB) = 52°$

, $m(\angle C) = m(\angle DAE) = 52°$

(corresponding angles)

4 $\Delta ABC \equiv \Delta CDA$

because $\begin{cases} m(\angle DAC) = m(\angle BCA) \\ AD = CB \\ \overline{AC} \text{ is a common side} \end{cases}$

Then : $m(\angle BAC) = m(\angle DCA)$

and they are alternate angles

Then : $\overline{AB} \parallel \overline{CD}$

5 The two right-angled triangles ABC , EDA are congruent

because $\begin{cases} BC = DA \\ AC = EA \end{cases}$

and we deduce that : $m(\angle C) = m(\angle DAE) = 35°$

6 Since : $\overleftrightarrow{AF} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$

, $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ are two transversals to them

, $AD = DX = XB$

Then : $AE = EY = YC = \frac{9}{3} = 3$ cm.

Then : $AY = 6$ cm.

7 Since : $m(\angle A) + m(\angle C) = 180°$

(two interior angles in the same side of the transversal)

Then : $m(\angle C) = 180° - 110° = 70°$

, $m(\angle D) = m(\angle C) = 70°$ (alternate angles)

8 Since : $m(\angle CAE) = m(\angle BAE) = 24°$

Then : $m(\angle BAC) = 2 \times 24° = 48°$

Then : m (∠ BAC) = m (∠ C) = 48°
and they are alternate angles
Then : $\overrightarrow{AB}$ // $\overrightarrow{CD}$

9 m (∠ C) = m (∠ D) = 70° (alternate angles)
Then : m (∠ C) + m (∠ A) = 70° + 110° = 180°
and they are interior angles in the same side of the transversal
Then : $\overrightarrow{AB}$ // $\overline{CD}$

10 Since :
m (∠ D) = m (∠ DCE) = 65° (alternate angles)
Then : m (∠ D) + m (∠ A) = 65° + 115° = 180°
and they are interior angles in the same side of the transversal
Then : $\overline{AB}$ // $\overline{DC}$

11 Since :
m (∠ DCB) = m (∠ B) = 45° (alternate angles)
, m (∠ DCE) = m (∠ E) = 50° (alternate angles)
Then : m (∠ BCE) = 45° + 50° = 95°

12 Since :
m (∠ ACD) = m (∠ A) = 30° (alternate angles)
, m (∠ DCF) + m (∠ F) = 180°
(interior angles in the same side of the transversal)
Then : m (∠ DCF) = 180° – 120° = 60°
Then : m (∠ ACF) = 30° + 60° = 90°

13 m (∠ AMD) = m (∠ CMB) = 116° (V.O.A.)
, m (∠ AMC) = 180° – 116° = 64°
Since : $\overrightarrow{ME}$ bisects ∠ AMC
Then : m (∠ AME) = $\frac{64°}{2}$ = 32°

14 Since : $\overrightarrow{BD}$ bisects ∠ ABC
Then : m (∠ ABD) = m (∠ DBC) = 30°
, since : m (∠ ADB) = 180° – 110° = 70°
Then : in Δ ABD
m (∠ A) = 180° – (70° + 30°) = 80°

15 Δ XLZ ≡ Δ YLZ

because $\begin{cases} XL = YL \\ XZ = YZ \\ \overline{LZ} \text{ is a common side} \end{cases}$

Then : m (∠ XLZ) = m (∠ YLZ) = $\frac{360° - 130°}{2}$
= 115°

16 Mention by yourself

17 m (∠ LMN) = m (∠ XYZ) = 50° (alternate angles)

1 The conditions of congurency of the two triangles XYZ , LMN are :

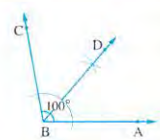
$\begin{cases} m (\angle X) = m (\angle L) = 100° \\ m (\angle XYZ) = m (\angle LMN) = 50° \\ XY = LM \end{cases}$

2 m (∠ N) = m (∠ Z) = 180° – (100° + 50°) = 30°

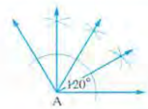
18 1 AB = MD = 5 cm.
, AD = MF = 8 cm.
Then : AM = 8 – 5 = 3 cm.

2 m (∠ B) = m (∠ MDE)
, m (∠ F) = m (∠ ADC)
, m (∠ MDE) + m (∠ ADC) = 180°
Then : m (∠ B) + m (∠ F) = 180°

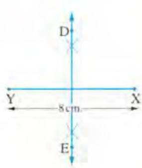
19



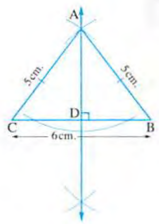
20



21

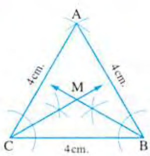


22



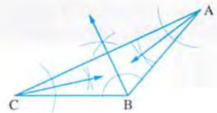
From the figure : AD = 4 cm.

23



From the figure : m (∠ BMC) = 120°

24



Answers of the school book models on Geometry

Model 1

1

1 its axis of symmetry
2 40°
3 255°
4 40°
5 the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

2

1 b　2 d　3 d　4 b　5 a　6 c

3

[a] The conditions of congruency of the two right-angled traingles Δ ABD and Δ CBD are

$\begin{cases} AB = CB \\ \overline{BD} \text{ is a common hypotenuse} \end{cases}$

, CD = AD = 3 cm.

[b] AY = 6 cm.

4

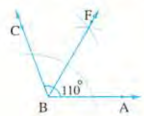
[a] m (∠ ACE) = 95°　　**[b]** m (∠ BMC) = 120°

5

[a] The conditions that make Δ AMB ≡ Δ DMC are :

$\begin{cases} AM = DM \\ BM = CM \\ m(\angle AMB) = m(\angle DMC) \text{ (V.O.A.)} \end{cases}$

[b]



Model 2

1

1 360°
2 equal in measure
3 250°
4 the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.
5 supplementary.

2

1 a　2 d　3 a　4 b　5 b　6 a

3

[a] Mention by yourself.

[b] Prove by yourself , m (∠ ABD) = 60°

4

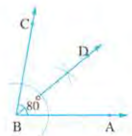
[a] m (∠ C) = m (∠ D) = 70° "Alternate angles"

Yes , $\overrightarrow{AB} \parallel \overline{CD}$

because : m (∠ A) + m (∠ C) = 110° + 70° = 180°

(Two interior angles in the same side of the transversal).

[b]



5

[a] X = 65°　　**[b]** m (∠ A) = 85°

Model examination for the merge students

1

1 260°   2 40°   3 parallel

4 the included angle between them of one triangle are congruent to the corresponding parts of the other triangle.

5 C

2

1 d   2 a   3 c   4 a   5 a

3

1 ✗   2 ✓   3 (a) ✗   (b) ✓   (c) ✗

4

[a] $\overrightarrow{BA} \parallel \overrightarrow{CD}$

, then m (∠ ABC) = m (∠ BCD) "alternate angles"

, m (∠ BCD) = 50°

[b] 1 DCM   2 5   3 C

5

[a] 1 60°   2 80°   3 40°   4 50°

[b] 1

Answers of the schools examinations on Geometry

1 Cairo

1

1 40°   2 C   3 equal in measure

4 360°   5 perpendicular

2

1 c   2 a   3 c   4 d   5 b   6 b

3

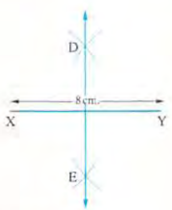
[a] m (∠ CMD) = 360° − (110° + 30° + 90°) = 130°

[b] Δ AMB ≡ Δ DMC

because $\begin{cases} MB = MC \\ MA = MD \\ m(\angle AMB) = m(\angle DMC) \quad (V.O.A.) \end{cases}$

4

[a]



[b] Since : $\overrightarrow{AD} \parallel \overline{CB}$

, then : m (∠ BAD) = m (∠ B) = 52° (alternate angles)

, since $\overrightarrow{AD}$ bisects ∠ BAE

, then m (∠ EAD) = 52°

, m (∠ C) = m (∠ EAD) = 52° (corresponding angles)

5

[a] m (∠ ABD) = 180° − 135° = 45°

, since $\overrightarrow{BA}$ bisects ∠ DBE

, then m (∠ DBE) = 2 × 45° = 90°

, then m (∠ CBE) = 360° − (135° + 45° + 45°) = 135°

[b] Since : $\overleftrightarrow{AF} \parallel \overline{DE} \parallel \overline{XY} \parallel \overline{BC}$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them

, AD = DX = XB

Then : AE = EY = YC = $\frac{9}{3}$ = 3 cm.

Then : AY = 6 cm.

2 Cairo

1

1 b   2 d   3 c   4 b   5 c

2

1 equal in measure   2 parallel

3 on the same straight line

4 55°   5 90°   6 180°

3

[a] m (∠ DMA) = 360° − (50° + 80° + 90°) = 140°

[b] Δ ABD ≡ Δ ACD

because $\begin{cases} AB = AC \\ BD = DC \\ \overline{AD} \text{ is a common side} \end{cases}$

and we deduce that :

m (∠ CAB) = 40° + 40° = 80°

4

[a]

[b] Since : $\overrightarrow{AB} \parallel \overrightarrow{CD}$

, then : m (∠ ACD) = m (∠ A) = 30° (alternate angles)

, since : $\overrightarrow{CD}$ bisects ∠ ACE

, then : m (∠ DCE) = m (∠ ACD) = 30°

Since : $\overrightarrow{CD} \parallel \overrightarrow{EF}$

, then : m (∠ CEF) = 180° − m (∠ DCE) = 180° − 30° = 150°

(interior angles in the same side of the transversal)

5

[a] The two right-angled triangles ABD , CBD are congruent

because $\begin{cases} AB = CB \\ \overline{BD} \text{ is a common hypotenuse} \end{cases}$

and we deduce that : CD = AD = 3 cm.

[b] Since : $\overleftrightarrow{AF} // \overleftrightarrow{DE} // \overleftrightarrow{BC}$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them

, AD = DB

Then : AE = EC

Then : AC = AE + EC = 4 + 4 = 8 cm.

3 Cairo

1

1 c 2 a 3 c 4 b 5 a 6 c

2

1 5 cm. 2 55° 3 20° , 110°

4 180° 5 supplementary

3

[a] The two right-angled triangles ABD , CBD are congruent

because $\begin{cases} AD = DC \\ \overline{BD} \text{ is a common hypotenuse} \end{cases}$

and we deduce that :

m (∠ ABD) = m (∠ CBD) = 180° – (90° + 30°)
= 60°

Then : m (∠ ABC) = 120°

, AB = CB = 3 cm.

[b] Yes , because : $\overrightarrow{AB} // \overrightarrow{CD}$

, then : m (∠ ACD) = 180° – m (∠ BAC)
= 180° – 130° = 50°

(interior angles in the same side of the transversal)

, since : $\overrightarrow{CF}$ bisects ∠ DCA

, then : m (∠ DCF) = $\frac{50°}{2}$ = 25°

, then : m (∠ CDE) = m (∠ DCF)

and they are alternate angles.

then : $\overrightarrow{DE} // \overrightarrow{CF}$

4

[a] Since : $\overrightarrow{AB} // \overrightarrow{CD}$

, then : m (∠ ACD) = 180° – m (∠ A)
= 180° – 60° = 120°

(interior angles in the same side of the transversal)

since : $\overrightarrow{AB} // \overrightarrow{CD}$, $\overrightarrow{AB} // \overrightarrow{EF}$

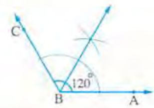
, then : $\overrightarrow{CD} // \overrightarrow{EF}$

, then : m (∠ DCE) = 180° – m (∠ E)
= 180° – 35° = 145°

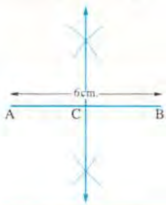
(interior angles in the same side of the transversal)

, then : m (∠ ACE) = 360° – (120° + 145°) = 95°

[b] 1



2



5

[a] Since : m (∠ AMD) = 360° – (40° + 100° + 120°)
= 100°

Then : 4 x = 100°

Then : $x = \frac{100°}{4} = 25°$

[b] Yes , the two right-angled triangles BAE , DCB are congruent.

because $\begin{cases} AE = CB \\ BE = BD \end{cases}$

and we deduce that :

m (∠ CBD) = m (∠ E) = 180° – (90° + 70°) = 20°

Then : m (∠ EBD) = 180° – (70° + 20°) = 90°

4 Giza

1

1 equal in measure 2 90°

3 perpendicular 4 250°

5 axis of symmetry

2

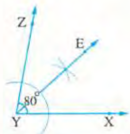
1 c   2 c   3 b

4 a   5 c   6 c

3

[a] m (∠ AMB) = 360° – (110° + 45° + 90°) = 115°

[b]



4

[a] Mention by yourself.

[b] Since : $\overleftrightarrow{AF} \,//\, \overline{ED} \,//\, \overline{XY} \,//\, \overline{BC}$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them

, AE = EY = YC

Then : AD = DX = XB = $\frac{12}{3}$ = 4 cm.

Then : AX = 8 cm.

5

Δ CBD ≡ Δ ABD

because $\begin{cases} AB = BC \\ AD = DC \\ \overline{BD} \text{ is a common side} \end{cases}$

and we deduce that : m (∠ ABD) = m (∠ CBD) = 40°

5 Giza

1

1 The axis of symmetry   2 equal in measure

3 245°   4 supplementary angles

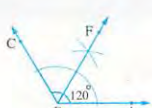
5 Z

2

1 b   2 c   3 a   4 d   5 d   6 b

3

[a]



[b] m (∠ ABD) = 50°

because : m (∠ DBC) + m (∠ ABD) = 180°

and m (∠ DBE) = 100°

because : $\overrightarrow{BA}$ bisects ∠ DBE

Then : m (∠ ABD) = m (∠ ABE)

4

[a] Yes , Δ ABD ≡ Δ CBD

because $\begin{cases} AB = CB \\ AD = CD \\ \overline{BD} \text{ is a common side} \end{cases}$

and we deduce that :

m (∠ ADC) = 25° + 25° = 50°

[b] Since : $\overrightarrow{BA} \,//\, \overrightarrow{CD}$

Then : m (∠ BCD) = m (∠ ABC) = 30°

(alternate angles)

Since : $\overrightarrow{CD} \,//\, \overrightarrow{EO}$

Then : m (∠ DCE) = 180° – m (∠ CEO)

= 180° – 110° = 70°

(interior angles in the same side of the transversal)

Then : m (∠ BCE) = 30° + 70 = 100°

5

[a] Since : $\overleftrightarrow{AF} \,//\, \overline{ED} \,//\, \overline{YX} \,//\, \overline{CB}$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them

, AD = DX = XB

Then : AE = EY = YC = $\frac{9}{3}$ = 3 cm.

Then : AY = 6 cm.

[b] m (∠ DMC) = 360° – (90° + 50° + 60° + 40°)

= 120°

6 Alexandria

1

1 a   2 c   3 d   4 a   5 a   6 c

2

1 360°   2 240°   3 90°

4 ⊂   5 equal in measure

3

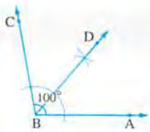
[a] Mention by yourself.

[b] $\Delta ABD \equiv \Delta ACD$

because $\begin{cases} AB = AC \\ BD = DC \\ \overline{AD} \text{ is a common side} \end{cases}$

4

[a]



[b] Since : $m(\angle AMD) = 360° - (50° + 90° + 120°)$
$= 100°$

Then : $2x = 100°$

Then : $x = \frac{100°}{2} = 50°$

5

[a] Since : $\overrightarrow{DE} \,//\, \overline{AC}$

Then : $m(\angle ACD) = m(\angle D) = 65°$ (alternate angles)

Since : $\overrightarrow{AB} \,//\, \overline{CD}$

Then : $m(\angle A) = 180° - m(\angle ACD)$
$= 180° - 65° = 115°$

(interior angles in the same side of the transversal)

[b] $\Delta ABD \equiv \Delta CBD$

because $\begin{cases} AB = CB \\ AD = CD \\ \overline{BD} \text{ is a common side} \end{cases}$

and we deduce that :

$m(\angle A) = m(\angle C) = 180° - (65° + 30°) = 85°$

7 Alexandria

1

1 c 2 b 3 c 4 a 5 a 6 b

2

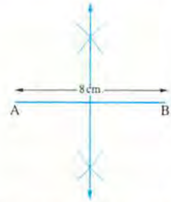
1 360° 2 12 3 200

4 and the included angle of one triangle are congruent to the corresponding parts of the other triangle.

5 $\overline{XZ}$

3

[a]



[b] $m(\angle BMC) = 360° - (120° + 90° + 40°) = 110°$

4

[a] Yes , $\Delta AMB \equiv \Delta DMC$

because $\begin{cases} AM = DM \\ BM = CM \\ m(\angle AMB) = m(\angle CMD) \quad \text{(V.O.A.)} \end{cases}$

[b] Since : $\overrightarrow{DH} \,//\, \overline{AC}$

Then : $m(\angle C) = m(\angle D) = 50°$ (alternate angle)

Since : $\overrightarrow{AB} \,//\, \overline{CD}$

Then : $m(\angle A) = 180° - m(\angle C) = 180° - 50°$
$= 130°$

(interior angles in the same side of the transversal)

5

[a] Since : $\overrightarrow{AB} \,//\, \overrightarrow{CD}$

Then : $m(\angle ACD) = m(\angle A) = 50°$ (alternate angle)

Since : $\overrightarrow{CD} \,//\, \overrightarrow{HO}$

Then : $m(\angle DCH) = 180° - m(\angle H) = 180° - 110°$
$= 70°$

(interior angles in the same side of the transversal)

Then : $m(\angle ACH) = 50° + 70° = 120°$

[b] Since : $\overleftrightarrow{AO} \,//\, \overleftrightarrow{HD} \,//\, \overleftrightarrow{XY} \,//\, \overleftrightarrow{BC}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them

, $AD = DX = XB$

Then : $AH = HY = YC = \frac{9}{3} = 3$ cm.

Then : $AY = 6$ cm.

8 El-Kalyoubia

1

1 b 2 a 3 b 4 a 5 b 6 d

2

1 360° 2 side 3 65°

4 $m(\angle ZYX)$ 5 125°

3

[a] m (∠ BMD) = 180° – 108° = 72°

Since : $\overrightarrow{ME}$ bisects ∠ BMD

Then : m (∠ EMD) = $\frac{72°}{2}$ = 36°

, m (∠ AMC) = m (∠ BMD) = 72° (V.O.A.)

, m (∠ BMC) = m (∠ AMD) = 108° (V.O.A.)

[b] The two right-angled triangles ABC and EDA are congruent (Hypotenuse and one side)

because $\begin{cases} AB = DE \\ AC = EA \end{cases}$

and we deduce that :

m (∠ EAD) = m (∠ ACB) = 180° – (90° + 52°) = 38°

, m (∠ EAC) = 180° – (52° + 38°) = 90°

4

[a] m (∠ BCO) = m (∠ FDO) = 120°

, m (∠ FAO) = m (∠ BAO) = $\frac{150°}{2}$ = 75°

, m (∠ AOD) = m (∠ AOC) = $\frac{180°}{2}$ = 90°

, CO = DO = $\frac{8}{2}$ = 4 cm.

[b] Since : $\overrightarrow{BA}$ // $\overrightarrow{CF}$

Then : m (∠ BCF) = m (∠ ABC) = 70° (alternate angles)

Since : m (∠ DCF) = m (∠ BCF) = 70°

Then : m (∠ CDE) + m (∠ DCF) = 110° + 70° = 180°

and they are two interior angles in the same side of the transversal.

Then : $\overrightarrow{CF}$ // $\overrightarrow{DE}$

5

[a] Since : $\overleftrightarrow{AF}$ // $\overleftrightarrow{XY}$ // $\overleftrightarrow{DE}$ // $\overleftrightarrow{BC}$

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them.

, AY = YE = EC

Then : AX = XD = DB = 3 cm.

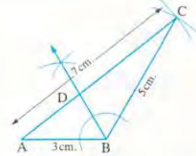
Then : AC = 3 × 4 = 12 cm. , AB = 3 × 3 = 9 cm.

Since : perimeter of Δ ABC

= AB + BC + AC = 30 cm.

Then : BC = 30 – (9 + 12) = 9 cm.

[b]



m (∠ ABD) = 60°

9 El-Sharkia

1

1 b　2 b　3 a　4 a　5 d　6 b

2

1 360°　2 equal　3 90°

4 equal in measure　5 80°

3

[a] m (∠ AMD) = 360° – (40 + 100° + 120°) = 100°

[b] Yes , Δ ADC ≡ Δ ADB

because $\begin{cases} AB = AC \\ BD = CD \\ \overline{AD} \text{ is a common side} \end{cases}$

and we deduce that :

m (∠ CAD) = m (∠ BAD) = 30°

Then : m (∠ BAC) = 30° + 30° = 60°

4

[a] Since : $\overrightarrow{BA}$ // $\overline{CD}$

Then : m (∠ C) = m (∠ B) = 60° (alternate angles)

Since : $\overline{CB}$ // $\overrightarrow{DE}$

Then : m (∠ D) = 180° – m (∠ C) = 180° – 60° = 120°

(interior angles in the same side of the transversal)

[b] m (∠ AMC) = 180° – 100° = 80°

Since : $\overrightarrow{ME}$ bisects ∠ AMC

Then : m (∠ EMA) = $\frac{80°}{2}$ = 40°

, m (∠ AMD) = m (∠ CMB) = 100° (V.O.A.)

Then : m (∠ EMD) = 40° + 100° = 140°

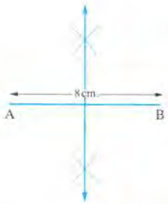
5

[a] 1 m (∠ EDC) = m (∠ EDZ) = $\frac{180°}{2}$ = 90°

2 Since : ZD = CD = 5 cm.

Then : ZC = 5 + 5 = 10 cm.

[b]



10 El-Gharbia

1

1 c 2 b 3 b 4 d 5 c 6 a

2

1 The square , the rhombus

2 The axis of symmetry

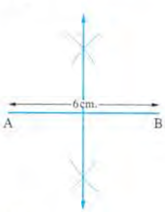
3 the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle

4 90° 5 zero

3

[a] state by yourself

[b]



[c] Δ ABC ≡ Δ ADC

because $\begin{cases} AB = AD \\ BC = DC \\ \overline{AC} \text{ is a common side} \end{cases}$

and we deduce that :

m (∠ BAC) = m (∠ DAC) = 180° – (100° + 20°)
= 60°

4

[a] The conditions of the congruency of the two triangles ABM , DCM are

$\begin{cases} MB = MC \\ MA = MD \\ m(\angle AMB) = m(\angle DMC) \quad \text{(V.O.A.)} \end{cases}$

and we deduce that : m (∠ B) = m (∠ C) = 80°

[b] Since : $\overrightarrow{AE}$ // $\overline{CB}$

Then : m (∠ B) = m (∠ EAB) = 55° (alternate angles)

, m (∠ C) = m (∠ DAE) = 70° (corresponding angles)

, m (∠ BAC) = 180° – (55° + 70°) = 55°

5

[a] Yes , since : $\overrightarrow{AB}$ // $\overline{CD}$

Then : m (∠ C) = 180° – m (∠ A)
= 180° – 115° = 65°

(interior angles in the same side of the transversal)

Since : m (∠ C) = m (∠ D) = 65°

and they are alternate angles

Then : $\overline{AC}$ // $\overrightarrow{DE}$

[b] m (∠ XML) = 360° – (110° + 90° + 50°) = 110°

11 El-Dakahlia

1

1 MN 2 4 3 360°

4 100° 5 equal in measure

2

1 d 2 c 3 d 4 b 5 b 6 d

3

[a] Since : $\overleftrightarrow{AF}$ // $\overline{XY}$ // $\overline{ZL}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them , AX = XZ = ZB

Then : AY = YL = LC = $\frac{9}{3}$ = 3 cm.

Then : AL = 6 cm.

[b] Yes , Δ MSR ≡ Δ MNL

because $\begin{cases} MR = ML \\ MS = MN \\ m(\angle RMS) = m(\angle LMN) \quad \text{(V.O.A.)} \end{cases}$

4

[a] Since : m (∠ CMB) = 360° – (120° + 90°) = 150°

Then : 3 X = 150°

Then : $X = \frac{150°}{3} = 50°$

[b] The conditions of the congruency of the two right-angled triangles LOM , LON are

$\begin{cases} LM = LN \\ \overline{LO} \text{ is a common side} \end{cases}$

and we deduce that :

m (∠ MLO) = m (∠ NLO) = 30°

Then : m (∠ MLN) = 30° + 30° = 60°

5

[a] Since : $\overrightarrow{RO} \,//\, \overline{ML}$

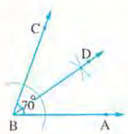
Then : m (∠ L) = m (∠ R) = 80° (alternate angles)

Yes , $\overrightarrow{MS} \,//\, \overline{RL}$

because : m (∠ L) + m (∠ M) = 80° + 100° = 180°

and they are interior angles in the same side of the transversal.

[b]



12 Suez

1

1 b 2 b 3 a 4 d 5 a 6 c

2

1 200° 2 80° 3 parallel
4 15 cm. 5 equal in measure

3

[a] Since : $\overrightarrow{AD} \,//\, \overline{BC}$

Then : m (∠ B) = m (∠ BAD) = 60° (alternate angles)

, in Δ ABC : m (∠ BAC) = 180° – (60° + 40°) = 80°

[b] m (∠ CMD) = 360° – (30° + 110° + 90°) = 130°

4

[a] The two right-angled triangles ABD , CBD are congruent because $\begin{cases} AB = CB \\ \overline{BD} \text{ is a common hypotenuse.} \end{cases}$

and we deduce that : CD = DA = 5 cm.

, m (∠ CBD) = m (∠ ABD) = 180° – (90° + 30°) = 60°

[b] Since : m (∠ BMD) = m (∠ AMC) = 40° (V.O.A.)

, since : $\overrightarrow{MD}$ bisect ∠ BME

Then : m (∠ BME) = 2 × 40° = 80°

, then m (∠ AME) = 180° – m (∠ BME) = 180° – 80° = 100°

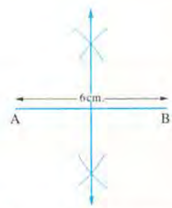
5

[a] Since : $\overleftrightarrow{OA} \,//\, \overline{HD} \,//\, \overline{XY} \,//\, \overline{BC}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two transversals to them , AD = DX = XB

Then : AH = HY = YC = $\frac{15}{3}$ = 5 cm.

Then : HC = 6 cm.

[b]



13 El-Beheira

1

1 d 2 d 3 b 4 b 5 c 6 a

2

1 330° 2 on the same straight line
3 the axis of symmetry 4 supplementary
5 40°

3

[a] Since : m (∠ AMB) = m (∠ HMD) = 50° (V.O.A.)

, m (∠ BMD) = 180° – m (∠ AMB) = 180° – 50° = 130°

, since : $\overrightarrow{MC}$ bisects ∠ BMD

, then : m (∠ BMC) = $\frac{130°}{2}$ = 65°

, then m (∠ AMC) = 50° + 65° = 115°

[b] Since : $\overrightarrow{AD} \parallel \overline{BC}$

, then : m (∠ C) = m (∠ CAD) = 30°

(alternate angles)

, m (∠ B) = m (∠ EAD) = 40°

(corresponding angles)

4

[a] The two right-angled triangles

ABC , EDA are congruent

because $\begin{cases} BC = DA \\ AC = EA \end{cases}$

and we deduce that :

m (∠ EAD) = m (∠ ACB) = 180° – (90° + 50°)

= 40°

[b] Since : $\overleftrightarrow{AF} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$

are two transversals to them.

AD = DX = XB

Then : AE = EY = YC = $\frac{9}{3}$ = 3 cm.

Then : AY = 6 cm.

5

[a] Δ AMB ≡ Δ DMC

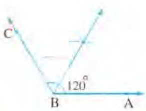
because $\begin{cases} BM = MC \\ AM = MD \\ m(\angle AMB) = m(\angle DMC) \quad \text{(V.O.A.)} \end{cases}$

and we deduce that : m (∠ B) = m (∠ C)

and they are alternate angles

Then : $\overline{AB} \parallel \overline{CD}$

[b]



14 Souhag

1

1 b 2 d 3 d 4 c 5 d 6 a

2

1 The axis of symmetry 2 55°

3 360° 4 parallel

5 255°

3

[a] m (∠ ABE) = 360° – (130° + 80° + 90°) = 60°

[b] Since : $\overline{BC} \parallel \overrightarrow{DE}$

Then : m (∠ C) = m (∠ D) = 50° (alternate angles)

Since : $\overrightarrow{BA} \parallel \overline{CD}$

Then : m (∠ B) = 180° – m (∠ C)

= 180° – 50° = 130°

(interior angles in the same side of the transversal)

4

[a] Yes , Δ AMC ≡ Δ BMD

because $\begin{cases} AM = BM \\ CM = DM \\ m(\angle AMC) = m(\angle BMD) \quad \text{(V.O.A.)} \end{cases}$

and we deduce that :

BD = AC = 3 cm. , CM = DM = $\frac{4}{2}$ = 2 cm.

[b] Since : $\overrightarrow{AE} \parallel \overrightarrow{BD}$

Then : m (∠ ABD) = 180° – m (∠ A)

= 180° – 105° = 75°

(interior angles in the same side of the transversal)

Since : $\overrightarrow{BD} \parallel \overrightarrow{CH}$

Then : m (∠ CBD) = 180° – m (∠ C)

= 180° – 100° = 80°

(interior angles in the same side of the transversal)

Then : m (∠ ABC) = 75° + 80° = 155°

5

[a] Δ ABC ≡ Δ ADC

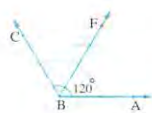
because $\begin{cases} BC = DC \\ \overline{AC} \text{ is a common side} \\ m(\angle ACB) = m(\angle ACD) = 90° \end{cases}$

and we deduce that : AD = AB = 5 cm.

, m (∠ DAC) = m (∠ BAC)

= 180° – (90° + 57°) = 33°

[b]



15 New Valley

1

1 b   2 b   3 a   4 d   5 b   6 a

2

1 $250°$ 2 $40°$ 3 zero

4 $60°$ 5 equal in measure

3

[a] Mention by yourself.

[b] $\Delta CBD \equiv \Delta ABD$

because $\begin{cases} AB = BC \\ AD = DC \\ \overline{BD} \text{ is a common side} \end{cases}$

and we deduce that :

$m(\angle ABD) = m(\angle CBD)$

$= 180° - (80° + 40°) = 60°$

4

[a] Since : $\overline{AC} \parallel \overrightarrow{DE}$

Then : $m(\angle C) = m(\angle D) = 70°$ (alternate angles)

Since : $m(\angle C) + m(\angle A) = 70° + 110° = 180°$

and they are two interior angles in the same side of the transversal.

Then : $\overrightarrow{AB} \parallel \overline{CD}$

[b] Since : $\overleftrightarrow{AF} \parallel \overline{DE} \parallel \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$

are two transversals to them , $AD = DX = XB$

Then : $AE = EY = YC = \frac{9}{3} = 3$ cm.

Then : $AY = 6$ cm.

5

[a] $m(\angle BMC) = 360° - (110° + 90° + 40°) = 120°$

[b]